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Mathematics, 2017 EC, entrance exam, version 2

Grade 12 · Mathematics · 2017 E.C.

Time allowed: 120 minutes **Questions:** 54 **Total marks:** 54

Name _____ Grade _____ ID No. _____ Date _____

1. If $x < 0$, then, the simplest form of $f(x) = \frac{4x+10|x|}{2x}$ is equal to: (1)

A. 3 **B.** 7

C. -3 **D.** -7

2. For the greatest integer function $f(x) = |x|$, if $|x|^2 + 5|x| + 6 = 0$, what is the value of x? (1)

A. [-3,0) **B.** [0,2)

C. [-3,-1) **D.** [-1,0)

3. Which of the following intervals is the solution set of the inequality $|2 - x| < 8$? (1)

A. (-10,6) **B.** (-6,10)

C. (-8,8) **D.** (-6,12)

4. Which one of the following is false about the relation $R = \{(x, y)|x, y \in \Re, y \leq -x^2 + 4$ and $y \geq 2x - 4$? (1)

A. Domain of R is [-4,2] **B.** Range of R is [-12,4]

C. p(2,1) is a point on the graph of R^{-1} **D.** Range of R^{-1}[-4,4]

5. Which of the following is equal to $f(x) = \sqrt{(x-1)^2}$, for every $x \in \mathbb{R}$? (1)

A. g(x) = x + 1 **B.** g(x) = x - 1

C. g(x) = |x - 1| **D.** g(x) = |x| + 1

6. The domain and range of the function $f(x) = 2x^{\frac{2}{3}}$ are respectively (1)

A. $\mathbb{R}$ and $\mathbb{R}$ **B.** [0,∞) and $\mathbb{R}$

C. $\mathbb{R}$ and [0,∞) **D.** [0,∞) and [0,∞)

7. If the point (a, b) is on the graph of $y = f(x)$, which point is on the graph of $y = f^{-1}(x)$? (1)

A. (1/a, b) **B.** (b, a)

C. (-a, -b) **D.** (a, 1/b)

8. What is the equation of the tangent line to the graph of $f(x) = x^2 - x + 2$ at $(1, 2)$? (1)

A. x - y - 3 = 0 **B.** x - y + 3 = 0

C. x - y + 1 = 0 **D.** x + y + 1 = 0

9. If $f(x) = x^2 - ax + b$ and $g(x) = x^3 + c$ have the same tangent line at $p(1, 3)$, then the value of b is equal to: (1)

A. 1 **B.** -2

C. 0 **D.** 3

10. If $f(x) = x^2 + x + 1$, then, which one of the following is the equation of the normal line to the graph of f at $x = 1$? (1)

A. x + 3y + 10 = 0
B. 3x + y + 10 = 0
C. x - 3y + 10 = 0
D. x + 3y - 10 = 0

11. Let $f(x) = x^4 - 4x + 6$. For what value of x does the graph of f have a horizontal tangent line? (1)

A. 0
B. 1
C. -1
D. 2

12. Let l be a line that passes through the points $P(-1, 2)$ and $Q(5, -10)$ and it is tangent to the graph of $y = f(x)$ at $x = -1$, then, $f'(-1) =$ (1)

A. -2
B. -4
C. -5
D. 2

13. If $f(x) = 4x + 5$ and $(fog)(x) = x^2 + 2x + 5$, then, what is the value of $g(4)$? (1)

A. 6
B. 4
C. 29
D. 21

14. Given $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{2x}$, which one of the following is the domain of fog? (1)

A. {x∈ℝ: -1≤x≤2}
B. {x∈ℝ: 0≤x≤2}
C. {x∈ℝ: x≤0}
D. {x∈ℝ: x≥0}

15. If $f(x) = 3 + \frac{1}{e^x}$, then, $f^{-1}(e^2 + 3) =$ (1)

A. 1/e
B. 2
C. -2
D. 1

16. Let $f(x) = \frac{1-3x}{x-2}$. Then, what is the range of $f(x)$? (1)

A. ℝ\{2}
B. ℝ
C. ℝ\{-3}
D. ℝ\{1/3}

17. If f is the greatest integer function and g is an absolute value function, then, what is the value of $(fog)\left(\frac{7}{2}\right) + (gof)\left(-\frac{10}{3}\right)$? (1)

A. 6
B. 7
C. 0
D. 8

18. Let $f(x) = x^{\frac{1}{m}}$ and $g(x) = x^{\frac{1}{n}}$ be power functions where m and n are even natural numbers and $m > n$. Which one of the following is true about f and g ? (1)

A. If 0≤x≤1, f(x)≤g(x)
B. If x≥1, f(x)≥g(x)
C. If 0≤x≤1, f(x)≥g(x)
D. The domain of f and g is ℝ

19. Let $f(x) = \frac{x-1}{x+1}$ and $g(x) = \sqrt{3 - 2x}$ be two functions. Which one of the following is false about the combination values of f and g at $x = -3$? (1)

A. (f g)(-3) = 6
B. (f/g)(-3) = 2/3
C. (f + g)(-3) = 5
D. (f - g)(-3) = -5

20. If $x < 0$ and $f(x) = |x|$, then, $x - f(f(x))$ is equal to (1)

A. 0
B. 2x
C. -2x
D. x + x^2

21. Which one of the following is the sum of the series $\sum_{n=1}^{\infty}\left(\frac{3^n+4^n}{6^n}\right)$? (1)

A. 0
B. 5/4
C. 3
D. ∞

22. If $2^k \times 2^{k^2} \times 2^{k^3} \times \ldots \times 2^{k^n} \times \ldots = \frac{1}{8}$, then, $k =$ (1)

A. 2/3
B. 1
C. -3/4
D. 3/2

23. If $\sum_{n=1}^{k}\left(\frac{n-13}{2}\right) = -25$, then, the value of $k =$ (1)

A. 3
B. 4
C. 7
D. 20

24. Let $G_n = r^n$ be a geometric progression with a common ratio r. which one of the following is false about the sequence G_n ? (1)

A. If |r|<1, as n→∞ G_n converges to 1
B. If r≤-1 as n→∞ G_n is vibrant
C. If r=1 as n→∞ G_n→1
D. If r>1 as n→∞ G_n→∞

25. What is the simplified form of $\frac{a^{-4}-b^{-4}}{a^{-2}+b^{-2}}$ (1)

A. (b^2 - a^2)/(ab)^2
B. (ab/(b-a))^2
C. ((b-a)/(ab))^2
D. (ab)^2/(b^2 - a^2)

26. Which one of the following is true about the function $f(x) = \frac{x^3-x}{x^2-x}$? (1)

A. f has a vertical asymptote
B. x=-1 is zero of f
C. f has a horizontal asymptote
D. x=1 is zero of f

27. If $p(x) = 3x^2$ and $q(x) = x^2 + x$, then, what is the solution set of $\frac{p(x)}{3q(x)} = \frac{1}{x} - \frac{1}{q(x)}$ (1)

A. {-1,0}
B. {1}
C. {0,1}
D. {}

28. The value(s) of x where the graph of the function $y = \frac{x^3-1}{x^4+1}$ crosses its horizontal asymptote is(are): (1)

A. x=1
B. x=-1 and x=1
C. x=0
D. x=-1

29. If $\frac{1-x}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ then, $A + B + C =$ (1)

A. -1
B. 0
C. 1
D. -2

30. Which one of the following functions has no vertical asymptote? (1)

A. f(x)=(x^2-1)/(x^2-x)
B. f(x)=x/(x^3+x)
C. f(x)=4/(3x+1)
D. f(x)=x^{-3}

31. Which of the following is false about the graph of $f(x) = \frac{x^2-5x+6}{x^2-4}$? (1)

A. The graph has a hole at x=2
B. The vertical asymptotes of the graph are x=2 and x=-2
C. The horizontal asymptote of the graph is y=1
D. The graph has y-intercept at (0, -3/2)

32. Let A and B be 3×3 matrices such that A = [[2,0,0],[1,5,0],[0,-1,1/2]] and |B| = 1/2. Which of the following is equal to |AB^{-1}| ? (1)

A. 1 **B.** 4

C. 10 **D.** 5/2

33. If A is 3×3 matrix with |A| = 1/2, then, |2 adj A| = (1)

A. 1 **B.** 2

C. 4 **D.** 8

34. Let A and B be 3×3 matrices with |A| = m, and |B| = 1/n, m≠0, n≠0. What is the value of det(A^T B^{-1})? (1)

A. m/n **B.** mn

C. n/m **D.** 2mn

35. If A = [[3,1],[5,2]] then, A^{-1} is equal to (1)

A. [[2,-1],[-5,3]] **B.** [[-2,1],[5,-3]]

C. [[2,1],[5,3]] **D.** [[-2,-1],[-5,-3]]

36. Let A = [[2, x+y],[6,4]] and B = [[2,8],[6, x-y]]. If A = B, then x^2 - y^2 = (1)

A. 32 **B.** 36

C. 4 **D.** 48

37. What is the sum of the series $\sum_{n=3}^{101}(-1)^n\left(\frac{1}{n}+\frac{1}{n+1}\right)$? (1)

A. 35/101 **B.** 35/101

C. 35/101 **D.** 35/102

38. If the fourth term of an arithmetic sequence is -1 and the tenth term is 35, what is the sum of the first 20 terms? (1)

A. 1520 **B.** 1420

C. 860 **D.** 760

39. If the sum of the first n-terms of an arithmetic progression is S_n = n + 2n^2, which of the following is the 21th term (a_21) of the sequence? (1)

A. 903 **B.** 820

C. 83 **D.** 63

40. If x, x+2, and x+6 are consecutive terms of a geometric progression, then the value of x is equal to (1)

A. 1 **B.** 2

C. 3 **D.** 4

41. Which one of the following is a false statement? (1)

A. Σ(a_i - b_i)= Σa_i - Σb_i **B.** Σ(a_i × b_i)= Σa_i × Σb_i

C. Σ(c a_i)= c Σa_i **D.** Σ(i=1 to n) c = n c

42. If a, b, c, d are the four geometric means between 1/3 and 81, what is the common ratio of the geometric progression 1/3, a, b, c, d, 81? (1)

A. √3 **B.** 1/3

C. 3 **D.** 9

43. If {a_n} is an arithmetic sequence whose third term and the tenth term are 3 and 24 respectively, then, the 50th term of the sequence, a_50, is equal to (1)

A. 156 **B.** 150

C. 144 **D.** 141

44. A single square is made from 4 matchsticks. To make two squares in a row takes 7 Matchsticks, while three squares in a row takes 10 matchsticks. How many matchsticks are in a row of 90 squares? (1)

A. 211 **B.** 271

C. 291 **D.** 298

45. $\sum_{i=1}^{80}(\log_3^{i+1}) =$ (1)

A. 1/81 **B.** 4

C. -4 **D.** 3

46. The fifth and tenth terms of a geometric sequence are 640 and 20 respectively, then, the common ratio of the sequence is equal to: (1)

A. 2 **B.** 1/2

C. -1/2 **D.** -2

47. Which one of the following is the fractional form of the recurring decimal 0.23? (1)

A. 23/100 **B.** 23/90

C. 23/99 **D.** 23/900

48. If Σ_{n=1}^∞ a_n is a convergent series such that Σ_{n=1}^∞ (a_n - 3/4^n) = 12, then Σ_{n=1}^∞ a_n is equal to: (1)

A. 11 **B.** 112

C. 13 **D.** 14

49. A certain meeting hall has n rows of seats and 120 total numbers of seats. There are 3 seats in the first row, 5 seats in the second row, 7 seats in the third row, and so on. How many seats are there in the last row (nth row) of the hall? (1)

A. 21 **B.** 23

C. 32 **D.** 41

50. If f(x) = √π, then f'(4) is equal to: (1)

A. 1/4 **B.** 1/2

C. 0 **D.** 1/(2√π)

51. Let f(x) = 6x/(x+a). For what value of a is f'(2a)=2? (1)

A. 1/3 **B.** 2/3

C. 3/2 **D.** 3

52. If h(x) = f(x)/g(x) + f(x)g(x), f(1)=3, f'(1)=-2, g'(1)=4, g(1)=2, then h'(1) is equal to: (1)

A. 4 **B.** -10.4

C. 5 **D.** 16

53. If g(x) = f(x^2+4x), f(-3)=6, f'(-3)=-5, then g'(-1) = (1)

A. -2 **B.** -10

C. -30 **D.** -5

54. If f(x) = 4 + |x+6| for all x, then what is the value of f'(x) at x = -6? (1)

A. -1

B. 4

C. 1

D. does not exist

— END OF EXAMINATION —