Grade 12 Mathematics revision cards

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Statistics formulas

Use these whenever a question hands you a list of numbers and asks what is typical or how spread out they are. Work out the mean first, because variance and standard deviation are both built on it and neither can be found without it.

RULE	FORMULA
Mean	$\mu = (\Sigma x_i)/n$
Median (odd count)	the middle value of the ordered list
Median (even count)	$(x_{n/2} + x_{n/2+1})/2$
Mode	the value that occurs most often
Range	$x_{max} - x_{min}$
Variance	$\sigma^2 = (\Sigma (x_i - \mu)^2)/n$
Standard deviation	$\sigma = \sqrt{((\Sigma (x_i - \mu)^2)/n)}$
Mean of grouped data	$\mu = (\Sigma f_i x_i)/(\Sigma f_i)$

WATCH OUT
Divide by n for a whole population and by n - 1 for a sample. Exam questions say which one they mean in the wording, not in the numbers, so read the sentence before you reach for the formula.

Probability rules

Start every probability question by naming the sample space and counting it. Almost every rule here is a way of avoiding counting the same outcome twice, and you cannot see the double counting until you know what you are counting from.

RULE	FORMULA
Probability of an event	$\mathrm{P(E) = (n(E))/(n(S))}$
Range of a probability	$0 \leq \mathrm{P(E)} \leq 1$
Certain and impossible	$\mathrm{P(S) = 1, P(\emptyset) = 0}$
Complement	$\mathrm{P(E') = 1 - P(E)}$
Addition rule	$\mathrm{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$
Mutually exclusive	$\mathrm{P(A \cup B) = P(A) + P(B)}$
Conditional probability	$\mathrm{P(A \mid B) = (P(A \cap B))/(P(B))}$
Multiplication rule	$\mathrm{P(A \cap B) = P(B) \cdot P(A \mid B)}$
Independent events	$\mathrm{P(A \cap B) = P(A) \cdot P(B)}$

WATCH OUT
Mutually exclusive and independent are not the same thing and are the pair students swap most. Mutually exclusive events cannot happen together; independent events do not affect each other's chances. Two events with real probabilities can never be both.

Set counting formulas

Reach for these when a question counts people or things that belong to more than one group at once, which is what every Venn diagram question really is. Draw the diagram, fill the innermost region first, and work outwards.

RULE	FORMULA
Two sets	$\mathrm{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

RULE	FORMULA
Complement	$n(A') = n(U) - n(A)$
In A only	$n(A) - n(A \cap B)$
Three sets	$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
Disjoint sets	$n(A \cup B) = n(A) + n(B)$
De Morgan	$(A \cup B)' = A' \cap B'$
De Morgan	$(A \cap B)' = A' \cup B'$
Subsets of a set	2^n for a set of n members

> WATCH OUT
> With three sets the signs alternate: subtract each pair, then add the triple back. Leaving that last term out is the single most common way a three-circle Venn question goes wrong, because the middle region ends up counted zero times instead of once.

Exponent and radical laws

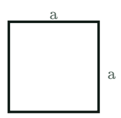
Turn every root into a fractional power before you do anything else. Once the whole expression is written as powers, the three multiply, divide and power-of-a-power rules handle it, and there is nothing left to remember.

RULE	FORMULA
Same base, multiply	$a^m \cdot a^n = a^{m+n}$
Same base, divide	$(a^m)/(a^n) = a^{m-n}$
Power of a power	$(a^m)^n = a^{mn}$
Power of a product	$(ab)^n = a^n b^n$
Power of a quotient	$(a/b)^n = (a^n)/(b^n)$
Zero exponent	$a^0 = 1$
Negative exponent	$a^{-n} = 1/(a^n)$
Root as a power	$\surd[n]a = a^{1/n}$
Fractional exponent	$a^{m/n} = \surd[n](a^m)$
Root of a product	$\surd[n]ab = \surd[n]a \cdot \surd[n]b$
Root of a quotient	$\surd[n](a/b) = (\surd[n]a)/(\surd[n]b)$

> WATCH OUT
> Every rule with a division line in it needs the base under that line to be something other than zero. Beyond that: these rules only combine powers of the SAME base, and only across multiplication and division. There is no rule for $a^m + a^n$, and $(a+b)^n$ is not $a^n + b^n$ for any n but 1.

Area and perimeter of plane shapes

Perimeter is the distance once around the edge and area is the surface inside it, so the two answer different questions and never share a unit. Use Heron's formula when a triangle gives you three sides and no height.

	SHAPE	PERIMETER	AREA
	**Square**	$4a$	a^2

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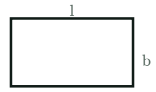
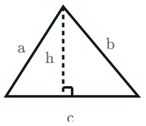
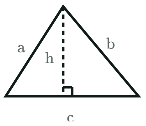
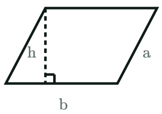
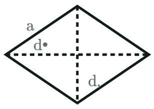
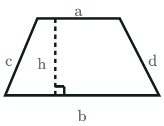
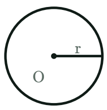
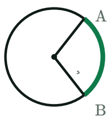
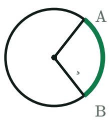
	SHAPE	PERIMETER	AREA
	**Rectangle**	$2(l + b)$	$l\,b$
	**Triangle**	$a + b + c$	$1/2\ b\ h$
	**Triangle from its sides**	$a + b + c$	$\sqrt{(s(s\text{-}a)(s\text{-}b)(s\text{-}c))}$
	**Parallelogram**	$2(a + b)$	$b\ h$
	**Rhombus**	$4a$	$1/2\ d_1\ d_2$
	**Trapezium**	$a + b + c + d$	$1/2(a + b)\ h$
	**Circle**	$2\ \pi\ r$	$\pi\ r^2$

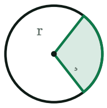
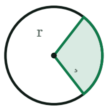
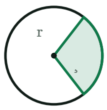
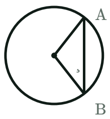
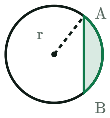
WATCH OUT

In a triangle, a parallelogram and a trapezium, h is the perpendicular distance, not the slanting side drawn next to it. Reaching for the slanting side is the single most common way marks are lost here, and the answer it gives is always too large.

Arcs, sectors and segments

Every formula here is the whole circle scaled down by how much of it the angle covers. Once you see that, there is only one thing to remember: what fraction of a full turn the angle is, and whether that turn is measured in degrees or radians.

	PART OF A CIRCLE	FORMULA
	**Arc length, angle in degrees**	$\theta/360 \times 2\ \pi\ r$
	**Arc length, angle in radians**	$r\ \theta$

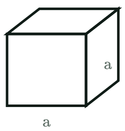
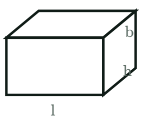
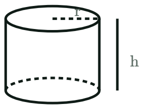
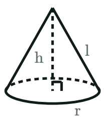
	PART OF A CIRCLE	FORMULA
	**Sector area, angle in degrees**	$\theta/360 \times \pi\, r^2$
	**Sector area, angle in radians**	$1/2\, r^2\, \theta$
	**Perimeter of a sector**	$2r + \theta/360 \times 2\, \pi\, r$
	**Chord length**	$2r\, \sin(\theta\ /\ 2)$
	**Segment area**	$\theta/360 \times \pi\, r^2$ - $1/2\, r^2 \sin\theta$

WATCH OUT

The degree forms divide by 360 and the radian forms do not. Putting a radian angle into a degree formula gives an answer roughly 57 times too small, and it will still look like a sensible number, so check the units on the angle before anything else.

Surface area and volume of solids

Read the question for the word total or the word curved before you pick a row, because most solids have both and the two differ by exactly the flat ends. With a cone, work out the slant height first: none of its surface formulas can be used without it.

	SOLID	CURVED SURFACE	TOTAL SURFACE	VOLUME
	**Cube**	—	$6a^2$	a^3
	**Cuboid**	—	$2(lb + bh + hl)$	$l\, b\, h$
	**Cylinder**	$2\, \pi\, r\, h$	$2\, \pi\, r\, (h + r)$	$\pi\, r^2\, h$
	**Cone**	$\pi\, r\, l$	$\pi\, r\, (l + r)$	$1/3\, \pi\, r^2\, h$

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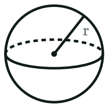
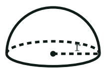
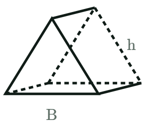
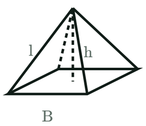
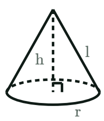
	SOLID	CURVED SURFACE	TOTAL SURFACE	VOLUME
	**Sphere**	—	$4 \pi r^2$	$4/3 \pi r^3$
	**Hemisphere**	$2 \pi r^2$	$3 \pi r^2$	$2/3 \pi r^3$
	**Prism**	$P h$	$2B + P h$	$B h$
	**Pyramid**	$1/2 P l$	$B + 1/2 P l$	$1/3 B h$
	**Slant height of a cone**	—	—	$l = \sqrt{(r^2 + h^2)}$

WATCH OUT
The slant height l and the perpendicular height h are different lengths, and a cone question usually gives you one while its formulas want the other. Surface area is measured in square units and volume in cubic ones, so an answer with the wrong unit is wrong even when the number is right.

Standard limits as x approaches zero

Substitute first, always. If putting the value in gives a number, that number is the limit and there is nothing else to do. These identities exist only for the case where substituting gives zero over zero, and each one is a way of recognising that shape.

LIMIT	VALUE
$(\sin x)/x$	1
$x/(\sin x)$	1
$(\tan x)/x$	1
$x/(\tan x)$	1
$(1 - \cos x)/(x^2)$	$1/2$
$(1 - \cos x)/x$	0
$(\sin ax)/x$	a
$(\sin ax)/(\sin bx)$	a/b
$(\tan ax)/(\tan bx)$	a/b
$(\sin ax)/(\tan bx)$	a/b
$(1 - \cos ax)/(x^2)$	$(a^2)/2$

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> WATCH OUT
> Every limit here needs the angle in RADIANS. In degrees (sin x)/x tends to π/180, which is about 0.017 rather than 1, and the working looks perfectly correct all the way to the wrong answer.

Exponential and logarithmic limits

Reach for these when an exponential or a logarithm sits over a plain x and substituting gives zero over zero. Each one is really the gradient of that function at a single point, written as a limit, which is why they all come out as tidy numbers.

AS	LIMIT	VALUE
$x \to 0$	$(e^x - 1)/x$	1
$x \to 0$	$(e^{ax} - 1)/x$	a
$x \to 0$	$(1 - e^{-x})/x$	1
$x \to 0$	$(a^x - 1)/x$	$\ln a$
$x \to 0$	$(\ln(1 + x))/x$	1
$x \to 0$	$(\ln(1 + ax))/x$	a
$x \to 1$	$(\ln x)/(x - 1)$	1
$x \to 0$	$(1 + x)^{1/x}$	e
$x \to \infty$	$(1 + 1/x)^x$	e
$x \to \infty$	$(1 + a/x)^x$	e^a
$x \to \infty$	$(1 + a/x)^{bx}$	e^{ab}

> WATCH OUT
> $(1 + x)^{1/x}$ tends to e as x approaches ZERO, while $(1 + 1/x)^x$ tends to e as x approaches INFINITY. The two look almost identical and their directions are opposite, so read which way the variable is going before you write e.

Limits as x approaches infinity

Divide every term by the highest power of x in the DENOMINATOR. Each term that then has an x underneath it goes to zero, and whatever is left standing is the answer. For a difference of two roots, multiply by the conjugate first so the leading terms cancel.

LIMIT	VALUE
$(a\,x^n + \ldots)/(b\,x^n + \ldots)$	a/b
$(x^p)/(x^q),\ p < q$	0
$(x^p)/(x^q),\ p > q$	∞
$1/(x^p),\ p > 0$	0
$x^p,\ p > 0$	∞
$c^x,\ c > 1$	∞
$c^x,\ 0 < c < 1$	0
$(\ln x)/(x^p),\ p > 0$	0
$(x^p)/(e^x),\ p > 0$	0
$\sqrt{(x^2 + 1)} - x$	0

WATCH OUT
Only the highest powers decide the answer, so a lower term never changes it. The ordering to remember is that an exponential beats every power of x, and a logarithm loses to every one, which settles most of these with no working at all.

Arithmetic mean between two numbers

Use this to find the single intermediate term between two terms that forms an arithmetic sequence.

$$m = (a + b)/2$$

WATCH OUT
When inserting multiple arithmetic means, find the common difference first rather than averaging the two endpoints.

Arithmetic sequence test

Use this when checking whether a general formula defines an arithmetic sequence.

WATCH OUT
Testing only the first few terms by substitution is not enough. You must subtract a_n from a_{n+1} and confirm that n cancels out completely.

Fibonacci and Mulatu sequences

Use this to recall the starting terms and recursion equations of Fibonacci and Mulatu sequences.

SEQUENCE	INITIAL TERMS	RECURSIVE RULE
Fibonacci	$F_0 = 1,\ F_1 = 1$	$F_n = F_{n-1} + F_{n-2}$
Mulatu	$M_0 = 4,\ M_1 = 1$	$M_n = M_{n-1} + M_{n-2}$

WATCH OUT
Notice the zeroth terms. Fibonacci starts with 1 and 1, whereas Mulatu sequence starts with 4 and 1.

nth term of an arithmetic sequence

Use this to calculate any term of an arithmetic sequence when the first term and common difference are known.

$$A_n = A_1 + (n - 1)d$$

WATCH OUT
Do not multiply d by n. The common difference is added n - 1 times to the first term.

Sign of common difference

Reach for this whenever finding the common difference of a decreasing arithmetic sequence.

WATCH OUT
Always calculate succeeding term minus preceding term. If terms decrease, the common difference is negative.

Average Rate of Change

Use to calculate the overall rate of change or the slope of the secant line between two endpoints on an interval [a, b].

$$(\Delta y)/(\Delta x) = (f(b) - f(a))/(b - a)$$

WATCH OUT
Do not divide f(b) - f(a) by a - b. The order of points in the numerator and denominator must match.

Confusing Secant and Tangent Rates

Use when distinguishing between the rate of change across an interval and the rate of change at an exact point.

WATCH OUT
Do not compute the average rate of change formula over an interval when the problem asks for the instantaneous rate at an exact point. An instantaneous rate requires taking the limit as h approaches zero.

Geometric Meaning of Rate of Change

Use to interpret what the sign of a slope or derivative says about the graph and behavior of a function.

SLOPE VALUE	GRAPH DIRECTION	FUNCTION BEHAVIOR
Positive	Slopes upward from left to right	Increasing
Negative	Slopes downward from left to right	Decreasing
Zero	Horizontal line	Constant
Undefined	Vertical line	Undefined

Instantaneous Rate of Change

Use to find the exact rate of change or the slope of the tangent line of a function at a single specific point x_0.

$$\lim_{h \to 0} (f(x_0 + h) - f(x_0))/h$$

WATCH OUT
Do not substitute h = 0 before simplifying the expression, as this produces an undefined division by zero.

Absolute vs Relative Dispersion

Use this comparison to decide whether to report spread in original units or use a unitless ratio to compare different data sets.

MEASURE TYPE	UNIT OF MEASUREMENT	MAIN PURPOSE	EXAMPLES
Absolute dispersion	Same unit as the data	Measures the spread of a single data set	Range, Inter-quartile range, Mean deviation
Relative dispersion	Unitless (dimensionless)	Compares variability between two or more data sets	Coefficient of range, Coefficient of variation

WATCH OUT
Never use absolute measures of dispersion to compare two data sets that have different measurement units.

Mean Deviation for Ungrouped Data

Use this formula to find the average absolute distance of raw data points from the mean, median, or mode.

$$MD = (\Sigma |x_i - A|)/n$$

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WATCH OUT
Always take the absolute value of each difference before summing. Summing deviations without absolute values about the mean always yields zero.

Quartiles for Grouped Data

Use this formula to calculate the first quartile, median, or third quartile in a grouped frequency table.

$$Q_k = L + (((k\ n)/4 - cf)/f)\ w$$

WATCH OUT
The term cf is the cumulative frequency of the class before the quartile class, not the quartile class itself.

Range for Grouped Data

Use this formula to calculate the range of a continuous grouped frequency distribution.

$$R = B_U(H) - B_L(L)$$

WATCH OUT
Do not subtract raw class limits. Always find the true class boundaries first.

Using Class Limits Instead of Class Boundaries

Review this when finding range, class width, or quartile positions in grouped distributions.

WATCH OUT
Discrete class limits leave gaps between classes. Calculate class boundaries by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit for integer data before computing range or quartiles.

Boundary Lines and Inequality Types

Use this table to choose the correct line style and determine whether the boundary line is included in the solution region.

INEQUALITY TYPE	SYMBOLS	BOUNDARY LINE	POINTS ON LINE INCLUDED
Strict	$<, >$	Dashed or broken	No
Slack	le, ge	Solid	Yes

WATCH OUT
Drawing a solid line for a strict inequality or a dashed line for a slack inequality is penalized in coordinate graphs.

Inequality Sign Inversion

Use this rule whenever multiplying or dividing both sides of an inequality by a negative number.

WATCH OUT
Multiplying or dividing by a negative number reverses the inequality sign, but adding or subtracting any number leaves the sign unchanged.

Test Point Selection for Half Planes

Use this technique to determine which side of the boundary line to shade for a linear inequality in two variables.

> WATCH OUT
> Never select a test point that lies on the boundary line. If the line passes through the origin, test a point like (0, 1) or (1, 0) instead of (0, 0).

Translating At Most and At Least

Use when translating word problems and everyday constraints into linear inequalities.

> WATCH OUT
> At most translates to less than or equal to, not greater than. At least translates to greater than or equal to, not less than.

Cross Product of Proportion

Use this relation to solve for an unknown value when two ratios are set equal, where b and d are non-zero.

$$a \cdot d = b \cdot c$$

> WATCH OUT
> Multiply the extremes together and the means together. Multiplying terms belonging to the same ratio produces a false result.

Direct and Inverse Variation

Use this reference to set up the correct variation formula based on whether two quantities vary directly or inversely.

VARIATION TYPE	PROPORTIONALITY	CONSTANT FORM	CONSTANT PROPERTY
Direct variation	$\mathrm{y\ propto\ x}$	$\mathrm{y = k\,x}$	Ratio remains constant
Inverse variation	$\mathrm{y\ propto\ 1/x}$	$\mathrm{x\,y = k}$	Product remains constant

> WATCH OUT
> For inverse variation, the product of the two variables remains constant, not their quotient.

Rate of Change

Use this formula to calculate the relative increase or decrease of a quantity compared to its starting level.

$$r = (A_f - A_i)/(A_i)$$

> WATCH OUT
> Always divide by the original amount in the denominator, never the final amount. A negative result represents a rate of decrease.

Ratios with Unmatched Units

Review this whenever forming a ratio or a rate from physical quantities given in different measurement units.

> WATCH OUT
> Never divide numbers before converting both quantities into identical units. For instance, comparing 6 Birr to 50 cents requires converting 6 Birr to 600 cents first, giving 600 to 50.