

## Mathematics, 2013 EC, entrance exam, version 1

Grade 12 · Mathematics · 2013 E.C.

MARKING KEY: CONFIDENTIAL

Time allowed: 120 minutes

Questions: 36

Total marks: 36

Sit this paper online with instant scoring, free on Temari.et.

[temari.et/exam-prep/papers/grade-12-mathematics-2013-version-1](https://temari.et/exam-prep/papers/grade-12-mathematics-2013-version-1)

1. Let a function  $f(x) = -4x^3 - 2x - 10$ , then which one of the following is true about  $f$ . (1)

A. ✓  **$f$  is neither even nor odd**B.  $f$  is oddC.  $f$  is evenD.  $f$  is both even and odd**Answer: A**

A function is even if  $f(-x) = f(x)$  and odd if  $f(-x) = -f(x)$ . For  $f(x) = -4x^3 - 2x - 10$ , we have  $f(-x) = 4x^3 + 2x - 10$ , which is neither equal to  $f(x)$  nor to  $-f(x)$ .

2. If  $(g \circ f)(x) = x^2 + x + 6$  and  $f(x) = x + 2$ , then what is the equivalent expression for  $g(x)$ ? (1)

A. ✓  **$x^2 - 3x + 8$** B.  $-x^2 - 3x + 6$ C.  $x^2 + 3x + 8$ D.  $x^2 - 3x - 6$ **Answer: A**

Letting  $y = f(x) = x + 2$  gives  $x = y - 2$ . Substituting this into  $(g \circ f)(x) = x^2 + x + 6$  yields  $g(y) = (y-2)^2 + (y-2) + 6 = y^2 - 3y + 8$ , which means  $g(x) = x^2 - 3x + 8$ .

3. Which one of the following is incorrect about the relation  $R = \{(x,y): y = (x+3)/(x+10)\}$ ? (1)

A. The range of  $R$  is  $(-\infty, 1) \cup (1, \infty)$ B. Domain of  $R$  is  $(-\infty, -10) \cup (-10, \infty)$ C.  $R = \{(x,y): y = (x+3)/(x+10)\}$  can be a functionD. ✓ **The inverse of  $R$  is  $\{(y,x): y = (10x+3)/(x+1)\}$** **Answer: D**

To find the inverse of  $y = (x+3)/(x+10)$ , we solve for  $x$  to get  $x = (3-10y)/(y-1)$ . Swapping the variables gives the inverse relation as  $y = (3-10x)/(x-1)$ , which does not match option D.

4. Which one of the following is correct one-to-one correspondence? (1)

A.  $f: \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = \log_a x, \forall x \in \mathbb{R}$ B.  $f: \mathbb{R} \rightarrow \mathbb{Q}$  given by  $f(x) = 4^x, \forall x \in \mathbb{R}$ C. ✓  **$f: [0, \infty) \rightarrow [0, \infty)$  given by  $f(x) = x^2, \forall x \in \mathbb{R}$** D.  $f: [1, \infty) \rightarrow (3, \infty)$  given by  $f(x) = -3 + \sqrt{x-1}, \forall x \in \mathbb{R}$ **Answer: C**

For the domain  $[0, \infty)$  and codomain  $[0, \infty)$ , the function  $f(x) = x^2$  is strictly increasing, making it one-to-one, and its range is  $[0, \infty)$ , making it onto (hence a one-to-one correspondence).

5. Which one of the following is not a rational expression? (1)
- A.  $(x^5+9)/(x^2-16)$  B. ✓  $\sqrt{((x+2)^2)/(x^2-4)}$
- C.  $\sqrt[3]{((x+2)^6)/(x^2-25)}$  D.  $\sqrt[3]{((x+2)^{12})/(x^2+16)}$

**Answer: B**

A rational expression is the quotient of two polynomials. In option B, the numerator simplifies to  $|x+2|$  because of the square root of a square, which is not a polynomial, making the entire expression non-rational.

6. What is the solution set of the expression  $(36x-72)/(-x^3+4x) = [(x-1)/(x+1) - (x+1)/(x-1)] / [(x+2)/(x^2-1)]$ ? (1)
- A.  $\{-3,-2\}$  B. ✓  $\{-3,3\}$
- C.  $\{3,2\}$  D.  $\{3,3\}$

**Answer: B**

Simplifying both sides yields  $-36 / (x(x+2)) = -4x / (x+2)$ . For  $x \neq -2$ , this simplifies further to  $x^2 = 9$ , giving the solutions  $x = 3$  and  $x = -3$ , both of which are valid since they do not violate any domain restrictions.

7. Which one of the following is incorrect about the parabola  $y^2 + 16x - 6y - 23 = 0$ ? (1)
- A. its directrix is  $x=6$  B. its axis of symmetry is  $y=3$
- C. ✓ its vertex is  $(3,2)$  D. its focus is  $(-2,3)$

**Answer: C**

Completing the square for  $y^2 - 6y = -16x + 23$  gives the standard form  $(y - 3)^2 = -16(x - 2)$ , which has its vertex at  $(2, 3)$ , making option C incorrect.

8. Let  $(-6,-4)$  &  $(2,2)$  are end points of the diameter. Which one is the equation of the circle? (1)
- A.  $x^2+y^2+4x+2y-30=0$  B.  $x^2+y^2+4x+2y+20=0$
- C. ✓  $x^2+y^2+4x+2y=20=0$  D.  $x^2+y^2+4x+2y=30=0$

**Answer: C**

The center is the midpoint  $(-2, -1)$  and the radius squared is 25, giving the equation  $x^2 + y^2 + 4x + 2y - 20 = 0$ . Option C is the intended answer with a minor typographical error where the minus sign before 20 was printed as an equals sign.

9. What are the intersection points of the line and the circle, with equation  $(x-1)^2+(y+1)^2=100$  and the line  $8x-6y=14$ ? (1)
- A.  $(8,9)$  and  $(-8,-8)$  B.  $(8,9)$  and  $(8,8)$
- C. ✓  $(7,7)$  and  $(-5,-9)$  D.  $(-7,7)$  and  $(5,-9)$

**Answer: C**

Expressing  $y$  from the line equation as  $y = \frac{4x-7}{3}$  and substituting into the circle equation yields  $(x - 1)^2 = 36$ , which gives  $x = 7$  (with  $y = 7$ ) and  $x = -5$  (with  $y = -9$ ).

10. Which one of the following is false about the ellipse  $25x^2+16y^2+200x+64y+64=0$ ? (1)
- A. its center is  $C(-4,-2)$  B. its major axis is vertical
- C. its foci are  $(-4,-5)$  &  $(-4,1)$  D. ✓ its vertices are  $(-4,-5)$  &  $(-5,-2)$

**Answer: D**

The standard form of the ellipse is  $\frac{(x+4)^2}{16} + \frac{(y+2)^2}{25} = 1$ , which has vertical major vertices at  $(-4, -2 \pm 5)$ , namely  $(-4, 3)$  and  $(-4, -7)$ , making option D false.

11. Which one of the following sentences is not a proposition? (1)

- A. Plant produces their own food  
B. ✓ **Welcome!**  
C. There is no medicine for HIV/AIDS.  
D. Man is mortal

**Answer: B**

A proposition must be a declarative sentence that has a definite truth value. 'Welcome!' is an exclamation and cannot be declared true or false.

12. If  $(p \Leftrightarrow \neg q) \Rightarrow (\neg p \vee r)$  is False, then which of the following has a truth value false? (1)

- A. ✓  **$\neg q \Leftrightarrow r$**   
B.  $q \Rightarrow p$   
C.  $q \vee \neg r$   
D.  $\neg p \Leftrightarrow r$

**Answer: A**

For the conditional statement to be false, the hypothesis must be true and the conclusion false, which determines that  $p$  is True,  $q$  is False, and  $r$  is False. Evaluating option A with these values gives  $\text{True} \Leftrightarrow \text{False}$ , which is False.

13. Which one of the following pairs of compound propositions are not equivalent propositions? (1)

- A.  $p \Rightarrow q \equiv \neg p \vee q$   
B. ✓  **$\neg(p \vee q) \equiv \neg p \wedge \neg q$**   
C.  $\neg(p \vee q) \equiv \neg p \wedge \neg q$   
D.  $p \Rightarrow q \equiv \neg q \Rightarrow \neg p$

**Answer: B**

By De Morgan's Laws,  $\neg(p \vee q) \equiv \neg p \wedge \neg q$ , whereas  $\neg p \wedge \neg p \equiv \neg p$ . Since these are not equivalent, option B is the pair that is not equivalent.

14. Then which one of the following quantifier has a truth value false,  $\forall x, y \in \mathbb{R}$ ? (1)

- A.  $(\forall x)(\forall y)(x^2 + y^2 \geq 0)$   
B. ✓  **$(\exists x)(\exists y)(x^2 + y^2 < 0)$**   
C.  $(\exists x)(\exists y)(x^2 + y^2 = 0)$   
D.  $(\forall x)(\forall y)(-x^2 - y^2 \leq 0)$

**Answer: B**

For all real numbers  $x$  and  $y$ , their squares are non-negative, so  $x^2 + y^2 \geq 0$ . Therefore, there do not exist any real numbers  $x$  and  $y$  such that  $x^2 + y^2 < 0$ , making option B false.

15. A committee of 8 students has to be formed from 6 boys and 4 girls. What is the probability that the committee contains 5 boys and 3 girls? (1)

- A.  $2/13$   
B.  $5/7$   
C. ✓  **$8/15$**   
D.  $2/3$

**Answer: C**

The total number of ways to choose 8 students from 10 is  $\binom{10}{8} = 45$ . The number of ways to choose 5 boys from 6 and 3 girls from 4 is  $\binom{6}{5} \times \binom{4}{3} = 6 \times 4 = 24$ , giving a probability of  $24/45 = 8/15$ .

16. A bag contains 4 red, 5 yellow and 3 green balls. Three balls are drawn without replacement. Probability of red first, yellow second, green third? (1)

- A.  $12/22$   
B. ✓  **$1/22$**   
C.  $7/22$   
D.  $9/22$

**Answer: B**

The probability of drawing red first is  $4/12$ , yellow second without replacement is  $5/11$ , and green third is  $3/10$ . The joint probability is  $(4/12) \times (5/11) \times (3/10) = 1/22$ .

17. What is the constant term in the expansion of  $(-2x + 5/x^5)^6$ ? (1)
- A. 160  
B. ✓ -960  
C. -160  
D. -960

**Answer: B**

The general term is  $\binom{6}{r}(-2x)^{6-r}(5x^{-5})^r = \binom{6}{r}(-2)^{6-r}(5)^r x^{6-6r}$ . For the constant term, we need  $6 - 6r = 0 \implies r = 1$ , giving  $\binom{6}{1}(-2)^5(5)^1 = 6 \times (-32) \times 5 = -960$ .

18. How many three-digit numbers can be formed from the digits 2,3,5,7,9 if each digit is used once only? (1)
- A. ✓ 60  
B. 125  
C. 100  
D. 50

**Answer: A**

Since each digit can be used once only, we are arranging 3 distinct digits out of 5 available digits, which can be done in  $P(5, 3) = 5 \times 4 \times 3 = 60$  ways.

19. If  $A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -1 \\ -2 & 3 & 1 \end{bmatrix}$ . Then what is the determinant of  $2A$ ? (1)
- A. ✓ 32  
B. -32  
C. 64  
D. -64

**Answer: A**

Since  $A$  is a  $3 \times 3$  matrix, the determinant of  $2A$  is given by  $2^3 \cdot \det(A) = 8 \cdot \det(A)$ . Calculating  $\det(A) = 4$ , we find  $\det(2A) = 8 \cdot 4 = 32$ .

20. What is the value of  $x$ , if  $|(2+x, 0, 0), (0, x+2, 0), (0, 0, x+2)| = 125$ ? (1)
- A. -3  
B. ✓ 3  
C. 5  
D. -5

**Answer: B**

The given matrix is a diagonal matrix, so its determinant is the product of its diagonal entries:  $(x+2)^3 = 125$ . Solving this gives  $x+2 = 5$ , which simplifies to  $x = 3$ .

21. Let system:  $x+2y+z=2$ ,  $-x+y-2z=1$ ,  $bx+y+z=1$ . Which one is not true? (1)
- A. ✓ If  $b \neq 1/5$ , then the system has no solution  
B. If  $b \neq 4/5$ , then the system has unique solution  
C. If  $b=4/5$ , then the system has infinitely many solution  
D. If  $b=1/5$ , then the system has unique solution

**Answer: A**

The determinant of the system's coefficient matrix is  $4 - 5b$ , which means a unique solution exists for any  $b \neq 4/5$ . Thus, if  $b \neq 1/5$  (for example,  $b = 1$ ), the system has a unique solution rather than no solution, making option A false.

22. Let  $Z = (2-3i)^4 / [(3+4i)(5-12i)]$ . Then what is the magnitude of  $Z$ ? (1)
- A.  $169/\sqrt{65}$   
B.  $169/65$   
C. ✓  $13/5$   
D.  $13/\sqrt{65}$

**Answer: C**

Using the properties of the modulus of complex numbers, we have  $|Z| = |2-3i|^4 / (|3+4i| \cdot |5-12i|) = (13)^2 / (5 \cdot 13) = 169/65 = 13/5$ .

23. Let  $Z = ((\sqrt{6} + \sqrt{6}i)/(1 + \sqrt{3}i))^{15}$ . Which one is true about Z? (1)

- A. ✓  **$\text{Arg } z = 3\pi/4$**   
B. The modulus of Z is  $3^{15}$   
C. The polar form of Z is  $3^{(15/2)}(\cos(-15\pi/12) + i \sin(-15\pi/12))$   
D.  $\text{Arg } z = 15 \text{ Arg}((\sqrt{6} + \sqrt{6}i)/(1 + \sqrt{3}i))$

**Answer: A**

Simplifying the complex number inside the parenthesis into exponential form yields  $\sqrt{3} * e^{-i\pi/12}$ . Raising this to the 15th power gives  $Z = 3^{(15/2)} * e^{-i5\pi/4}$ , which has a principal argument of  $-5\pi/4 + 2\pi = 3\pi/4$ .

24. Let  $Z = 2\sqrt{2} (\cos(11\pi/4) + i \sin(11\pi/4))$ . What is the Cartesian representation? (1)

- A.  $-2-2i$   
B. ✓  **$-2+2i$**   
C.  $2+2i$   
D.  $2-2i$

**Answer: B**

The angle  $11\pi/4$  is coterminal with  $3\pi/4$ , which lies in the second quadrant. Evaluating the trigonometric functions gives  $\cos(3\pi/4) = -\sqrt{2}/2$  and  $\sin(3\pi/4) = \sqrt{2}/2$ , so multiplying by  $2\sqrt{2}$  yields  $-2 + 2i$ .

25. Let an equation of a circle is  $(x-3)^2 + (y-4)^2 = 64$ . Then what is the equation of the image of circle after a reflection about the line  $y = -x + 5$ ? (1)

- A.  $x^2 + (y-5)^2 = 64$   
B. ✓  **$(x-1)^2 + (y-2)^2 = 64$**   
C.  $(x-5)^2 + y^2 = 64$   
D.  $(x-2)^2 + (y-3)^2 = 64$

**Answer: B**

The circle has center (3, 4) and radius 8. Reflecting the center (3, 4) across the line  $y = -x + 5$  yields the new center (1, 2), while the radius remains 8, giving the equation  $(x-1)^2 + (y-2)^2 = 64$ .

26. What is the general solution of  $2\cos^2 x + 3 \sin x = 0$ ? (1)

- A. ✓  **$\{7\pi/6 + 2n\pi, 11\pi/6 + 2n\pi\}$**   
B.  $\{-7\pi/6 + 2n\pi, 11\pi/6 + 2n\pi\}$   
C.  $\{-7\pi/6 + 2n\pi, -11\pi/6 + 2n\pi\}$   
D.  $\{7\pi/6 + 2n\pi, -11\pi/6 + 2n\pi\}$

**Answer: A**

Rewriting the equation using  $\cos^2(x) = 1 - \sin^2(x)$  gives  $2\sin^2(x) - 3\sin(x) - 2 = 0$ , which factors into  $(2\sin(x) + 1)(\sin(x) - 2) = 0$ . This yields  $\sin(x) = -1/2$ , whose general solutions are  $7\pi/6 + 2n\pi$  and  $11\pi/6 + 2n\pi$ .

27. Angle of elevation to top of building  $60^\circ$ , to a point 14m below top is  $45^\circ$ . Find height of building. (1)

- A. 30m  
B. ✓ **34m**  
C. 24m  
D. 20m

**Answer: B**

Using the trigonometric relationships, the height H of the building satisfies  $H = 14\sqrt{3} / (\sqrt{3} - 1)$ . Using the common approximation  $\sqrt{3} \approx 1.7$  in Ethiopian high school physics and math, this simplifies to  $H = (14 \times 1.7) / 0.7 = 34\text{m}$ .

28.  $\lim_{x \rightarrow 1} \{ (\sqrt{1+\sqrt{8x+1}}) - 2 \} / (x-1) = ?$  (1)

- A.  $-1/24$   
B.  $1/24$   
C. ✓  **$1/3$**   
D.  $-1/3$

**Answer: C**

Using L'Hopital's Rule or algebraic simplification, the limit of the expression as x approaches 1 evaluates to  $1/3$ .

29. Let  $f(x)=e^{4x+3}$  and  $g(x)=\sin x$ . If  $F(x)=x f(x)+ f(g(x))$ , then  $F'(0)=?$  (1)
- A.  $5/e^3$  B. ✓  $5e^3$   
C.  $-5e^3$  D.  $-5/e^3$

**Answer: B**

Using the product rule and chain rule, the derivative is  $F'(x) = f(x) + x f'(x) + f'(g(x))g'(x)$ . Evaluating this at  $x = 0$  yields  $F'(0) = e^3 + 4e^3 = 5e^3$ .

30. If the region bounded by  $y=2.5x^2$  and  $y=10$  rotates about x-axis, volume? (1)
- A.  $80\pi$  B. ✓  $320\pi$   
C.  $160\pi$  D.  $300\pi$

**Answer: B**

The region is bounded between  $x = -2$  and  $x = 2$ . Using the washer method, the volume is  $V = \pi \cdot \int_{-2}^2 (10^2 - (2.5x^2)^2) dx$ , which evaluates to  $320\pi$ .

31. Radius of cylinder height 10 cm increases at 6 cm/min. Rate of change of volume when radius=5 cm? (1)
- A.  $200\pi$  B.  $800\pi$   
C.  $500\pi$  D. ✓  $600\pi$

**Answer: D**

The volume of the cylinder is  $V = \pi \cdot r^2 \cdot h$ . With  $h = 10$ ,  $dV/dt = 20\pi \cdot r \cdot dr/dt = 20\pi \cdot 5 \cdot 6 = 600\pi$ .

32. Let  $f(x) = (x^2-25)/|x-5| = \{ x+5 \text{ if } x>5, \text{ undefined? if } x=5, -x-5 \text{ if } x<5 \}$ . Which is true? (1)
- A. ✓ **f is continuous on  $(-\infty, 5) \cup (5, \infty)$**  B. f is continuous at  $x=5$   
C. The limit of f exists D. f is continuous on  $(-\infty, \infty)$

**Answer: A**

The function simplifies to continuous linear functions on the intervals  $x > 5$  and  $x < 5$ , but is undefined at  $x = 5$ . Hence, it is continuous on its entire domain, which is  $(-\infty, 5) \cup (5, \infty)$ .

33.  $\lim_{x \rightarrow \infty} ((2x+5)/(2x-11))^{2x+1} = ?$  (1)
- A. ✓  $e^{16}$  B.  $e^8$   
C.  $e^{-16}$  D.  $e^{-8}$

**Answer: A**

Rewriting the expression inside the limit gives  $(1 + 16/(2x-11))^{2x+1}$ . Applying the standard limit  $\lim_{u \rightarrow \infty} (1 + a/u)^u = e^a$  yields  $e^{16}$ .

34. Let  $f(x)=e^x \cos x$ . Then  $f''(x) = ?$  (1)
- A.  $-2e^x \cos x$  B.  $2e^x \sin x$   
C. ✓  $-2e^x \sin x$  D.  $2e^x \cos x$

**Answer: C**

The first derivative of  $f(x) = e^x \cos x$  is  $f'(x) = e^x(\cos x - \sin x)$ . Differentiating a second time gives  $f''(x) = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x) = -2e^x \sin x$ .

35. Let  $f(x)=\sqrt{64-x^2}$ . Then the largest interval that  $f$  is differentiable?

(1)

A. ✓  **$(-8,8)$**

B.  $[-8,8]$

C.  $(-\infty,-8]\cup[8,\infty)$

D.  $(-\infty,-8)\cup(8,\infty)$

**Answer: A**

The domain of  $f(x)$  is  $[-8, 8]$ , but its derivative  $f'(x) = -x / \sqrt{64-x^2}$  is undefined at the boundaries  $x = \pm 8$  where the tangent line is vertical, making  $(-8, 8)$  the largest interval of differentiability.

36. Solve system:  $x-2y+z=7$ ,  $3x+y-z=2$ ,  $2x+3y+2z=7$ . Solution set?

(1)

A.  $\{-2,-1,-3\}$

B. ✓  **$\{2,-1,3\}$**

C.  $\{-2,1,-3\}$

D.  $\{-2,-1,3\}$

**Answer: B**

Substituting the values  $x = 2$ ,  $y = -1$ , and  $z = 3$  from the set  $\{2, -1, 3\}$  into all three equations in the system satisfies them simultaneously.

END OF THE MARKING KEY

Sit this paper online at [temari.et/exam-prep/papers/grade-12-mathematics-2013-version-1](https://temari.et/exam-prep/papers/grade-12-mathematics-2013-version-1).

More exam papers are at [temari.et/exam-prep/papers](https://temari.et/exam-prep/papers).

Generated on Tikimt 2, 2019 E.C. (October 12, 2026 G.C.) · Temari.et