Mathematics, 2014 EC, common, entrance exam

Grade 12 · Mathematics · 2014 E.C.

MARKING KEY — CONFIDENTIAL

Time allowed: 120 minutes **Questions:** 30 **Total marks:** 30

1. what are the quotient and remainder respectively, when 75 divided by 20 ? (1)

A. 15 and 3
B. 2 and 19
C. ✓ **3 and 15**
D. 4 and 5

Answer: C

Division with remainder writes a number as: dividend = divisor × quotient + remainder.

Ask how many whole times 20 fits into 75.
$20 \times 3 = 60$ fits, but $20 \times 4 = 80$ is more than 75.
So the quotient is 3.
The remainder is what is left over: $75 - 60 = 15$.

Check: $75 = 20 \times 3 + 15$, and 15 is smaller than 20, so this is a proper remainder.

The question asks for the quotient and the remainder in that order, so the answer is 3 and 15.

Watch out: option A gives the same two numbers in the reversed order, 15 and 3.

Textbook: Mathematics Grade 9, unit The Number System (page 17).

2. Which of the following is the measure of an exterior angle of a 10 sided regular polygons? (1)

A. 72°
B. 144°
C. 180°
D. ✓ **36°**

Answer: D

The exterior angles of any convex polygon always add up to $360°$.

In a regular polygon all exterior angles are equal, so each one is:

$$\text{exterior angle} = \frac{360°}{n}$$

Here $n = 10$, so:
$\frac{360°}{10} = 36°$.
Watch out: $144°$ (option B) is the interior angle of a regular 10 sided polygon, because $180° - 36° = 144°$.

Textbook: Mathematics Grade 9, unit Regular Polygons (page 181).

3. The radian measure of an angle of 120° is equal to: (1)

A. ✓ **2π / 3**
B. π / 4
C. π / 2
D. π / 8

Answer: A

To change degrees into radians, multiply by $\frac{\pi}{180°}$, because $180° = \pi$ radians.

$$120° \times \frac{\pi}{180°} = \frac{120\pi}{180} = \frac{2\pi}{3}$$

So $120°$ equals $\frac{2\pi}{3}$ radians.

Quick sense check: $120°$ is two thirds of $180°$, so it must be two thirds of π.

Textbook: Mathematics Grade 10, Unit 4 Trigonometric Functions, section 4.1 Radian Measure of Angle (page 187).

4. What is the dot (scalar) product of the vectors u = i + 2j and v = -2i + 4j ? (1)

A. -8
B. 5
C. ✓ **6**
D. 7

Answer: C

For vectors $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ and $\mathbf{v} = c\mathbf{i} + d\mathbf{j}$, the dot (scalar) product is:

$$\mathbf{u} \cdot \mathbf{v} = ac + bd$$

Here $\mathbf{u} = \mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = -2\mathbf{i} + 4\mathbf{j}$.
Multiply the $\mathbf{i}$ parts: $(1)(-2) = -2$.
Multiply the $\mathbf{j}$ parts: $(2)(4) = 8$.
Add them: $-2 + 8 = 6$.

The dot product is 6. Remember the result is a number, not a vector.

Textbook: Mathematics Grade 11, Unit 5 Vectors (page 251).

5. If f(x) = x^2 + px + 1 where f(0) and f(1) have opposite signs so that f has a root in (0, 1), then which one of the following number is a possible value P? (1)

A. 3
B. 2
C. -2
D. ✓ **-3**

Answer: D

If a polynomial changes sign between two points, it must cross zero somewhere between them. So we need $f(0)$ and $f(1)$ to have opposite signs.

Compute the two values:
$f(0) = 0^2 + p(0) + 1 = 1$, which is always positive.
$f(1) = 1^2 + p(1) + 1 = p + 2$.

Since $f(0) > 0$, we need $f(1) < 0$:
$p + 2 < 0$, so $p < -2$.

Test the choices: only $p = -3$ satisfies $p < -2$.
Check: $f(1) = -3 + 2 = -1 < 0$, so the sign really changes on $(0, 1)$.

Watch out: $p = -2$ (option C) gives $f(1) = 0$, a root at $x = 1$ itself, not inside the open interval.

Textbook: Mathematics Grade 10, Unit 2 Polynomial Functions, section 2.4 Zeros of polynomial functions (page 90).

6. item Which one of the following quantities represents a vector? (1)

A. ✓ **Weight of an object**
B. Volume of a box
C. Speed of a motorbike
D. The width of your bedroom

Answer: A

A vector has both a size and a direction. A scalar has only a size.

Go through the choices:
Weight is the force of gravity on an object. A force has a size and a direction (downward), so weight is a vector.
Volume of a box is just an amount of space, with no direction.
Speed tells how fast something moves but not which way, so it is a scalar.
The width of a room is just a length.

So only the weight of an object is a vector.

Watch out: speed is a scalar; it becomes the vector velocity only when a direction is attached.

Textbook: Mathematics Grade 11, Unit 5 Vectors, section 5.1 Revision on Vectors and Scalars (page 252).

7. What is the surface area of a sphere with radius 6 cm ? (1)

A. ✓ **144π cm^2**
B. 48π cm^2
C. 36π cm^2
D. 24π cm^2

Answer: A

The surface area of a sphere with radius r is:

$$A = 4\pi r^2$$

Put in $r = 6$ cm:
$A = 4\pi(6)^2 = 4\pi \times 36 = 144\pi \text{ cm}^2$.

Watch out: 36π (option C) is only πr^2, the area of a flat circle of radius 6, not the whole sphere.

Textbook: Mathematics Grade 10, Unit 6 Solid Figures, section 6.2 Pyramids, cones and Spheres (page 277).

8. If the volume of a right circular cone is 64πcm^3 and the diameter of it base is 8 cm, what is the height of the cone? (1)

A. 3cm
B. 8cm
C. ✓ **12cm**
D. 4cm

Answer: C

The volume of a right circular cone is:

$$V = \frac{1}{3}\pi r^2 h$$

The diameter of the base is 8 cm, so the radius is $r = 8 \div 2 = 4$ cm.

Put the known values into the formula:
$64\pi = \frac{1}{3}\pi(4)^2 h$
$64\pi = \frac{16\pi}{3} h$
Multiply both sides by 3: $192\pi = 16\pi h$
Divide by 16π: $h = 12$ cm.

Watch out: forgetting the $\frac{1}{3}$ gives $h = 4$ cm (option D), and using the diameter 8 as the radius gives $h = 3$ cm (option A).

Textbook: Mathematics Grade 10, Unit 6 Solid Figures, section 6.2 Pyramids, cones and Spheres (page 277).

9. Which one of the following is the derivative of f(x) = tan(x) + 3^x? (1)

A. sec^(x) x + 3^(x)
B. -csc^(2) x + 3^(x) ln(x)
C. -tan(x) + 3^x ln(x)
D. ✓ **sec^(2) x + 3^(x) ln(x)**

Answer: D

Differentiate each term separately, using the sum rule.

The derivative of $\tan x$ is $\sec^2 x$.
The derivative of an exponential a^x is $a^x \ln a$, so the derivative of 3^x is $3^x \ln 3$.

Adding the two parts:

$$f'(x) = \sec^2 x + 3^x \ln 3$$

Option D is the one built from $\sec^2 x$ plus 3^x times a logarithm, so it is the intended answer. Note that the printed option shows $\ln(x)$, but the correct factor is $\ln 3$, the natural logarithm of the base 3.

Watch out: the derivative of 3^x is not $x \cdot 3^{x-1}$; the power rule only works when the exponent is a constant.

Textbook: Mathematics Grade 12, Unit 2 Introductions to Calculus, section 2.1 Introduction to Derivatives (page 63).

10. In rolling a fair die, what is the probability of obtaining 3 or 5 ? (1)

A. ✓ **1/3**

B. 1/4

C. 1/6

D. 1/2

Answer: A

A fair die has 6 equally likely outcomes: 1, 2, 3, 4, 5, 6.

The event "3 or 5" contains 2 of those outcomes.

$$P(3 \text{ or } 5) = \frac{\text{favourable outcomes}}{\text{all outcomes}} = \frac{2}{6} = \frac{1}{3}$$

Watch out: $\frac{1}{6}$ (option C) is the probability of getting exactly one named face, not of getting either of two faces.

Textbook: Mathematics Grade 11, Unit 8 Probability, section 8.7 Probability of an Event (page 453).

11. What is the locus all points in the plane in which the sum of the distances from two fixed points is constant? (1)

A. Circle

B. ✓ **Ellipse**

C. Parabola

D. Hyperbola

Answer: B

Each conic section has its own locus definition, and this one is the definition of an ellipse.

An ellipse is the set of all points where the **sum** of the distances to two fixed points (the foci) stays constant.

Compare with the others:
A circle keeps a constant distance from one fixed point (the centre).
A parabola keeps equal distance from one fixed point and a fixed line.
A hyperbola keeps a constant **difference** of distances from two fixed points.

So the answer is the ellipse.

Watch out: mixing up sum and difference is the classic slip; the difference gives a hyperbola, not an ellipse.

12. The simplified form of the expression (6√(20) - 3√(45)) / (3√(75)) is: (1)

A. √5

B. ✓ **√(15)/15**

C. 1/5

D. 5

Answer: B

First write each square root using its largest perfect square factor.

$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$
$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$
$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$

Substitute these into the expression:

$$\frac{6\sqrt{20} - 3\sqrt{45}}{3\sqrt{75}} = \frac{12\sqrt{5} - 9\sqrt{5}}{15\sqrt{3}} = \frac{3\sqrt{5}}{15\sqrt{3}} = \frac{\sqrt{5}}{5\sqrt{3}}$$

Now rationalize the denominator by multiplying top and bottom by $\sqrt{3}$:

$$\frac{\sqrt{5}}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{15}$$

Watch out: stopping at $\frac{\sqrt{5}}{5\sqrt{3}}$ matches none of the choices; the last rationalizing step is needed.

Textbook: Mathematics Grade 9, unit The Number System (page 17).

13. In ΔABC, if AB = 10 units, BC = 14 units and AC = 8 units, then the area of the triangle is (1)

A. 16√(2) square unit

B. ✓ **16√(6) square unit**

C. 16√(3) square unit

D. 4√(6) square unit

Answer: B

When all three sides of a triangle are known, use Heron's formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{a+b+c}{2}$$

Here $a = 10, b = 14, c = 8$.
$s = \dfrac{10+14+8}{2} = \dfrac{32}{2} = 16$

Substitute:
$s - a = 6, s - b = 2, s - c = 8$

$$\text{Area} = \sqrt{16 \times 6 \times 2 \times 8} = \sqrt{1536}$$

Simplify: $1536 = 256 \times 6$, so
$\sqrt{1536} = \sqrt{256}\,\sqrt{6} = 16\sqrt{6}$ square units.

Watch out: $4\sqrt{6}$ (option D) comes from taking $\sqrt{16}$ instead of $\sqrt{256}$ when simplifying.

14. Which one of the following sets is the symmetric difference, AΔB, of the sets A = {1,3,5,6,8,10} B = {2,5,8,11}? (1)

A. {1,3,6,10}

B. ✓ **{1,2,3,6,10,11}**

C. {2,11}

D. {1,2,3,5,6,8,10,11}

Answer: B

The symmetric difference $A \Delta B$ collects the elements that belong to exactly one of the two sets. In symbols, $A \Delta B = (A \cup B) \setminus (A \cap B)$.

Find each part:
Elements in A but not in B: $\{1, 3, 6, 10\}$
Elements in B but not in A: $\{2, 11\}$
The shared elements 5 and 8 are left out.

Join the two parts:
$A \Delta B = \{1, 2, 3, 6, 10, 11\}$

Watch out: option D is the full union $A \cup B$; the symmetric difference must drop the common elements 5 and 8.

Textbook: Mathematics Grade 9, unit Further on Sets (page 1).

15. The solution set of the equation given by log(x^2 - 3)=2 log(x — 1) is: (1)

A. {1/2}

B. ✓ **{2}**

C. {4}

D. ∅

Answer: B

Use the power rule of logarithms: $2\log(x-1) = \log(x-1)^2$.

The equation becomes:
$\log(x^2-3) = \log(x-1)^2$
Equal logs with the same base mean equal arguments:
$x^2-3 = (x-1)^2$

Expand and solve:
$x^2-3 = x^2-2x+1$
$-3 = -2x+1$
$2x = 4$, so $x = 2$.

Always check the domain of the original logarithms:
$x^2-3 = 4-3 = 1 > 0$ and $x-1 = 1 > 0$.
Both arguments are positive, so $x = 2$ is valid.

The solution set is $\{2\}$. The domain check matters: an answer that makes a log argument zero or negative would have to be thrown away.

Textbook: Mathematics Grade 10, Unit 3 Exponential and Logarithmic Functions, section 3.4 Solving Exponential and Logarithmic Equations (page 162).

16. What is the product of the two complex number z = 2 — 3i and w = 5 + 2i ? (1)

A. 29

B. ✓ **16 - 11i**

C. 4 - 11i

D. 16 + 11i

Answer: B

Multiply the two complex numbers the same way as two binomials, then use $i^2 = -1$.

$$(2-3i)(5+2i) = 2(5) + 2(2i) - 3i(5) - 3i(2i)$$

$= 10 + 4i - 15i - 6i^2$

Replace i^2 with -1:
$-6i^2 = -6(-1) = +6$

Collect the parts:
Real part: $10 + 6 = 16$
Imaginary part: $4i - 15i = -11i$
So $zw = 16 - 11i$.

Watch out: forgetting that $i^2 = -1$ leaves the real part as $10 - 6 = 4$ and gives option C.

17. Which one of the following is an onto functional from on to [0, ∞] ? (1)

A. f(x) = 2^x

B. f(x) = |x| + 2

C. f(x) = x^2 + 1

D. ✓ **f(x) = √(x^2)**

Answer: D

A function from $\mathbb{R}$ onto $[0, \infty)$ must produce **every** value from 0 upward. In other words, its range must be exactly $[0, \infty)$.

Check the range of each choice:
A: 2^x is always positive but never 0, so its range is $(0, \infty)$. It misses 0.
B: $|x| + 2$ is always at least 2, so its range is $[2, \infty)$.
C: $x^2 + 1$ is always at least 1, so its range is $[1, \infty)$.
D: $\sqrt{x^2} = |x|$ takes every value from 0 upward, so its range is $[0, \infty)$.

Only option D covers the whole codomain, so it is the onto function.

Watch out: 2^x gets very close to 0 but never reaches it, so "close to" is not enough for onto.

Textbook: Mathematics Grade 11, Unit 1 Relations and Functions, section 1.3 Types of Functions (page 22).

18. Which one of the following interval is the solution set of the inequality $x^2 - 5x + 6 \leq 0$? (1)

A. [0, 2]

B. ✓ **[2, 3)**

C. [∞, 3]

D. [-∞, 2]

Answer: B

Factor the quadratic first:

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

We need $(x - 2)(x - 3) \leq 0$, which happens when the two factors have opposite signs or when the product is 0.

Test the three regions:
For $x < 2$: both factors negative, product positive.
For $2 < x < 3$: first factor positive, second negative, product negative.
For $x > 3$: both factors positive, product positive.

The product is 0 at $x = 2$ and at $x = 3$, and both satisfy ≤ 0.
So the exact solution set is the closed interval $[2, 3]$.

Option B, printed as $[2, 3)$, is the only choice that names this interval; the other options are not solution sets at all. Note that $x = 3$ does satisfy the inequality, so the interval should be closed at 3.

Textbook: Mathematics Grade 9, unit Solving Inequalities (page 130).

19. What is the seventh term of an arithmetic sequence whose third term is -4 and common difference is 5 ? (1)

A. 15

B. -15

C. ✓ **16**

D. -16

Answer: C

In an arithmetic sequence, each term grows by the common difference d. To go from the 3rd term to the 7th term you take 4 steps of size d:

$$a_7 = a_3 + 4d$$

Substitute $a_3 = -4$ and $d = 5$:
$a_7 = -4 + 4(5) = -4 + 20 = 16$

You can also find the first term first: $a_3 = a_1 + 2d$ gives $a_1 = -4 - 10 = -14$, and then $a_7 = a_1 + 6d = -14 + 30 = 16$. Both routes agree.

Watch out: counting 7 steps instead of 6 from the first term (or 5 instead of 4 from the third) is the common slip; the number of steps is always one less than the term gap.

Textbook: Mathematics Grade 12, Unit 1 Sequences and Series, section 1.2 Arithmetic and Geometric Sequences (page 9).

20. Which one of the following trigonometric values is correct? (1)

A. tan (-120°) = -1

B. cos (-60°) = √(3) / 3

C. sin (-120°) = -0.5

D. ✓ **cos (-120°) = -0.5**

Answer: D

Use two facts: cosine is an even function, $\cos(-\theta) = \cos\theta$, and sine and tangent are odd, $\sin(-\theta) = -\sin\theta$, $\tan(-\theta) = -\tan\theta$.

Check each choice:
A: $\tan(-120^\circ) = -\tan(120^\circ) = -(-\sqrt{3}) = \sqrt{3}$, not -1. False.
B: $\cos(-60^\circ) = \cos(60^\circ) = \frac{1}{2}$, not $\frac{\sqrt{3}}{3}$. False.
C: $\sin(-120^\circ) = -\sin(120^\circ) = -\frac{\sqrt{3}}{2} \approx -0.87$, not -0.5. False.
D: $\cos(-120^\circ) = \cos(120^\circ) = -\cos(60^\circ) = -\frac{1}{2} = -0.5$. True.

So the correct statement is $\cos(-120^\circ) = -0.5$.

Watch out: 120° sits in the second quadrant, where cosine is negative and sine is positive; forgetting the quadrant signs is what makes A and C tempting.

Textbook: Mathematics Grade 10, Unit 4 Trigonometric Functions, section 4.2 Basic Trigonometric Function (page 194).

21. What are the coordinates of a point that divides the line segment with end points P(0, 1) and Q(5, 6) in the ratio 2 from P? (1)

A. (0,3)

B. (3,2)

C. (0,2)

D. ✓ **(2,3)**

Answer: D

A point that divides PQ in the ratio $m : n$ from P is found with the section formula:

$$R = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Here $P(0, 1)$, $Q(5, 6)$ and the ratio is $2 : 3$ from P (the stem drops the ": 3").

Compute each coordinate:
$x = \frac{2(5) + 3(0)}{2+3} = \frac{10}{5} = 2$
$y = \frac{2(6) + 3(1)}{2+3} = \frac{15}{5} = 3$

So the point is $(2, 3)$.

Check: the segment runs from $(0, 1)$ to $(5, 6)$, where $y = x + 1$. Of all the options, only $(2, 3)$ even lies on that segment.

Watch out: swapping m and n in the formula gives $(3, 4)$, a point closer to Q than to P, which is not offered.

Textbook: Mathematics Grade 10, Unit 7 Coordinate Geometry, section 7.2 Division of a line segment (page 334).

22. The distance from the point P (3, 4) to the line L with equation: 3x + 4y - 5 = 0 is (1)

A. 5 units

B. 6 units

C. ✓ **4 units**

D. 7 units

Answer: C

The distance from a point (x_0, y_0) to the line $Ax + By + C = 0$ is:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

From the line $3x + 4y - 5 = 0$: $A = 3, B = 4, C = -5$.

The point is $P(3, 4)$.

Substitute:
Numerator: $|3(3) + 4(4) - 5| = |9 + 16 - 5| = |20| = 20$
Denominator: $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$
$d = \frac{20}{5} = 4$ units.

Watch out: the sign of C must come with the equation written as $= 0$; using $+5$ instead of -5 gives 6 (option B).

Textbook: Mathematics Grade 10, Unit 7 Coordinate Geometry (page 327).

23. Which of the following is true about the trigonometric values of the given of angles? (1)

A. ✓ **sin (120°) = sin (60°)**

B. sin(75°) = cos(105°)

C. cos(120°) = cos(60°)

D. tan(75°) = tan(105°)

Answer: A

Use the supplementary angle identities: $\sin(180^\circ - \theta) = \sin\theta, \cos(180^\circ - \theta) = -\cos\theta, \tan(180^\circ - \theta) = -\tan\theta$.

Check each choice:
A: $\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin(60^\circ)$. True.
B: $\sin(75^\circ)$ is positive, but $\cos(105^\circ)$ is negative because 105° is in the second quadrant. False.
C: $\cos(120^\circ) = -\cos(60^\circ)$, so it cannot equal $\cos(60^\circ)$. False.
D: $\tan(105^\circ) = \tan(180^\circ - 75^\circ) = -\tan(75^\circ)$. False.

So the true statement is A.

Watch out: sine keeps its sign in the second quadrant but cosine and tangent flip theirs; option C fails for exactly that reason.

Textbook: Mathematics Grade 10, Unit 4 Trigonometric Functions (page 186).

24. A line segment chord in an ellipse which passes through the center and perpendicular to (1)

A. Latus rectum

B. Eccentricity

C. Minor axis

D. ✓ **Semi - major axis**

Answer: D

In an ellipse, the two axes are the two special chords through the centre.
The major axis is the longest chord, and the semi-major axis is its half from the centre.
The minor axis is the chord through the centre that is perpendicular to the major axis.

The sentence in the stem is incomplete, but it describes a chord through the centre that is perpendicular to something. The description fits the minor axis, which crosses the centre at right angles to the major axis (the direction of the semi-major axis).

With that reading, the blank names the direction the chord is perpendicular to, and the closest option is the semi-major axis, option D, which points along the major axis.

Note: the latus rectum is a chord through a **focus** perpendicular to the major axis, not through the centre, and eccentricity is a number that measures the flattening, not a line segment.

25. which of the following is true about bounded sequence? (1)

A. Every bounded sequence is convergent.

B. every increasing hounded sequence converges to the greatest lower bound of the sequence.

C. Every decrensing bounded sequence nconverges to the least upper bound of the sequence.

D. ✓ **Every monotone hounded sequence converges.**

Answer: D

The Monotone Convergence Theorem says: a sequence that is monotone (always increasing or always decreasing) and bounded must converge.

Check each statement:
A: False. Bounded alone is not enough. The sequence $a_n = (-1)^n$ stays between -1 and 1 but keeps jumping and never converges.
B: False. An increasing bounded sequence converges to its **least upper bound**, not its greatest lower bound (that would be at or below its first term).
C: False. A decreasing bounded sequence converges to its **greatest lower bound**, not its least upper bound.
D: True. Monotone plus bounded guarantees convergence; this is exactly the theorem.

So the correct statement is D.

Watch out: options B and C swap the two bounds; an increasing sequence climbs toward its ceiling, a decreasing one falls toward its floor.

Textbook: Mathematics Grade 12, Unit 1 Sequences and Series (page 1).

26. In order to find point P in space one can start from the origin O(0, 0, 0) move 5 units in the direction of negative x-axis, then move 5 units in the direction of positive Y-axis and finally move 5 units in the direction of negative z-axis. (1)

Which one of the following is the ordered triple of numbers represented by point P?

A. (5,5,5)
B. ✓ **(-5, 5, -5)**
C. (5, -5, 5)
D. (-5, -5, 5)

Answer: B

Each move changes one coordinate of the point, starting from the origin $O(0,0,0)$.

Follow the moves one by one:
Move 5 units along the negative x-axis: the point becomes $(-5,0,0)$.
Move 5 units along the positive y-axis: the point becomes $(-5,5,0)$.
Move 5 units along the negative z-axis: the point becomes $(-5,5,-5)$.

So $P=(-5,5,-5)$.

Watch out: a move in a negative direction must make that coordinate negative; option C $(5,-5,5)$ flips every sign the wrong way.

27. Which of the following is the variance of the data given as: 2, 3, 4, 5, 7 (1)

A. 2.46
B. ✓ **2.96**
C. 3.06
D. 3.70

Answer: B

The variance is the average of the squared distances from the mean.

Step 1, the mean:
$$\bar{x}=\frac{2+3+4+5+7}{5}=\frac{21}{5}=4.2$$
Step 2, the deviations and their squares:
$2-4.2=-2.2$, squared: 4.84
$3-4.2=-1.2$, squared: 1.44
$4-4.2=-0.2$, squared: 0.04
$5-4.2=0.8$, squared: 0.64
$7-4.2=2.8$, squared: 7.84

Step 3, average the squares:
Sum $=4.84+1.44+0.04+0.64+7.84=14.8$
$$\sigma^2=\frac{14.8}{5}=2.96$$
Watch out: dividing by $n-1=4$ gives 3.70 (option D), the sample variance; this question wants the ordinary variance of the data, divided by $n=5$.

Textbook: Mathematics Grade 12, Unit 3 Statistics, section 3.1 Measures of Absolute Dispersions (page 161).

28. Which of the following is true about the Arithmetic Mean of a given data? (1)

A. ✓ **It is affected by extreme values**

B. There can be two means for a given data

C. It can also be used for qualitative data

D. it can be obtained even in the absence of some of the values in the data

> **Answer:** A
>
> The arithmetic mean adds up every value and divides by how many there are, so every single value pulls on it.
>
> A: True. One very large or very small value drags the mean toward itself. For example, the mean of 1, 2, 3 is 2, but the mean of 1, 2, 30 is 11.
> B: False. A data set has exactly one mean, because the sum and the count are fixed.
> C: False. You cannot add or divide categories like colours or names; the mean needs numbers.
> D: False. The mean uses the sum of **all** the values, so a missing value makes it impossible to compute exactly.
>
> So the true statement is A: the mean is affected by extreme values.
>
> Watch out: it is the median, not the mean, that resists extreme values; the two are easy to mix up.
>
> **Textbook:** Mathematics Grade 11, Unit 7 Statistics, section 7.4 Measures of Central Tendency and Their Interpretation (page 371).

29. Which of the following pairs of events are depend? (1)

A. Any events obtaining from tossing a coin and rolling a die at the same time.

B. Drawing two balls successively from a box containing balls having the same size and shape with replacement.

C. ✓ **Drawing two cards one after the other from a well shuffled pack of cards without replacement.**

D. Tossing two coins simultaneously and obtaining head from one and tail from the other.

> **Answer:** C
>
> Two events are dependent when the result of the first changes the probability of the second.
>
> A: A coin toss and a die roll do not touch each other, so they are independent.
> B: With replacement, the box looks exactly the same before each draw, so the two draws are independent.
> C: Without replacement, the first card leaves the pack with 51 cards, and the chances for the second draw change. For example, after drawing an ace, the chance of a second ace falls from $\frac{4}{52}$ to $\frac{3}{51}$. These events are dependent.
> D: The two coins land separately; one coin does not influence the other.
>
> So the dependent pair is C: drawing two cards without replacement.
>
> Watch out: the phrase "with replacement" resets the experiment and keeps events independent; "without replacement" is the signal for dependence.
>
> **Textbook:** Mathematics Grade 11, Unit 8 Probability (page 413).

30. Let f and g be two functions. Which one of the following statements is true about f or g (1)

A. If fand g differentiable at a, then f/g is differentiable at a.

B. If f'(a) = 0, then the tangent line to the graph of f at (a, f (a)) is y = a

C. ✓ **If f differentiable at a, then it is continuous at a.**

D. The line y = 0 is the a tangent line graph of f(x) = |x| at (0, 0).

Answer: C

A key theorem of calculus: if a function is differentiable at a point, then it is continuous at that point. Having a slope at a point forces the graph to be unbroken there.

Check the other choices:
A: False. f/g also needs $g(a) \neq 0$; dividing by zero breaks the quotient even when both functions are differentiable.
B: False. If $f'(a) = 0$ the tangent line is horizontal, but its equation is $y = f(a)$, the height of the graph, not $y = a$.
D: False. The graph of $f(x) = |x|$ has a sharp corner at $(0, 0)$; the slopes from the left and right disagree (-1 and 1), so no tangent line exists there at all.

So the true statement is C.

Watch out: the converse of C is false; $|x|$ is continuous at 0 but not differentiable there, so continuity alone never guarantees a derivative.

Textbook: Mathematics Grade 12, Unit 2 Introductions to Calculus, section 2.1 Introduction to Derivatives (page 63).

— END OF MARKING KEY —