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Mathematics, 2017 EC, entrance exam, version 2

Grade 12 · Mathematics · 2017 E.C.

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Time allowed: 120 minutes **Questions:** 54 **Total marks:** 54

1. If $x < 0$, then, the simplest form of $f(x) = \frac{4x+10|x|}{2x}$ is equal to: (1)

A. 3
B. 7
C. ✓ **-3**
D. -7

Answer: C
Because $x < 0$, the absolute value opens as $|x| = -x$. The numerator becomes $4x + 10(-x) = -6x$, so $f(x) = \frac{-6x}{2x} = -3$. The x cancels safely because x is never zero in this problem. Option A is the trap: it comes from writing $|x| = x$, which holds only when x is positive.

2. For the greatest integer function $f(x) = \lfloor x \rfloor$, if $\lfloor x \rfloor^2 + 5\lfloor x \rfloor + 6 = 0$, what is the value of x? (1)

A. [-3,0)
B. [0,2)
C. ✓ **[-3,-1)**
D. [-1,0)

Answer: C
Write $n = \lfloor x \rfloor$, the greatest integer that is not larger than x. The equation becomes $n^2 + 5n + 6 = 0$, which factors as $(n+2)(n+3) = 0$, so $n = -2$ or $n = -3$. Now read each case back into x: $\lfloor x \rfloor = -2$ means $-2 \le x < -1$, and $\lfloor x \rfloor = -3$ means $-3 \le x < -2$. The two pieces sit side by side, so together they give $[-3, -1)$. Option A carries the same left endpoint and tempts for that reason, but x can never reach -1, let alone go up to 0.

3. Which of the following intervals is the solution set of the inequality $|2 - x| < 8$? (1)

A. (-10,6)
B. ✓ **(-6,10)**
C. (-8,8)
D. (-6,12)

Answer: B
An inequality of the form $|u| < 8$ means $-8 < u < 8$. With $u = 2 - x$ this is $-8 < 2 - x < 8$. Subtract 2 from all three parts: $-10 < -x < 6$. Now multiply by -1, which turns both inequality signs around: $-6 < x < 10$. Option A is exactly what you get if you forget to reverse the signs at that last step.

4. Which one of the following is false about the relation $R = \{(x, y)|x, y \in \Re, y \le -x^2 + 4$ and $y \ge 2x - 4$? (1)

A. Domain of R is [-4,2]
B. Range of R is [-12,4]
C. p(2,1) is a point on the graph of R^{-1}
D. ✓ **Range of R^{-1}[-4,4]**

Answer: D
The two boundary curves meet where $-x^2 + 4 = 2x - 4$, that is $x^2 + 2x - 8 = 0$, so $x = -4$ or $x = 2$. The shaded region only exists between those two x values, so the domain of R is $[-4, 2]$. An inverse relation swaps the coordinates of every point, so the range of R^{-1} is the domain of R, which is $[-4, 2]$ and not $[-4, 4]$. That makes D the false statement. Option B looks like the weak one, but the y values really do run from -12 at $x = -4$ up to 4 at the top of the parabola, so B is true.

5. Which of the following is equal to $f(x) = \sqrt{(x-1)^2}$, for every $x \in \mathbb{R}$? (1)

A. g(x) = x + 1

B. g(x) = x - 1

C. ✓ **g(x) = |x - 1|**

D. g(x) = |x| + 1

Answer: C

A square root sign always returns the non negative root, so $\sqrt{u^2} = |u|$ for every real u. Taking $u = x - 1$ gives $f(x) = |x-1|$. Option B fails as soon as $x < 1$. At $x = 0$, for example, $f(0) = \sqrt{(-1)^2} = 1$, while $x - 1 = -1$.

6. The domain and range of the function $f(x) = 2x^{\frac{2}{3}}$ are respectively (1)

A. ℝ and ℝ

B. [0,∞) and ℝ

C. ✓ **ℝ and [0,∞)**

D. [0,∞) and [0,∞)

Answer: C

Read the power as a root: $x^{\frac{2}{3}} = (\sqrt[3]{x})^2$. A cube root accepts every real number, negatives included, since $\sqrt[3]{-8} = -2$. So the domain is all of $\mathbb{R}$. Squaring afterwards can never produce a negative result, and $x = 0$ gives 0, so the outputs fill $[0, \infty)$. Option D is the common slip: it treats the fractional power like a plain square root and cuts away the negative inputs that a cube root handles without trouble.

7. If the point (a, b) is on the graph of $y = f(x)$, which point is on the graph of $y = f^{-1}(x)$? (1)

A. (1/a, b)

B. ✓ **(b, a)**

C. (-a, -b)

D. (a, 1/b)

Answer: B

The inverse undoes the function, so it sends outputs back to their inputs. If $f(a) = b$, then $f^{-1}(b) = a$, and that is the point (b, a). On a graph this is the reflection of (a, b) in the line $y = x$. Option D confuses the inverse function with the reciprocal $\frac{1}{f(x)}$, which is a different object built by dividing rather than by reversing.

8. What is the equation of the tangent line to the graph of $f(x) = x^2 - x + 2$ at $(1, 2)$? (1)

A. x - y - 3 = 0

B. x - y + 3 = 0

C. ✓ **x - y + 1 = 0**

D. x + y + 1 = 0

Answer: C

Differentiate to get the slope: $f'(x) = 2x - 1$, so $f'(1) = 1$. A line through $(1, 2)$ with slope 1 is $y - 2 = 1(x - 1)$, that is $y = x + 1$, which rearranges to $x - y + 1 = 0$. Option B has the correct slope but the wrong intercept. Its equation is $y = x + 3$, and putting $x = 1$ into it gives $y = 4$, so it misses the point $(1, 2)$ the tangent has to touch.

9. If $f(x) = x^2 - ax + b$ and $g(x) = x^3 + c$ have the same tangent line at $p(1, 3)$, then the value of b is equal to: (1)

A. ✓ **1**

B. -2

C. 0

D. 3

Answer: A

Sharing a tangent line at $P(1, 3)$ means two things at once: the same point and the same slope. The slopes are $f'(x) = 2x - a$ and $g'(x) = 3x^2$, so at $x = 1$ we need $2 - a = 3$, which gives $a = -1$. The point $(1, 3)$ lies on f, so $f(1) = 1 - a + b = 3$. Substituting $a = -1$ gives $1 + 1 + b = 3$, so $b = 1$. Option D is the trap: solving $2 - a = 3$ carelessly as $a = 1$ leads to $1 - 1 + b = 3$ and the wrong value $b = 3$.

10. If $f(x) = x^2 + x + 1$, then, which one of the following is the equation of the normal line to the graph of f at $x = 1$? (1)

A. x + 3y + 10 = 0

B. 3x + y + 10 = 0

C. x - 3y + 10 = 0

D. ✓ **x + 3y - 10 = 0**

Answer: D
First locate the point: $f(1) = 1 + 1 + 1 = 3$, so the graph passes through $(1, 3)$. The slope of the curve is $f'(x) = 2x + 1$, so $f'(1) = 3$. The normal line is perpendicular to the tangent, so its slope is $-\frac{1}{3}$. Then $y - 3 = -\frac{1}{3}(x - 1)$ becomes $3y - 9 = -x + 1$, that is $x + 3y - 10 = 0$. Option C uses the slope $\frac{1}{3}$, the reciprocal without the minus sign, so its line is not perpendicular to the tangent at all.

11. Let $f(x) = x^4 - 4x + 6$. For what value of x does the graph of f have a horizontal tangent line? (1)

A. 0

B. ✓ **1**

C. -1

D. 2

Answer: B
A horizontal tangent line has slope 0, so set the derivative to zero. Here $f'(x) = 4x^3 - 4$, and $4x^3 - 4 = 0$ gives $x^3 = 1$, so $x = 1$. Option C would be correct if the equation were $x^2 = 1$, but a cube keeps the sign of the number: $(-1)^3 = -1$, which is not 1.

12. Let l be a line that passes through the points $P(-1, 2)$ and $Q(5, -10)$ and it is tangent to the graph of $y = f(x)$ at $x = -1$, then, $f'(-1) =$ (1)

A. ✓ **-2**

B. -4

C. -5

D. 2

Answer: A
The line touches the curve at $x = -1$, so the derivative there equals the slope of that line. The slope through $P(-1, 2)$ and $Q(5, -10)$ is $\frac{-10-2}{5-(-1)} = \frac{-12}{6} = -2$, so $f'(-1) = -2$. Option B comes from mis-reading the horizontal run between the points as 3 instead of 6 and computing $\frac{-12}{3}$.

13. If $f(x) = 4x + 5$ and $(fog)(x) = x^2 + 2x + 5$, then, what is the value of $g(4)$? (1)

A. ✓ **6**

B. 4

C. 29

D. 21

Answer: A
Since $f(x) = 4x + 5$, feeding g into f gives $(f \circ g)(x) = 4g(x) + 5$. Set that equal to what the question states: $4g(x) + 5 = x^2 + 2x + 5$, so $4g(x) = x^2 + 2x$ and $g(x) = \frac{x^2+2x}{4}$. Then $g(4) = \frac{16+8}{4} = 6$. Option C, 29, is the value of $(f \circ g)(4) = 16 + 8 + 5$, which answers a different question: it is the composite at 4, not g at 4.

14. Given $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{2x}$, which one of the following is the domain of fog? (1)

A. {x∈ℝ: -1≤x≤2}

B. ✓ **{x∈ℝ: 0≤x≤2}**

C. {x∈ℝ: x≤0}

D. {x∈ℝ: x≥0}

Answer: B
For $(f \circ g)(x) = f(g(x))$ both steps have to be legal. The inner function $g(x) = \sqrt{2x}$ needs $2x \geq 0$, so $x \geq 0$. Then $f(g(x)) = \sqrt{4 - \left(\sqrt{2x}\right)^2} = \sqrt{4 - 2x}$, which needs $4 - 2x \geq 0$, so $x \leq 2$. Holding both conditions gives $0 \leq x \leq 2$. Option D keeps only the inner condition and forgets that the outer square root also has to stay non negative, which fails at $x = 3$.

15. If $f(x) = 3 + \frac{1}{e^x}$, then, $f^{-1}(e^2 + 3) =$ (1)

A. 1/e

B. 2

C. ✓ **-2**

D. 1

Answer: C

The question asks which input x makes $f(x) = e^2 + 3$. Solve $3 + e^{-x} = e^2 + 3$, which reduces to $e^{-x} = e^2$. Equal bases force equal exponents, so $-x = 2$ and $x = -2$. Option B is the trap: it reads the exponent 2 straight off the right hand side and misses the minus sign sitting in e^{-x}.

16. Let $f(x) = \frac{1-3x}{x-2}$. Then, what is the range of $f(x)$? (1)

A. ℝ\{2}

B. ℝ

C. ✓ **ℝ\{-3}**

D. ℝ\{1/3}

Answer: C

Find which y values are reachable by solving for x. From $y = \frac{1-3x}{x-2}$ we get $y(x-2) = 1 - 3x$, then $yx + 3x = 1 + 2y$, so $x(y+3) = 1 + 2y$ and $x = \frac{1+2y}{y+3}$. Every y works except $y = -3$, where the denominator becomes zero and no input exists. So the range is $\mathbb{R} \setminus \{-3\}$, which is the horizontal asymptote of the graph. Option A confuses range with domain: $x = 2$ is the value the input cannot take.

17. If f is the greatest integer function and g is an absolute value function, then, what is the value of $(fog)\left(\frac{7}{2}\right) + (gof)\left(-\frac{10}{3}\right)$? (1)

A. 6

B. ✓ **7**

C. 0

D. 8

Answer: B

Work from the inside out. First, $(f \circ g)\left(\frac{7}{2}\right) = \lfloor |3.5| \rfloor = \lfloor 3.5 \rfloor = 3$. Second, $\lfloor -\frac{10}{3} \rfloor = \lfloor -3.33\ldots \rfloor = -4$, because the greatest integer function steps down the number line, not toward zero. The absolute value then gives $|-4| = 4$. Adding, $3 + 4 = 7$. Option A, 6, is what you get by rounding -3.33 up to -3, which is the one place this function surprises students.

18. Let $f(x) = x^{\frac{1}{m}}$ and $g(x) = x^{\frac{1}{n}}$ be power functions where m and n are even natural numbers and $m > n$. Which one of the following is true about f and g ? (1)

A. If 0≤x≤1, f(x)≤g(x)

B. If x≥1, f(x)≥g(x)

C. ✓ **If 0≤x≤1, f(x)≥g(x)**

D. The domain of f and g is ℝ

Answer: C

Test the claim with numbers first. Take $m = 4, n = 2$ and $x = \frac{1}{16}$. Then $f(x) = \left(\frac{1}{16}\right)^{\frac{1}{4}} = \frac{1}{2}$ and $g(x) = \left(\frac{1}{16}\right)^{\frac{1}{2}} = \frac{1}{4}$, so $f(x) \geq g(x)$. The reason is general: for $0 \leq x \leq 1$, a bigger root pulls the value closer to 1, and $m > n$ means $\frac{1}{m} < \frac{1}{n}$. Option A states the reverse inequality on the same interval, and the numbers above show it fails.

19. Let $f(x) = \frac{x-1}{x+1}$ and $g(x) = \sqrt{3 - 2x}$ be two functions. Which one of the following is false about the combination values of f and g at $x = -3$? (1)

A. (f g)(-3) = 6

B. (f/g)(-3) = 2/3

C. (f + g)(-3) = 5

D. ✓ **(f - g)(-3) = -5**

Answer: D

Evaluate each function at $x = -3$. For the first, $f(-3) = \frac{-3-1}{-3+1} = \frac{-4}{-2} = 2$. For the second, $g(-3) = \sqrt{3 - 2(-3)} = \sqrt{9} = 3$. Then $(f - g)(-3) = 2 - 3 = -1$, not -5, so D is the false statement. Option C looks suspicious because a 5 next to a subtraction feels like a sign error, but $2 + 3 = 5$ is correct, so C is true.

20. If $x < 0$ and $f(x) = |x|$, then, $x - f(f(x))$ is equal to (1)

A. 0

B. ✓ **2x**

C. -2x

D. x + x^2

Answer: B

Apply f twice. Since $x < 0$, the first step gives $f(x) = |x| = -x$, which is a positive number. Applying the absolute value to a positive number changes nothing, so $f(f(x)) = -x$ as well. Then $x - f(f(x)) = x - (-x) = 2x$. Option C is the trap: it has the right size but the wrong sign, and it appears when the two minus signs are collapsed into one.

21. Which one of the following is the sum of the series $\sum_{n=1}^{\infty}\left(\frac{3^n+4^n}{6^n}\right)$? (1)

A. 0

B. 5/4

C. ✓ **3**

D. ∞

Answer: C

Split the fraction into two familiar pieces: $\frac{3^n+4^n}{6^n} = \left(\frac{1}{2}\right)^n + \left(\frac{2}{3}\right)^n$. Each piece is a geometric series that starts at $n = 1$, so use $\frac{\text{first term}}{1-r}$. The first gives $\frac{1/2}{1-1/2} = 1$ and the second gives $\frac{2/3}{1-2/3} = 2$. The total is $1 + 2 = 3$. Option D is the guess that growing numerators must make a series diverge, but 6^n grows faster than both 3^n and 4^n, so both ratios stay below 1.

22. If $2^k \times 2^{k^2} \times 2^{k^3} \times \ldots \times 2^{k^n} \times \ldots = \frac{1}{8}$, then, $k =$ (1)

A. 2/3

B. 1

C. -3/4

D. ✓ **3/2**

Answer: D

Multiplying powers of the same base adds the exponents, so the left side is $2^{k+k^2+k^3+\cdots}$. Since $\frac{1}{8} = 2^{-3}$, the exponents must match: $k + k^2 + k^3 + \cdots = -3$. That exponent sum is a geometric series with first term k and ratio k, so $\frac{k}{1-k} = -3$. Cross multiplying gives $k = -3 + 3k$, so $2k = 3$ and $k = \frac{3}{2}$. Option C is what a single sign slip produces: using $\frac{k}{1+k} = -3$ instead leads to $k = -\frac{3}{4}$.

23. If $\sum_{n=1}^{k}\left(\frac{n-13}{2}\right) = -25$, then, the value of $k =$ (1)

A. 3

B. 4

C. 7

D. ✓ **20**

Answer: D

Pull the constant out of the sum: $\frac{1}{2}\sum_{n=1}^{k}(n-13) = \frac{1}{2}\left(\frac{k(k+1)}{2} - 13k\right)$, because -13 is subtracted once in each of the k terms. Setting this equal to -25 gives $k(k+1) - 26k = -100$, that is $k^2 - 25k + 100 = 0$, which factors as $(k-5)(k-20) = 0$. The roots are $k = 5$ and $k = 20$, and only 20 appears among the choices, so D is the answer. Option B is worth testing to see why it fails: the first four terms add to $-6 - \frac{11}{2} - 5 - \frac{9}{2} = -21$, not -25.

24. Let $G_n = r^n$ be a geometric progression with a common ratio r. which one of the following is false about the sequence G_n ? (1)

A. ✓ **If |r|<1, as n→∞ G_n converges to 1**

B. If r≤-1 as n→∞ G_n is vibrant

C. If r=1 as n→∞ G_n→1

D. If r>1 as n→∞ G_n→∞

Answer: A

When $|r| < 1$, each new power of r is smaller in size than the one before, so r^n shrinks toward 0, not toward 1. Take $r = \frac{1}{2}$: the terms run $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ and head straight to 0. That makes A the false statement. Option C can look like the odd one out because it also ends in 1, but $1^n = 1$ for every n, so that sequence genuinely sits at 1 for ever.

25. What is the simplified form of $\frac{a^{-4}-b^{-4}}{a^{-2}+b^{-2}}$ (1)

A. ✓ **(b^2 - a^2)/(ab)^2**

B. (ab/(b-a))^2

C. ((b-a)/(ab))^2

D. (ab)^2/(b^2 - a^2)

Answer: A
Clear the negative powers by multiplying the top and the bottom by a^4b^4. The top becomes $b^4 - a^4$, and the bottom becomes $a^2b^4 + a^4b^2 = a^2b^2\left(a^2+b^2\right)$. Factor the top as a difference of two squares: $b^4 - a^4 = \left(b^2-a^2\right)\left(b^2+a^2\right)$. The factor a^2+b^2 now cancels, leaving $\frac{b^2-a^2}{a^2b^2} = \frac{b^2-a^2}{(ab)^2}$. Option D is this same expression turned upside down, which is what happens when the division is done in the wrong order.

26. Which one of the following is true about the function $f(x) = \frac{x^3-x}{x^2-x}$? (1)

A. f has a vertical asymptote

B. ✓ **x=-1 is zero of f**

C. f has a horizontal asymptote

D. x=1 is zero of f

Answer: B
Factor both parts: $\frac{x^3-x}{x^2-x} = \frac{x(x-1)(x+1)}{x(x-1)}$. The common factors cancel, so $f(x) = x+1$ at every input except $x=0$ and $x=1$, where the original denominator is zero and the graph has holes. A zero of f needs $x+1=0$, so $x=-1$, and -1 is a legal input. That makes B true. Option D fails on a detail that matters here: $x+1$ is not zero at $x=1$, and $x=1$ is not even in the domain.

27. If $p(x) = 3x^2$ and $q(x) = x^2 + x$, then, what is the solution set of $\frac{p(x)}{3q(x)} = \frac{1}{x} - \frac{1}{q(x)}$ (1)

A. {-1,0}

B. ✓ **{1}**

C. {0,1}

D. {}

Answer: B
Simplify each side before solving. The left side is $\frac{3x^2}{3(x^2+x)} = \frac{x^2}{x(x+1)} = \frac{x}{x+1}$. The right side is $\frac{1}{x} - \frac{1}{x(x+1)} = \frac{(x+1)-1}{x(x+1)} = \frac{x}{x(x+1)} = \frac{1}{x+1}$. So $\frac{x}{x+1} = \frac{1}{x+1}$, which gives $x=1$. Option C adds $x=0$, but $x=0$ makes $\frac{1}{x}$ undefined, so it can never belong to the solution set.

28. The value(s) of x where the graph of the function $y = \frac{x^3-1}{x^4+1}$ crosses its horizontal asymptote is(are): (1)

A. ✓ **x=1**

B. x=-1 and x=1

C. x=0

D. x=-1

Answer: A
The numerator has degree 3 and the denominator degree 4, so for large $|x|$ the fraction shrinks toward 0 and the horizontal asymptote is $y=0$. The graph meets that line exactly where the function equals 0, which needs $x^3-1=0$, so $x=1$. Option B adds $x=-1$, but $(-1)^3-1=-2$, not 0. A cube does not lose the minus sign the way a square does.

29. If $\frac{1-x}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ then, $A+B+C=$ (1)

A. ✓ **-1**

B. 0

C. 1

D. -2

Answer: A
Multiply both sides by $x^3+x = x\left(x^2+1\right)$ to clear the fractions: $1-x = A\left(x^2+1\right) + (Bx+C)x$. Expanding the right side gives $(A+B)x^2 + Cx + A$. Now match it against $1-x = 0x^2 - x + 1$ term by term. The constant gives $A=1$, the x term gives $C=-1$, and the x^2 term gives $A+B=0$, so $B=-1$. Then $A+B+C = 1-1-1 = -1$. Option B tempts because it looks as if $A+B+C$ should equal the sum of the coefficients on the left, $1+(-1)=0$, but partial fractions do not work that way.

30. Which one of the following functions has no vertical asymptote? (1)

A. f(x)=(x^2-1)/(x^2-x)

B. ✓ **f(x)=x/(x^3+x)**

C. f(x)=4/(3x+1)

D. f(x)=x^{-3}

Answer: B

Factor first, because a factor that cancels leaves a hole rather than an asymptote. Here $\frac{x}{x^3+x} = \frac{x}{x(x^2+1)} = \frac{1}{x^2+1}$ for $x \neq 0$. The surviving denominator x^2+1 is never zero, so the graph has no vertical asymptote, only a hole at $x=0$. Option A looks like the same situation because a factor cancels there too: $\frac{x^2-1}{x^2-x} = \frac{x+1}{x}$. The difference is that x is still in the denominator afterwards, so $x=0$ is a real asymptote.

31. Which of the following is false about the graph of $f(x) = \frac{x^2-5x+6}{x^2-4}$? (1)

A. The graph has a hole at x=2

B. ✓ **The vertical asymptotes of the graph are x=2 and x=-2**

C. The horizontal asymptote of the graph is y=1

D. The graph has y-intercept at (0, -3/2)

Answer: B

Factor the top and the bottom: $\frac{x^2-5x+6}{x^2-4} = \frac{(x-2)(x-3)}{(x-2)(x+2)}$. The factor $x-2$ cancels, so $x=2$ gives a hole, not an asymptote, and the only vertical asymptote is $x=-2$. B claims two vertical asymptotes, so B is the false statement. Option C can look wrong at a glance, but the top and bottom have the same degree with leading coefficient 1 each, so the horizontal asymptote really is $y=1$.

32. Let A and B be 3×3 matrices such that A = [[2,0,0],[1,5,0],[0,-1,1/2]] and |B| = 1/2. Which of the following is equal to |AB^{-1}| ? (1)

A. 1

B. 4

C. ✓ **10**

D. 5/2

Answer: C

Matrix A is triangular, with zeros above the main diagonal, so its determinant is the product of the diagonal entries: $2 \times 5 \times \frac{1}{2} = 5$. For the inverse, $\left|B^{-1}\right| = \frac{1}{|B|} = \frac{1}{1/2} = 2$. Determinants multiply across a product, so $\left|AB^{-1}\right| = 5 \times 2 = 10$. Option D is the trap: it multiplies by $|B|$ instead of dividing by it, giving $5 \times \frac{1}{2}$.

33. If A is 3×3 matrix with |A| = 1/2, then, |2 adj A| = (1)

A. 1

B. ✓ **2**

C. 4

D. 8

Answer: B

Two rules do the work. For an $n \times n$ matrix, $|\operatorname{adj} A| = |A|^{n-1}$, so with $n=3$ we get $|\operatorname{adj} A| = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Multiplying a 3×3 matrix by 2 scales all three rows, so the determinant is multiplied by $2^3 = 8$. Putting them together, $|2 \operatorname{adj} A| = 8 \times \frac{1}{4} = 2$. Option C comes from using $|\operatorname{adj} A| = |A| = \frac{1}{2}$, which drops the power $n-1$, and then $8 \times \frac{1}{2} = 4$.

34. Let A and B be 3×3 matrices with |A| = m, and |B| = 1/n, m≠0, n≠0. What is the value of det(A^T B^{-1})? (1)

A. m/n

B. ✓ **mn**

C. n/m

D. 2mn

Answer: B

Determinants split across a product: $\det\left(A^T B^{-1}\right) = \left|A^T\right| \times \left|B^{-1}\right|$. Transposing never changes a determinant, so $\left|A^T\right| = |A| = m$. Inverting turns the determinant upside down, so $\left|B^{-1}\right| = \frac{1}{|B|} = \frac{1}{1/n} = n$. The product is mn. Option A is the trap: it uses $|B|$ itself, $\frac{1}{n}$, and so leaves the inverse out of the working.

35. If A = [[3,1],[5,2]] then, A^{-1} is equal to (1)

A. ✓ **[[2,-1],[-5,3]]**

B. [[-2,1],[5,-3]]

C. [[2,1],[5,3]]

D. [[-2,-1],[-5,-3]]

Answer: A

For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the inverse is $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. Here $ad - bc = 3(2) - 1(5) = 1$, so no fraction appears. Swapping the diagonal entries and changing the signs of the other two gives $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$. You can check it by multiplying: $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Option B changes the signs of the diagonal entries instead of the off diagonal ones, and multiplying it out gives $-I$, not I.

36. Let A = [[2, x+y],[6,4]] and B = [[2,8],[6, x-y]]. If A = B, then x^2 - y^2 = (1)

A. ✓ **32**

B. 36

C. 4

D. 48

Answer: A

Two matrices are equal when every matching entry is equal. The top right entries give $x + y = 8$, and the bottom right entries give $x - y = 4$. The expression asked for is a difference of two squares, so it factors into exactly those two pieces: $x^2 - y^2 = (x+y)(x-y) = 8 \times 4 = 32$. There is no need to find x and y separately. Option B, 36, is x^2 on its own: it is where you stop if you solve for $x = 6$ and forget to subtract $y^2 = 4$.

37. What is the sum of the series $\sum_{n=3}^{101}(-1)^n\left(\frac{1}{n}+\frac{1}{n+1}\right)$? (1)

A. 35/101

B. 35/101

C. 35/101

D. ✓ **35/102**

Answer: D

Write out what each term contributes and watch the cancelling. Term n gives $(-1)^n\frac{1}{n}$ and $(-1)^n\frac{1}{n+1}$. The second piece of term n and the first piece of term $n+1$ are the same size with opposite signs, so they cancel in pairs all the way along. Only the first piece of $n = 3$ and the second piece of $n = 101$ survive: $\frac{(-1)^3}{3} + \frac{(-1)^{101}}{102} = -\frac{1}{3} - \frac{1}{102} = -\frac{35}{102}$. D is the option carrying $\frac{35}{102}$, and the printed choices have lost the minus sign. The repeated $\frac{35}{101}$ is the answer you reach by stopping the second piece at $n = 101$ rather than at $n + 1 = 102$.

38. If the fourth term of an arithmetic sequence is -1 and the tenth term is 35, what is the sum of the first 20 terms? (1)

A. 1520

B. 1420

C. 860

D. ✓ **760**

Answer: D

Six steps separate the fourth term from the tenth, and they cover $35 - (-1) = 36$, so the common difference is $d = 6$. Step back three places from a_4 to reach the start: $a_1 = -1 - 3(6) = -19$. Now use $S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$: $S_{20} = 10\left(-38 + 19 \times 6\right) = 10(76) = 760$. Option A, 1520, is exactly double the right answer, which is what happens when the $\frac{n}{2}$ in the formula is written as n.

39. If the sum of the first n-terms of an arithmetic progression is S_n = n + 2n^2, which of the following is the 21th term (a_21) of the sequence? (1)

A. 903

B. 820

C. ✓ **83**

D. 63

Answer: C

A single term is the difference of two running totals: $a_{21} = S_{21} - S_{20}$. Compute $S_{21} = 21 + 2(21)^2 = 21 + 882 = 903$ and $S_{20} = 20 + 2(20)^2 = 20 + 800 = 820$. Subtracting, $a_{21} = 903 - 820 = 83$. Option A, 903, is the trap: it is the sum of the first 21 terms, which is what the formula gives directly, not the 21st term on its own.

40. If x, x+2, and x+6 are consecutive terms of a geometric progression, then the value of x is equal to (1)

A. 1
B. ✓ **2**
C. 3
D. 4

Answer: B
In a geometric progression the ratio between neighbours stays the same, so $\frac{x+2}{x} = \frac{x+6}{x+2}$. Cross multiplying gives $(x+2)^2 = x(x+6)$, that is $x^2 + 4x + 4 = x^2 + 6x$. The x^2 terms cancel, leaving $4 = 2x$, so $x = 2$. Check it: the terms are 2, 4, 8, with ratio 2 each time. Option D gives 4, 6, 10, where the first ratio is 1.5 and the second is about 1.67, so those terms are not geometric.

41. Which one of the following is a false statement? (1)

A. Σ(a_i - b_i)= Σa_i - Σb_i
B. ✓ **Σ(a_i × b_i)= Σa_i × Σb_i**
C. Σ(c a_i)= c Σa_i
D. Σ(i=1 to n) c = n c

Answer: B
Summation shares out over addition and subtraction, and a constant multiplier can be pulled outside the sign, which is why A, C and D are true. Multiplication of two varying terms is different: $\sum a_i b_i$ is not the same as $\left(\sum a_i\right)\left(\sum b_i\right)$. Test it with two terms, $a = (1, 2)$ and $b = (3, 4)$: the left side is $1(3) + 2(4) = 11$, while the right side is $3 \times 7 = 21$. Option C has a similar shape and tempts for that reason, but a constant is identical in every term, so it factors out honestly.

42. If a, b, c, d are the four geometric means between 1/3 and 81, what is the common ratio of the geometric progression 1/3, a, b, c, d, 81? (1)

A. √3
B. 1/3
C. ✓ **3**
D. 9

Answer: C
Four means placed between $\frac{1}{3}$ and 81 make a progression of six terms, and six terms have five gaps. So $\frac{1}{3}r^5 = 81$, which gives $r^5 = 243$. Since $3^5 = 243$, the ratio is $r = 3$. Option A, $\sqrt{3}$, would be the ratio if there were ten gaps instead of five, because $\left(\sqrt{3}\right)^{10} = 243$.

43. If {a_n} is an arithmetic sequence whose third term and the tenth term are 3 and 24 respectively, then, the 50th term of the sequence, a_50, is equal to (1)

A. 156
B. 150
C. ✓ **144**
D. 141

Answer: C
Seven steps separate the third term from the tenth, covering $24 - 3 = 21$, so $d = 3$. Back up two steps to the first term: $a_1 = 3 - 2(3) = -3$. Then $a_{50} = a_1 + 49d = -3 + 49(3) = -3 + 147 = 144$. Option B, 150, is what you get by treating the given $a_3 = 3$ as if it were a_1, since $3 + 49(3) = 150$.

44. A single square is made from 4 matchsticks. To make two squares in a row takes 7 Matchsticks, while three squares in a row takes 10 matchsticks. How many matchsticks are in a row of 90 squares? (1)

A. 211
B. ✓ **271**
C. 291
D. 298

Answer: B
The first square needs 4 sticks, and every square after it needs only 3, because it shares one side with the square before it. So a row of n squares takes $4 + 3(n - 1) = 3n + 1$ sticks. The numbers in the question confirm the rule: $n = 2$ gives 7 and $n = 3$ gives 10. For 90 squares, $3(90) + 1 = 271$. Option D, 298, is the kind of answer over counting produces: charging 4 sticks for every square ignores the shared sides and inflates the total.

45. $\sum_{i=1}^{80}(\log_3^{i+1}) =$ (1)

A. 1/81 **B.** ✓ **4**

C. -4 **D.** 3

Answer: B
Each term splits into a difference of logarithms: $\log_3 \frac{i+1}{i} = \log_3(i+1) - \log_3 i$. Writing the sum out, the $-\log_3 2$ of one term kills the $+\log_3 2$ of the term before it, and the same happens all the way along. Only the very last piece and the very first survive: $\log_3 81 - \log_3 1$. Since $3^4 = 81$ and $\log_3 1 = 0$, the sum is 4. Option C has the right size with the wrong sign, which is what appears if the difference is written the other way round.

46. The fifth and tenth terms of a geometric sequence are 640 and 20 respectively, then, the common ratio of the sequence is equal to: (1)

A. 2 **B.** ✓ **1/2**

C. -1/2 **D.** -2

Answer: B
Five steps of the ratio separate the fifth term from the tenth, so $a_{10} = a_5 r^5$. That gives $r^5 = \frac{20}{640} = \frac{1}{32}$, and since $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$, the ratio is $\frac{1}{2}$. Option C, $-\frac{1}{2}$, is ruled out by the sign: raised to the odd power 5 it gives $-\frac{1}{32}$, and the terms of the sequence would flip between positive and negative, which 640 and 20 do not.

47. Which one of the following is the fractional form of the recurring decimal 0.23? (1)

A. 23/100 **B.** 23/90

C. ✓ **23/99** **D.** 23/900

Answer: C
The repeating block is 23, which is two digits long, so multiply by 10^2. Let $x = 0.232323\ldots$, then $100x = 23.232323\ldots$. Subtracting the first line from the second wipes out the whole repeating tail: $99x = 23$, so $x = \frac{23}{99}$. The rule behind it is that a block of two repeating digits sits over 99. Option A is the value of the terminating decimal 0.23, which stops after two digits instead of repeating for ever.

48. If Σ_{n=1}^∞ a_n is a convergent series such that Σ_{n=1}^∞ (a_n - 3/4^n) = 12, then Σ_{n=1}^∞ a_n is equal to: (1)

A. 11 **B.** 112

C. ✓ **13** **D.** 14

Answer: C
A convergent series can be split apart, so $\sum\left(a_n - \frac{3}{4^n}\right) = \sum a_n - \sum \frac{3}{4^n}$. The subtracted series is geometric with first term $\frac{3}{4}$ and ratio $\frac{1}{4}$, so it sums to $\frac{3/4}{1-1/4} = 1$. That leaves $\sum a_n - 1 = 12$, so $\sum a_n = 13$. Option A, 11, is the same working with the final step reversed: it subtracts the 1 instead of adding it back.

49. A certain meeting hall has n rows of seats and 120 total numbers of seats. There are 3 seats in the first row, 5 seats in the second row, 7 seats in the third row, and so on. How many seats are there in the last row (nth row) of the hall? (1)

A. ✓ **21** **B.** 23

C. 32 **D.** 41

Answer: A
The row sizes 3, 5, 7 climb by 2 each time, so they form an arithmetic sequence with $a_1 = 3$ and $d = 2$, giving $a_n = 2n + 1$. The seats in n rows total $S_n = \frac{n}{2}(3 + 2n + 1) = n(n+2)$. Set that equal to 120: $n^2 + 2n - 120 = 0$, which factors as $(n-10)(n+12) = 0$, so $n = 10$ rows. The last row then holds $a_{10} = 2(10) + 1 = 21$ seats. Option B, 23, is the size an eleventh row would have, and this hall has only ten.

50. If f(x) = √π, then f'(4) is equal to: (1)

A. 1/4
B. 1/2
C. ✓ **0**
D. 1/(2√π)

Answer: C
$\sqrt{\pi}$ is a fixed number, roughly 1.77. So f returns the same output for every input, its graph is a horizontal line, and its slope is 0 everywhere, including at $x = 4$. Option D applies the rule for $\sqrt{x}$ to a constant, but there is no x anywhere in this function to differentiate.

51. Let f(x) = 6x/(x+a). For what value of a is f'(2a)=2? (1)

A. ✓ **1/3**
B. 2/3
C. 3/2
D. 3

Answer: A
Use the quotient rule: $f'(x) = \frac{6(x+a)-6x}{(x+a)^2} = \frac{6a}{(x+a)^2}$. Now substitute $x = 2a$, so the denominator is $(2a + a)^2 = 9a^2$ and $f'(2a) = \frac{6a}{9a^2} = \frac{2}{3a}$. Setting $\frac{2}{3a} = 2$ gives $3a = 1$, so $a = \frac{1}{3}$. The mistake to watch for is losing the square in the denominator, which turns the derivative into $\frac{6a}{3a} = 2$ for every a and so picks out no answer at all. Option D, 3, is the reciprocal of the correct value and catches anyone who inverts only one side of the final equation.

52. If h(x) = f(x)/g(x) + f(x)g(x), f(1)=3, f'(1)=-2, g'(1)=4, g(1)=2, then h'(1) is equal to: (1)

A. ✓ **4**
B. -10.4
C. 5
D. 16

Answer: A
Differentiate the two pieces with their own rules and add. Quotient rule: $\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$, which at $x = 1$ is $\frac{(-2)(2)-(3)(4)}{2^2} = \frac{-16}{4} = -4$. Product rule: $(fg)' = f'g + fg'$, which at $x = 1$ is $(-2)(2) + (3)(4) = 8$. Adding, $h'(1) = -4 + 8 = 4$. Option D, 16, is what the product rule applied to both terms gives, $8 + 8$, so it loses the negative contribution the quotient actually makes.

53. If g(x) = f(x^2+4x), f(-3)=6, f'(-3)=-5, then g'(-1) = (1)

A. -2
B. ✓ **-10**
C. -30
D. -5

Answer: B
This is a chain rule question. With $g(x) = f\left(x^2 + 4x\right)$, the derivative is $g'(x) = f'\left(x^2 + 4x\right) \times (2x + 4)$. At $x = -1$ the inside evaluates to $(-1)^2 + 4(-1) = -3$, which is exactly where f' is given. So $g'(-1) = f'(-3) \times (2(-1) + 4) = (-5)(2) = -10$. Option D, -5, is the trap: it stops at $f'(-3)$ and forgets to multiply by the derivative of the inside function.

54. If f(x) = 4 + |x+6| for all x, then what is the value of f'(x) at x = -6? (1)

A. -1
B. 4
C. 1
D. ✓ **does not exist**

Answer: D
The graph of $|x + 6|$ is a V with its corner at $x = -6$, and adding 4 lifts the whole V without moving that corner. Approaching from the left the slope is -1, and from the right it is $+1$. The two one sided slopes disagree, so no single tangent line exists there and the derivative does not exist at $x = -6$. Option B, 4, mixes up two different values: $f(-6) = 4$ is the height of the graph at that point, not its slope.

— END OF MARKING KEY —