

Mathematics, 2016 EC, natural science, entrance exam, round 1, Tigray

Grade 12 · Mathematics · 2016 E.C.

MARKING KEY: CONFIDENTIAL

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Questions: 52

Total marks: 52

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1. Let p and q be propositions. Which one of the following propositions is equivalent to the proposition $\neg p \vee (p \wedge q)$? (1)

A. $\neg p \vee \neg q$

B. $\checkmark \ p \Rightarrow q$

C. $\neg p \wedge q$

D. $\neg p \Rightarrow q$

Answer: B

Using the distributive law, we have $\neg p \vee (p \wedge q) \equiv (\neg p \vee p) \wedge (\neg p \vee q) \equiv \text{True} \wedge (\neg p \vee q) \equiv \neg p \vee q$, which is equivalent to the conditional statement $p \Rightarrow q$.

2. Which one of the following is NOT true about the function $f(x) = \tan^{-1} x$? (1)

A. $\checkmark$ The line $x = -\frac{\pi}{2}$ is a vertical asymptote to the graph of f .B. The graph of f is the image of the graph of $y = \tan x$ after it is reflected with respect to the line $y = x$.C. The graph of f intersects the x -axis at $(0, 0)$.D. The line $y = \frac{\pi}{2}$ is a horizontal asymptote to the graph of f .**Answer: A**

The domain of $f(x) = \tan^{-1} x$ is the entire set of real numbers $\mathbb{R}$, and it has horizontal asymptotes at $y = \pm \frac{\pi}{2}$ but no vertical asymptotes. The line $x = -\frac{\pi}{2}$ is actually a vertical asymptote for $y = \tan x$, not for its inverse.

3. Which one of the following statements is true? (1)

A. If x and y are integers and x divides y , then y divides x .B. $\checkmark$ If x is an even integer and y is an odd integer, then $x^2 + y^2$ is odd.C. If x and y are even integers, then $x + y$ is an odd integer.D. If x and y are positive integers, then $\frac{x}{y}$ is a positive integer.**Answer: B**

If x is even, then x^2 is even; if y is odd, then y^2 is odd. The sum of an even number and an odd number, $x^2 + y^2$, is always odd.

4. Let p , q , and r be propositions. If p is True, q is False, and r is False, which one of the following propositions has truth value True? (1)

A. $\checkmark \ (p \wedge \neg q) \Leftrightarrow (p \vee r)$

B. $(q \wedge r) \Leftrightarrow (p \wedge \neg r)$

C. $p \Rightarrow (q \vee r)$

D. $(p \vee q) \Rightarrow r$

Answer: A

Substituting the truth values, we have $p \wedge \neg q \equiv \text{True} \wedge \text{True} \equiv \text{True}$, and $p \vee r \equiv \text{True} \vee \text{False} \equiv \text{True}$. Thus, $(p \wedge \neg q) \Leftrightarrow (p \vee r) \equiv \text{True} \Leftrightarrow \text{True}$, which is True.

5. Which one of the following is the equation of the tangent line to the graph of $f(x) = 2x^3 - 4x^2 + 5x + 3$ at the point $(0, 3)$? (1)
- A. $3x + 5y - 3 = 0$ B. $3x - 5y - 3 = 0$
 C. ✓ $5x - y + 3 = 0$ D. $5x + y + 3 = 0$

Answer: C

The derivative of $f(x)$ is $f'(x) = 6x^2 - 8x + 5$, which gives a tangent slope of $f'(0) = 5$ at $x = 0$. The equation of the tangent line is $y - 3 = 5(x - 0)$, which simplifies to $5x - y + 3 = 0$.

6. Let $v = \vec{AB}$ and $w = \vec{CD}$ be vectors in the plane, where $A(1, -1)$, $B(2, -2)$, $C(-1, 3)$, and $D(4, -3)$. Which one of the following is equal to $v - w$? (1)
- A. $5i - 6j$ B. $i - 15j$
 C. ✓ $-4i + 5j$ D. $4i - 5j$

Answer: C

The vector $v = B - A = (1, -1) = i - j$ and $w = D - C = (5, -6) = 5i - 6j$. Subtracting these gives $v - w = (i - j) - (5i - 6j) = -4i + 5j$.

7. Which one of the following is true about the graph of the function $f(x) = 4 \sin\left(\frac{2}{3}x\right)$? (1)
- A. The amplitude of the graph of f is 8. B. ✓ **The maximum value of the function is 4.**
 C. The range of the function is $\left[-\frac{2}{3}, \frac{2}{3}\right]$. D. The period of the graph of f is $\frac{2}{3}$.

Answer: B

For $f(x) = 4 \sin\left(\frac{2}{3}x\right)$, the amplitude is $|4| = 4$, which means the function oscillates between -4 and 4 , making its maximum value equal to 4.

8. Let $z_1 = 1 + 2i$ and $z_2 = 2 - 2i$ be complex numbers. Which one of the following is equal to $\frac{z_1}{z_2}$? (1)
- A. $\frac{1}{4} - \frac{3}{4}i$ B. ✓ $-\frac{1}{4} + \frac{3}{4}i$
 C. $\frac{1}{8} + \frac{1}{4}i$ D. $\frac{1}{8} - \frac{1}{4}i$

Answer: B

Multiplying the numerator and denominator by the conjugate of z_2 , we get $\frac{1+2i}{2-2i} = \frac{(1+2i)(2+2i)}{2^2+(-2)^2} = \frac{2+2i+4i-4}{8} = \frac{-2+6i}{8} = -\frac{1}{4} + \frac{3}{4}i$.

9. Let $z_1 = 3 - 4i$ and $z_2 = -4 + 3i$ be complex numbers. Which one of the following is equal to $z_1 - z_2$? (1)
- A. ✓ $7 - 7i$ B. $-7 + 7i$
 C. $1 - i$ D. $-1 + i$

Answer: A

Subtracting the complex numbers gives $z_1 - z_2 = (3 - (-4)) + (-4 - 3)i = 7 - 7i$.

10. Let $f(x) = x^2 + 2x - 5$. What is the meaning of the value $\frac{f(3)-f(1)}{3-1}$? It is the slope of the tangent line to the graph of f at: (1)
- A. $(2.5, f(2.5))$ B. $(3, f(3))$
 C. $(1, f(1))$ D. ✓ $(2, f(2))$

Answer: D

By the Mean Value Theorem, the average rate of change on $[1, 3]$ equals the instantaneous rate of change at some point $c \in (1, 3)$. For a quadratic function, this point is the midpoint $c = 2$, meaning the value represents the slope of the tangent line at $(2, f(2))$.

11. If θ is the principal argument of the complex number $z = 4 + 3i$, which one of the following is equal to $\tan \theta$? (1)
- A. $\frac{3}{5}$ B. $\frac{4}{5}$
 C. $\frac{3}{4}$ D. $\frac{4}{3}$

Answer: C

For any complex number $z = x + iy$ in the first quadrant, the principal argument θ satisfies $\tan \theta = \frac{y}{x}$. Here $z = 4 + 3i$, so $\tan \theta = \frac{3}{4}$.

12. Let p, q, r , and s be propositions. Which one of the following defines that the argument form $p, q, r \vdash s$ is a valid argument? (1)
- A. $\checkmark$ If $p \wedge (q \wedge r)$ is true, then s is true. B. If $p \wedge (q \vee r)$ is true, then s is true.
 C. If $p \vee (q \wedge r)$ is true, then s is true. D. If $p \vee (q \vee r)$ is true, then s is true.

Answer: A

An argument is valid if whenever all the premises are true, the conclusion is also true. Thus, if the conjunction of the premises $p \wedge (q \wedge r)$ is true, then the conclusion s must be true.

13. Let A and B be two nonempty sets. Which one of the following is true about an onto function $f : A \rightarrow B$? (1)
- A. For any $x \in A$, there exists $y \in B$ such that $f(y) = x$. B. For $x, y \in A$, if $f(x) = f(y)$, then $x = y$.
 C. The range of f is a proper subset of B . D. $\checkmark$ For any $y \in B$, there exists $x \in A$ such that $f(x) = y$.

Answer: D

By definition, a function $f : A \rightarrow B$ is onto (surjective) if for every element y in the codomain B , there exists at least one element x in the domain A such that $f(x) = y$.

14. Which one of the following is the standard form of the equation of a parabola with vertex $V(1, -4)$ and focus $F(1, 4)$? (1)
- A. $(x - 1)^2 = 16(y + 4)$ B. $(x - 1)^2 = 16(y - 4)$
 C. $\checkmark$ $(x - 1)^2 = 32(y + 4)$ D. $(x - 1)^2 = 32(y - 4)$

Answer: C

Since the vertex $V(1, -4)$ and focus $F(1, 4)$ share the same x -coordinate, the parabola opens vertically upwards with focal parameter $p = 4 - (-4) = 8$. The standard form is $(x - h)^2 = 4p(y - k)$, which gives $(x - 1)^2 = 32(y + 4)$.

15. Which of the following is the value of the tangent of the angle between the lines $\ell_1 : 3x + 2y = 3$ and $\ell_2 : 2x + 3y = 1$ measured counterclockwise from ℓ_1 to ℓ_2 ? (1)
- A. $\checkmark$ $\frac{5}{12}$ B. $-\frac{5}{12}$
 C. $\frac{5}{6}$ D. $-\frac{5}{6}$

Answer: A

The slopes of the lines are $m_1 = -\frac{3}{2}$ and $m_2 = -\frac{2}{3}$. Using the angle formula $\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$, we get $\tan \theta = \frac{-2/3 - (-3/2)}{1 + (-3/2)(-2/3)} = \frac{5/6}{2} = \frac{5}{12}$.

16. Let $p(n)$: For any natural number n , $n^2 + 3n + 33$ is divisible by 3. We can show that $p(1)$ is false. What kind of proof have we used? (1)
- A. Proof by contrapositive
B. ✓ **Proof by counterexample**
C. Proof by contradiction
D. Proof by direct proof

Answer: B

A single example that disproves a universal statement is called a counterexample. Thus, showing $p(1)$ is false to disprove the general claim is a proof by counterexample.

17. Which one of the following is true about a right-angled triangle $\triangle ABC$ with a right angle at B ? (1)
- A. ✓ $\cot(\angle ACB) = \frac{BC}{AB}$
B. $\sin(\angle BAC) = \frac{AC}{BC}$
C. $\csc(\angle BAC) = \frac{AB}{AC}$
D. $\csc(\angle BCA) = \frac{BC}{AC}$

Answer: A

In $\triangle ABC$ with a right angle at B , the side adjacent to angle ACB is BC , and the side opposite is AB . Therefore, $\cot(\angle ACB) = \frac{\text{adjacent}}{\text{opposite}} = \frac{BC}{AB}$.

18. Which one of the following is true about a rigid motion? (1)
- A. ✓ **Reflection is an example of a rigid motion.**
B. A rotation by 180 degrees is the identity rotation.
C. A rigid motion carries an isosceles triangle to a scalene triangle.
D. Every rigid motion preserves the orientation of plane figures.

Answer: A

A rigid motion is a transformation of the plane that preserves distance. Reflections, rotations, and translations are all standard examples of rigid motions.

19. Consider the sequence defined by $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Which one of the following is equal to a_{15} ? (1)
- A. 377
B. 233
C. 987
D. ✓ **610**

Answer: D

Following the Fibonacci recurrence relation $a_n = a_{n-1} + a_{n-2}$ with $a_1 = a_2 = 1$, the first 15 terms are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610. Thus, $a_{15} = 610$.

20. A radioactive substance of 1024 grams loses half of its weight per year. What is the weight of the substance that will be left at the end of the ninth year? (1)
- A. ✓ **2 grams**
B. $\frac{1}{2}$ grams
C. 4 grams
D. $\frac{1}{4}$ grams

Answer: A

The weight of the substance at the end of the ninth year is given by $1024 \times (\frac{1}{2})^9 = 2^{10} \times 2^{-9} = 2$ grams.

21. What is the value of

(1)

$$\lim_{x \rightarrow 0} \frac{\sqrt{4-x} - 2}{x^2 - x}?$$

A. ✓ $\frac{1}{4}$

B. $-\frac{1}{4}$

C. $\frac{1}{8}$

D. $-\frac{1}{8}$

Answer: A

Rationalizing the numerator gives $\frac{(\sqrt{4-x}-2)(\sqrt{4-x}+2)}{x(x-1)(\sqrt{4-x}+2)} = \frac{-x}{x(x-1)(\sqrt{4-x}+2)} = \frac{-1}{(x-1)(\sqrt{4-x}+2)}$. Evaluating the limit as $x \rightarrow 0$ yields $\frac{-1}{(-1)(4)} = \frac{1}{4}$.

22. Which one of the following is the distance between the points $P(1, -1, 2)$ and $Q(3, 1, -1)$ in space?

(1)

A. 7 units

B. 17 units

C. $\sqrt{7}$ units

D. ✓ $\sqrt{17}$ units

Answer: D

Using the 3D distance formula, the distance between $P(1, -1, 2)$ and $Q(3, 1, -1)$ is $\sqrt{(3-1)^2 + (1-(-1))^2 + (-1-2)^2} = \sqrt{2^2 + 2^2 + (-3)^2} = \sqrt{17}$.

23. Which one of the following numbers satisfies Rolle's Theorem for the function $f(x) = \frac{1}{4}x^4 - 2x^2$ on the closed interval $[-3, 3]$?

(1)

A. 3

B. 4

C. 1

D. ✓ 2

Answer: D

Since $f(-3) = f(3) = \frac{9}{4}$ and f is continuous and differentiable on the interval, Rolle's Theorem guarantees a point $c \in (-3, 3)$ where $f'(c) = c^3 - 4c = 0$. The roots are $c = 0, \pm 2$, of which 2 is an option.

24. Let $f(x) = x^3 + 2x^2 - 3$. Which one of the following is true about f ?

(1)

A. ✓ f is increasing on $(0, \infty)$.

B. f is decreasing on $(\frac{4}{3}, \infty)$.

C. f is increasing on $(-\frac{4}{3}, 0)$.

D. f is decreasing on $(-\infty, -\frac{4}{3})$.

Answer: A

The derivative of $f(x) = x^3 + 2x^2 - 3$ is $f'(x) = 3x^2 + 4x = x(3x + 4)$, which is strictly positive on $(0, \infty)$. Therefore, f is increasing on $(0, \infty)$.

25. Which one of the following is equal to $\int \cos(3x + 5) dx$?

(1)

A. ✓ $\frac{\sin(3x+5)}{3} + c$

B. $\frac{\sin(3x+5)}{5} + c$

C. $5 \sin(3x + 5) + c$

D. $3 \sin(3x + 5) + c$

Answer: A

Using the substitution method with $u = 3x + 5$ and $dx = \frac{du}{3}$, we get $\int \cos(3x + 5) dx = \frac{1}{3} \int \cos(u) du = \frac{\sin(3x+5)}{3} + c$.

26. Which one of the following is equal to $\int 3x^2(x^3 + 2)^4 dx$?

(1)

A. $\frac{x^3(x^3+2)^5}{15} + c$

B. ✓ $\frac{(x^3+2)^5}{5} + c$

C. $15x^3(x^3 + 2)^5 + c$

D. $5x^3(x^3 + 2)^5 + c$

Answer: B

Using the substitution $u = x^3 + 2$, we have $du = 3x^2 dx$. The integral becomes $\int u^4 du = \frac{u^5}{5} + c = \frac{(x^3+2)^5}{5} + c$.

27. Let f and g be continuous functions on an interval I . If $\int f(x) dx = F(x)$ and $\int g(x) dx = G(x)$ on I , which one of the following expressions is true? (1)

- A. $\int (f + g)(x) dx = (F + G)(x)$ B. $\int (fg)(x) dx = (FG)(x)$
 C. $\int \left(\frac{f}{g}\right)(x) dx = \frac{F(x)}{G(x)}$ D. $\int (2f)(x) dx = \frac{F(x)}{2}$

Answer: A

By the sum rule of integration, the integral of a sum of two functions is the sum of their individual integrals, so $\int (f + g)(x) dx = \int f(x) dx + \int g(x) dx = F(x) + G(x) = (F + G)(x)$.

28. What is the sum of the first 7 terms of a sequence defined by $G_n = 10^{n-4}$ for $n = 1, 2, 3, \dots$? (1)

- A. 1111.001 B. $\checkmark$ 1111.111
 C. 1110.111 D. 1110.111

Answer: B

The sum of the first 7 terms is $10^{-3} + 10^{-2} + 10^{-1} + 10^0 + 10^1 + 10^2 + 10^3 = 0.001 + 0.01 + 0.1 + 1 + 10 + 100 + 1000 = 1111.111$.

29. Which one of the following is the greatest lower bound of the sequence $\left\{\frac{2n-3}{3n+2}\right\}$? (1)

- A. $-\frac{3}{5}$ B. $-\frac{1}{2}$
 C. $-\frac{3}{2}$ D. $\checkmark$ $-\frac{1}{5}$

Answer: D

Since the function $f(x) = \frac{2x-3}{3x+2}$ has a positive derivative $f'(x) = \frac{13}{(3x+2)^2} > 0$, the sequence is strictly increasing, meaning its greatest lower bound is its first term: $a_1 = \frac{2(1)-3}{3(1)+2} = -\frac{1}{5}$.

30. Let $v = \vec{AB}$ and $w = \vec{CD}$ be vectors in the plane where $A(-1, 2)$, $B(2, 1)$, $C(3, 1)$, and $D(4, -3)$. Which one of the following is equal to the scalar (or dot) product of $\vec{v}$ and $\vec{w}$? (1)

- A. $\checkmark$ 7 B. 8
 C. 5 D. 3

Answer: A

The vectors are $\vec{v} = B - A = (3, -1)$ and $\vec{w} = D - C = (1, -4)$. Their dot product is $\vec{v} \cdot \vec{w} = 3(1) + (-1)(-4) = 7$.

31. Let $M = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 3 & 4 \\ -1 & -1 & 2 \end{pmatrix}$. Which one of the following is the $\det(M)$? (1)

- A. $\checkmark$ -10 B. 0
 C. 30 D. 10

Answer: A

Expanding the determinant along the first row gives $\det(M) = 1(6 - (-4)) - 2(6 - (-4)) + 1(-3 - (-3)) = 10 - 20 + 0 = -10$.

32. Let A , B , and C be three exhaustive and mutually exclusive events of a random experiment, and S be the sample space of the given experiment. Which one of the following is true? (1)

- A. $A \cap B \cap C = S$ B. $A \cap B = S \cup C$
 C. $\checkmark$ $A \cup B \cup C = S$ D. $A \cup B = S \cup C$

Answer: C

By definition, if events A , B , and C are exhaustive, their union covers the entire sample space S , so $A \cup B \cup C = S$.

33. Which one of the following is the contrapositive of the statement "If x is an even natural number, then x is divisible by 2"? (1)
- A. ✓ If x is a natural number that is not divisible by 2, then x is not even.
- B. If x is an even natural number, x is NOT divisible by 2.
- C. If x is a natural number divisible by 2, then x is even.
- D. If x is not an even natural number, then x is not divisible by 2.

Answer: A

The contrapositive of the conditional statement $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$. Thus, "If x is not divisible by 2, then x is not an even natural number" (or equivalently Option A) is the contrapositive.

34. Let $P(n, r)$ be the number of permutations of n objects taking r at a time, and $C(n, r)$ be the number of ways of selecting r objects from a set of n objects without considering the order of selection. Which of the following is true? (1)
- A. $C(10, 4) = P(10, 4) \cdot 4!$
- B. $P(10, 4) = P(10, 4) \cdot 4!$
- C. $C(10, 5) = P(10, 5) \cdot 5!$
- D. ✓ $P(10, 5) = C(10, 5) \cdot 5!$

Answer: D

The relationship between permutations and combinations is given by $P(n, r) = C(n, r) \cdot r!$. Substituting $n = 10$ and $r = 5$ yields $P(10, 5) = C(10, 5) \cdot 5!$.

35. Let $f(x) = 2 \sin\left(\frac{1}{2}x\right)$. Which one of the following is equal to $f^{(5)}(x)$? (1)
- A. $-\frac{1}{16} \sin\left(\frac{1}{2}x\right)$
- B. $-\frac{1}{16} \cos\left(\frac{1}{2}x\right)$
- C. $\frac{1}{16} \sin\left(\frac{1}{2}x\right)$
- D. ✓ $\frac{1}{16} \cos\left(\frac{1}{2}x\right)$

Answer: D

Repeatedly differentiating $f(x) = 2 \sin\left(\frac{1}{2}x\right)$ gives $f'(x) = \cos\left(\frac{1}{2}x\right)$, $f''(x) = -\frac{1}{2} \sin\left(\frac{1}{2}x\right)$, $f'''(x) = -\frac{1}{4} \cos\left(\frac{1}{2}x\right)$, $f^{(4)}(x) = \frac{1}{8} \sin\left(\frac{1}{2}x\right)$, and $f^{(5)}(x) = \frac{1}{16} \cos\left(\frac{1}{2}x\right)$.

36. Which one of the following is the derivative of the function $f(x) = \frac{x^2 e^x}{x+1}$? (1)
- A. ✓ $\frac{(2x + 2x^2 + x^3)e^x}{(x+1)^2}$
- B. $\frac{(2x + 2x^2 + x^3)e^x}{x^2 + 1}$
- C. $\frac{(2x + x^2)e^x}{(x+1)^2}$
- D. $\frac{(2x + x^2)e^x}{x^2 + 1}$

Answer: A

Applying the quotient rule to $f(x) = \frac{x^2 e^x}{x+1}$ gives $f'(x) = \frac{(2xe^x + x^2 e^x)(x+1) - x^2 e^x}{(x+1)^2} = \frac{e^x(x^3 + 2x^2 + 2x)}{(x+1)^2}$.

37. What is the derivative of the function $f(x) = \ln(\sqrt{x^2 + 1})$? (1)
- A. $\frac{2x}{2\sqrt{x^2 + 1}}$
- B. $\frac{2x+1}{2x^2+1}$
- C. ✓ $\frac{x}{x^2+1}$
- D. $\frac{x}{\sqrt{x^2+1}}$

Answer: C

Simplifying the function yields $f(x) = \frac{1}{2} \ln(x^2 + 1)$. Using the chain rule, the derivative is $f'(x) = \frac{1}{2} \cdot \frac{2x}{x^2+1} = \frac{x}{x^2+1}$.

38. Let a and b be real numbers and $a < b$. Let f be a continuous function on $[a, b]$ and F be an antiderivative of f . Which one of the following is true? (1)
- A. $\int_a^b f(x) dx = F(a) - F(b)$ B. ✓ $\int_a^b f(x) dx = F(b) - F(a)$
C. $\int_a^b F(x) dx = f(b) - f(a)$ D. $\int_a^b F(x) dx = f(a) - f(b)$

Answer: B

According to the Fundamental Theorem of Calculus, if f is a continuous function on $[a, b]$ and F is an antiderivative of f (i.e., $F'(x) = f(x)$), then the definite integral $\int_a^b f(x) dx$ is evaluated as $F(b) - F(a)$.

39. Which one of the following is true in a standard coordinate space? (1)
- A. The intersection of the x -axis and the x -axis is the yz -plane.
B. The intersection of the xz -plane and the xy -plane is the xz -plane.
C. ✓ **The x -axis and the y -axis determine the xy -plane.**
D. The y -axis and the z -axis determine the xz -plane.

Answer: C

In standard coordinate space, any two distinct intersecting lines (like the x -axis and y -axis) uniquely determine a plane (the xy -plane).

40. Suppose S and T can complete a job in two hours if they work together. T needs 3 more hours than S to complete the job. If T works alone, in how many hours can he complete the job? (1)
- A. 3 hours B. ✓ **6 hours**
C. 9 hours D. 12 hours

Answer: B

Letting t be T 's time and $t - 3$ be S 's time, we solve $\frac{1}{t-3} + \frac{1}{t} = \frac{1}{2}$. This simplifies to $t^2 - 7t + 6 = 0$, giving $t = 6$ hours (since $t > 3$).

41. Which one of the following points is the intersection point of the line $l : 3x - 2y + 4 = 0$ and the circle $c : x^2 + y^2 - 2x - 4y - 5 = 0$? (1)
- A. $(-4, -4)$ B. ✓ **$(2, 5)$**
C. $(4, 8)$ D. $(-2, -1)$

Answer: B

Plugging the point $(2, 5)$ into the line gives $3(2) - 2(5) + 4 = 0$, and into the circle gives $4 + 25 - 4 - 20 - 5 = 0$. Since it satisfies both equations, it is their point of intersection.

42. Which one of the following is NOT true about the graph of the function $f(x) = \frac{x^2 - 4}{x^4 + 5x^2 + 6}$? (1)
- A. The graph of f is symmetric with respect to the y -axis.
B. ✓ **The graph of f is symmetric with respect to the origin.**
C. The x -intercepts of the graph of f are $(-2, 0)$ and $(2, 0)$.
D. The y -intercepts of the graph of f are $(0, -\frac{2}{3})$.

Answer: B

The function $f(x)$ is an even function because $f(-x) = f(x)$, meaning its graph is symmetric with respect to the y -axis, not the origin. Thus, statement B is false.

A function $f : A \rightarrow B$ has an inverse if and only if it is a bijection (one-to-one correspondence), which means it is both injective and surjective.

48. Let f be a function defined by:

(1)

$$f(x) = \begin{cases} 3 - bx^2, & \text{for } x \geq 1 \\ ax, & \text{for } -1 < x < 1 \\ 2x + b, & \text{for } x \leq -1 \end{cases}$$

What are the values of a and b so that f is continuous on the whole real line?

A. ✓ $a = -3$ and $b = 6$

B. $a = 3$ and $b = -6$

C. $a = 3$ and $b = 6$

D. $a = -3$ and $b = -6$

Answer: A

For f to be continuous at $x = 1$, we must have $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$, which means $a(1) = 3 - b(1)^2 \implies a + b = 3$. Among the choices, only Option A satisfies this condition ($a = -3, b = 6$).

49. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Which one of the following is true about minors and cofactors of entries of A ?

(1)

A. The minor of the entry 5 is 12.

B. The cofactor of the entry 7 is 3.

C. The minor of the element 9 is 3.

D. ✓ **The cofactor of the element 6 is 6.**

Answer: D

The element 6 is at row 2, column 3. Its minor is $M_{23} = 1(8) - 2(7) = -6$, so its cofactor is $C_{23} = (-1)^{2+3}(-6) = 6$.

50. Consider the following system of linear equations:

(1)

$$\begin{cases} x + y = 0 \\ y + z = 1 \\ x + z = 0 \end{cases}$$

Which one of the following is the solution set of the given system of linear equations?

A. $\left\{ \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right\}$

B. ✓ $\left\{ \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right\}$

C. $\left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \right\}$

D. $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$

Answer: B

From the first and third equations, we get $y = -x$ and $z = -x$. Substituting these into the second equation gives $-x - x = 1 \implies x = -1/2$, which means $y = 1/2$ and $z = 1/2$.

51. The angle of elevation from the top of a building of height 15 meters to the top of building A is 30° . If the distance between the two buildings is 60 meters, what is the height of building A?

(1)

A. $20\sqrt{3}$ meters

B. ✓ $(20\sqrt{3} + 15)$ meters

C. $60\sqrt{3}$ meters

D. $(60\sqrt{3} + 15)$ meters

Answer: B

Using the tangent of the angle of elevation, we have $\tan(30^\circ) = \frac{H-15}{60} \implies \frac{1}{\sqrt{3}} = \frac{H-15}{60}$, which gives $H - 15 = 20\sqrt{3}$, so the height $H = 20\sqrt{3} + 15$ meters.

52. Let f and g be two continuous functions on the closed interval $[a, b]$, k be a constant, and c be a real number between a and b . Which one of the following is NOT true about the functions f and g ? (1)

A. $\int_a^b f(x) dx = - \int_b^a f(x) dx$

B. ✓ $\int_a^b f(x) dx = \int_c^b f(x) dx - \int_a^c f(x) dx$

C. $\int_a^b (kf)(x) dx = k \int_a^b f(x) dx$

D. $\int_a^b (f + g)(x) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$

Answer: B

The additive property of definite integrals states that $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$, which makes the expression in Option B mathematically incorrect due to the minus sign.

END OF THE MARKING KEY

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