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# Mathematics, 2005 EC, entrance exam

Grade 12 · Mathematics · 2005 E.C.

MARKING KEY — CONFIDENTIAL

**Time allowed:** 120 minutes **Questions:** 60 **Total marks:** 60

**1.** If $\{a_n\}$ is a sequence such that $a_1 = 2$, and $a_{n+1} = a_n + 4$ for all $n \geq 1$, then $\sum_{n=1}^{35} a_n$ is equal to : (1)

**A.** 2460 **B.** 2458
**C.** ✓ **2450** **D.** 2442

**Answer:** C
The rule $a_{n+1} = a_n + 4$ means every term is 4 more than the one before it, so this is an arithmetic sequence with $a_1 = 2$ and $d = 4$. From the 1st term to the 35th there are 34 steps, so $a_{35} = 2 + 34(4) = 138$. The sum of an arithmetic sequence is $S_n = \frac{n}{2}(a_1 + a_n)$, so $S_{35} = \frac{35}{2}(2 + 138) = 35 \times 70 = 2450$. The common slip is to count 35 steps and write $a_{35} = 142$, which gives a total of 2520. A total that is not on the list is your signal to recount the steps.

**2.** If $f(x) = x^2 + 2\ln x$, then $\lim_{h\to 0} \dfrac{f(2+h) - f(2)}{h}$ ? (1)

**A.** ✓ **5** **B.** 4
**C.** 2 **D.** 0

**Answer:** A
That limit is the definition of the derivative of $f$ at $x = 2$, so find $f'$ and put 2 into it. For $f(x) = x^2 + 2\ln x$, the derivative is $f'(x) = 2x + \frac{2}{x}$. At $x = 2$ this gives $4 + 1 = 5$. Option B, 4, is what you get if you differentiate only the $x^2$ part and forget that $2\ln x$ contributes $\frac{2}{x}$.

**3.** If $f(x) = e^{2x} + x - 3\cos x$, then what is $f''(x)$? (1)

**A.** $e^{2x} + 1 - 3\sin(x)$ **B.** $e^{2x} + 1 + 3\sin(x)$
**C.** $4e^{2x} - 3\cos(x)$ **D.** ✓ $4e^{2x} + 3\cos(x)$

**Answer:** D
Differentiate twice. The first derivative is $f'(x) = 2e^{2x} + 1 + 3\sin x$: the chain rule pulls a factor of 2 out of $e^{2x}$, and the derivative of $-3\cos x$ is $+3\sin x$. Differentiating again, the constant 1 disappears and $3\sin x$ becomes $3\cos x$, so $f''(x) = 4e^{2x} + 3\cos x$. Option C keeps a minus sign in front of the cosine. That minus sign was already used up at the first step, when $-3\cos x$ turned into $+3\sin x$.

**4.** Which one of the following intervals does $f(x) = x^4 + 4x$ increase? (1)

**A.** $(-\infty, -1]$ **B.** $(-\infty, 0]$
**C.** ✓ $[-1, \infty)$ **D.** $(-\infty, \infty)$

**Answer:** C
A function increases on the interval where its derivative is not negative. Here $f'(x) = 4x^3 + 4 = 4(x^3 + 1)$, and $x^3 + 1 \geq 0$ exactly when $x^3 \geq -1$, which means $x \geq -1$. So $f$ increases on $[-1, \infty)$. Option A is the mirror image of the answer: on $(-\infty, -1]$ the derivative is negative, so the function is falling there.

**5.** Which one of the following is the simplest form of $|3+4i| - \dfrac{25i}{3+4i}$ (1)

**A.** 5 ? 5i

**B.** 5 + 5i

**C.** 1 + 3i

**D.** ✓ **1 � 3i**

**Answer:** D

Handle the two pieces separately. The modulus is $|3+4i| = \sqrt{9+16} = 5$. For the fraction, multiply top and bottom by the conjugate $3-4i$: $\frac{25i(3-4i)}{(3+4i)(3-4i)} = \frac{75i-100i^2}{25} = \frac{100+75i}{25} = 4+3i$. So the expression is $5-(4+3i) = 1-3i$. Option C, $1+3i$, comes from subtracting the real part but leaving the imaginary part with the sign it had before the subtraction.

**6.** If $Z = \cos(\pi/10) + i\sin(\pi/10)$, then what is the value of $Z^5$? (1)

**A.** $\pi/2 + \pi/2\, i$

**B.** $1/2 + 1/2\, i$

**C.** ✓ $i$

**D.** $1+i$

**Answer:** C

The number is already in the form $\cos\theta + i\sin\theta$ with $\theta = \frac{\pi}{10}$, so use de Moivre's rule: raising to the power 5 multiplies the angle by 5. That gives $Z^5 = \cos\frac{5\pi}{10} + i\sin\frac{5\pi}{10} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$. Since $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$, the value is $i$. Option B comes from raising the two numbers $\cos\frac{\pi}{10}$ and $\sin\frac{\pi}{10}$ to the power 5 one by one, instead of multiplying the angle by 5.

**7.** What is the value of $|x| + 2x$ if $x \leq 0$ ? (1)

**A.** $-3x$

**B.** $3x$

**C.** �x

**D.** ✓ $x$

**Answer:** D

The sign of $x$ decides what $|x|$ means. When $x \leq 0$ the number is negative or zero, so $|x| = -x$. Then $|x| + 2x = -x + 2x = x$. Option B, $3x$, comes from writing $|x| = x$, which holds only when $x$ is positive or zero.

**8.** Suppose $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ if x is a 2 x 2 matrix such that $AX - A^T = 2A$, then what is the value of x? (1)

**A.** $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

**B.** $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$

**C.** $\begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$

**D.** ✓ $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

**Answer:** D

Notice that $A$ is symmetric: swapping its rows and columns gives the same matrix back, so $A^T = A$. The equation $AX - A^T = 2A$ then becomes $AX = 2A + A = 3A$. Since $\det A = 1(3) - 2(2) = -1$, which is not zero, $A$ has an inverse, and multiplying both sides by it leaves $X = 3I = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$. Option C is $3A$ itself, which is the right hand side of the equation rather than the matrix that solves it.

**9.** What is the $50^{th}$ term of the sequence 3, 10, 17, 24, 31,�? (1)

**A.** 310

**B.** ✓ **346**

**C.** 510

**D.** 531

**Answer:** B

The differences are $10-3=7$ and $17-10=7$, so this is arithmetic with $a_1 = 3$ and $d = 7$. Use $a_n = a_1 + (n-1)d$ : $a_{50} = 3 + 49(7) = 3 + 343 = 346$. Every term here is 3 more than a multiple of 7, and that gives a quick check on the options. Option A, 310, fails it, because $310 - 3 = 307$ is not a multiple of 7, so 310 can never appear in this sequence.

**10.** For real members x and y, which one of the following statements is true? (1)

**A.** ✓ $(\forall x)(\exists y)(x^2 + y + 1 = 0)$

**B.** $(\exists x)(\forall y)(x^2 + y + 1 = 0)$

**C.** $(\exists y)(\forall x)(x^2 + y + 1 = 0)$

**D.** $(\forall y)(\exists x)(x^2 + y + 1 = 0)$

**Answer:** A

Read the quantifiers from left to right. Option A says that for every $x$ you can find some $y$ making $x^2 + y + 1 = 0$. Rearranging, $y = -x^2 - 1$, which is a real number for every real $x$, so A is true. Option C says one single $y$ works for every $x$ at the same time, and that cannot happen, because $-x^2 - 1$ changes as $x$ changes.

**11.** Which one of the following functions has NO vertical asymptote? (1)

**A.** $f(x) = \ln(x+1)$

**B.** $f(x) = \dfrac{x^2+1}{x^3+8}$

**C.** ✓ $f(x) = \dfrac{x^2-9}{x-3}$

**D.** $f(x) = \dfrac{x-1}{x^2-x}$

**Answer:** C

A vertical asymptote appears where the denominator is zero and the fraction cannot be reduced. In option C, $\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x + 3$ for every $x \neq 3$, so the graph is a straight line with one point missing. A missing point like that is called a hole, and it is not an asymptote. Option D looks similar, but $\frac{x-1}{x^2-x} = \frac{x-1}{x(x-1)} = \frac{1}{x}$, and the $x$ in the denominator survives, so D does have a vertical asymptote at $x = 0$.

**12.** Let p, q and r be propositions such that $p \Rightarrow (r \vee \neg q)$ is false. Then, which one of the following propositions is true? (1)

**A.** $p \Rightarrow r$

**B.** ✓ $\neg r \Rightarrow q$

**C.** $\neg q \Rightarrow q$

**D.** $q \Leftrightarrow r$

**Answer:** B

**13.** If $f(x) = \dfrac{1}{e^x + 1}$, then which one of the following is equal to $f^{-1}(x)$ for $0 < x < 1$? (1)

**A.** ✓ $\ln(1-x) - \ln(x)$

**B.** $e^{-x} + 1$

**C.** $\ln(1/x + 1)$

**D.** $1/e^{-x} + 1$

**Answer:** A

Set $y = \frac{1}{e^x+1}$ and make $x$ the subject. Turning both sides upside down gives $e^x + 1 = \frac{1}{y}$, so $e^x = \frac{1-y}{y}$. Taking natural logs, $x = \ln(1-y) - \ln y$, and renaming the letter gives $f^{-1}(x) = \ln(1-x) - \ln x$. Option C is close to a correct form, but the inside would have to be $\frac{1}{x} - 1$: the 1 crosses the equals sign, so its sign changes.

**14.** If $x^2 - 6x + y^2 + k = 0$ is equation of a circle with radius 2, then what is the value of k? (1)

**A.** 13

**B.** ✓ **5**

**C.** 4

**D.** -4

**Answer:** B

Complete the square in $x$. Since $x^2 - 6x = (x-3)^2 - 9$, the equation becomes $(x-3)^2 + y^2 = 9 - k$. For a circle of radius 2 the right hand side must equal $2^2 = 4$, so $9 - k = 4$ and $k = 5$. Option C, 4, is the value of $r^2$. That is what the right hand side has to become, not the constant $k$ sitting in the original equation.

**15.** If a line with angle of inclination of $3\pi/4$ passes through (0, 1), which one of the following is the equation of the line? (1)

**A.** ✓ $y = -x + 1$

**B.** $y = x + 1$

**C.** $y = -x - 1$

**D.** $y = x - 1$

> **Answer:** A
> The angle of inclination gives the slope through $m = \tan\theta$, so here $m = \tan\frac{3\pi}{4} = -1$. The line passes through $(0, 1)$, so its $y$ intercept is 1 and the equation is $y = -x + 1$. Option B has slope $+1$, which is $\tan\frac{\pi}{4}$. That is the first quadrant angle, while $\frac{3\pi}{4}$ leans the other way and gives a negative slope.

**16.** If $g(x) = xf(x) - \sqrt{f(x)}$ and $f(2) = f'(2) = 4$, then which of the following is equal to $g'(2)$? (1)

**A.** ✓ **11**

**B.** 8

**C.** 2

**D.** 0

> **Answer:** A
> Differentiate the two parts with the right rules. The product rule gives $\frac{d}{dx}[xf(x)] = f(x) + xf'(x)$, and the chain rule gives $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$. Putting $x = 2$ with $f(2) = f'(2) = 4$: $g'(2) = 4 + 2(4) - \frac{4}{2\sqrt{4}} = 4 + 8 - 1 = 11$. Option B, 8, is only the $xf'(x)$ piece. It leaves out the $f(x)$ the product rule adds and the square root term as well.

**17.** What is the sum of the series $\sum_{n=1}^{\infty} \dfrac{2^{2n+1}}{5^{n-1}}$ (1)

**A.** ✓ **40**

**B.** 20

**C.** 10

**D.** 8

> **Answer:** A
> Find the first term and the ratio. At $n = 1$ the term is $\frac{2^3}{5^0} = 8$. Dividing any term by the one before it gives $\frac{4}{5}$, because the top gains a factor of $2^2$ and the bottom gains a factor of 5. Since $|r| < 1$ the series converges, and $S = \frac{a}{1-r} = \frac{8}{1-\frac{4}{5}} = \frac{8}{\frac{1}{5}} = 40$. Option D, 8, is the first term on its own rather than the sum of all of them.

**18.** What is the value of $\lim_{x\to\infty}\left(1 + \dfrac{1}{x}\right)^{\frac{-x}{2}}$ ? (1)

**A.** ✓ $\dfrac{1}{\sqrt{e}}$

**B.** $\sqrt{e}$

**C.** $e^{-2}$

**D.** $\infty$

> **Answer:** A
> Start from the standard limit $\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e$. The exponent here is $-\frac{x}{2}$, which is $x$ multiplied by $-\frac{1}{2}$, so the expression is $\left[\left(1 + \frac{1}{x}\right)^x\right]^{-\frac{1}{2}}$ and the limit is $e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$. Option C, $e^{-2}$, comes from reading the exponent as $-2x$ instead of $-\frac{x}{2}$.

**19.** Among students who took a quiz, 15 students scored 6, 20 students scored 7, 10 students scored 8 and 5 students scored 10. What is the average score of the students? (1)

**A.** 7.8

**B.** 7.5

**C.** ✓ **7.2**

**D.** 7.0

> **Answer:** C
> This is a weighted mean, so multiply each score by the number of students who got it before dividing. The total of all the scores is $15(6) + 20(7) + 10(8) + 5(10) = 90 + 140 + 80 + 50 = 360$, and the number of students is $15 + 20 + 10 + 5 = 50$. The average is $\frac{360}{50} = 7.2$. Option A, 7.8, is the plain average of 6, 7, 8 and 10, which treats the four scores as equally common when far more students scored 7 than scored 10.

**20.** A parabola with focus (3, -1) has directrix $y = 3$. Which one of the following is the equation of the parabola? (1)

**A.** $(x-3)^2 = -4(y+1)$

**B.** ✓ $(x-3)^2 = -8(y-1)$

**C.** $(x-3)^2 = 4(y+1)$

**D.** $(x-3)^2 = 8(y-1)$

**Answer:** B

The vertex sits halfway between the focus and the directrix, so it is at $(3, 1)$, and the distance from the vertex to the focus is $p = 2$. The focus lies below the directrix, so the parabola opens downward and takes the form $(x-h)^2 = -4p(y-k)$. With $h = 3$, $k = 1$ and $p = 2$ this gives $(x-3)^2 = -8(y-1)$. Option A uses $-4$ where $-4p$ belongs, which would place the focus only one unit from the vertex.

**21.** How many four digit even numbers can be formed from 1, 2, 3, 4 and 5 if the numbers start with 3? (1)

**A.** 40

**B.** ✓ **50**

**C.** 100

**D.** 120

**Answer:** B

The first digit is fixed as 3, so there is 1 choice there. The last digit has to be even, and the only even digits available are 2 and 4, giving 2 choices. Nothing in the question forbids repeating a digit, so each of the two middle places has all 5 digits to choose from. The count is $1 \times 5 \times 5 \times 2 = 50$. Option D, 120, is $5 \times 4 \times 3 \times 2$, the number of four digit arrangements with no repeated digit, which ignores both the fixed 3 and the even ending.

**22.** A satellite moves along a hyperbolic curve whose horizontal transverse axis is 24 km and an asymptote $y = 5/12\ x + 2$. Then, what is the eccentricity of the hyperbola? (1)

**A.** 5/13

**B.** 12/13

**C.** ✓ **13/12**

**D.** 15/3

**Answer:** C

A horizontal transverse axis of 24 km means $2a = 24$, so $a = 12$. The asymptotes of such a hyperbola have slopes $\pm\frac{b}{a}$, and the given slope is $\frac{5}{12}$, so $b = 5$. For a hyperbola $c^2 = a^2 + b^2 = 144 + 25 = 169$, so $c = 13$ and the eccentricity is $e = \frac{c}{a} = \frac{13}{12}$. Option B, $\frac{12}{13}$, is that fraction upside down, and it can be ruled out on sight: the eccentricity of a hyperbola is always greater than 1.

**23.** What is an anti-derivative of $f(x) = \dfrac{2}{4x^2 + 4x + 1}$? (1)

**A.** $\dfrac{1}{2x+1}$

**B.** $\dfrac{-2}{2x+1}$

**C.** ✓ $-\dfrac{1}{2x+1}$

**D.** $\ln(4x^2\ \ 4x + 1)$

**Answer:** C

The denominator is a perfect square, since $4x^2 + 4x + 1 = (2x+1)^2$, so $f(x) = 2(2x+1)^{-2}$. Integrating with the power rule and the chain rule, $\int 2(2x+1)^{-2}\,dx = 2 \cdot \frac{(2x+1)^{-1}}{(-1)(2)} = -\frac{1}{2x+1}$. You can confirm it by differentiating $-\frac{1}{2x+1}$, which returns $\frac{2}{(2x+1)^2}$. Option A is the same size with the wrong sign, and differentiating it gives $-\frac{2}{(2x+1)^2}$.

**24.** At which valeu(s) of x does $f(x) = 0.25x^4 - 2x^2$ have a local maximum? (1)

**A.** $x = 4$

**B.** ✓ $x = 0$

**C.** $x = -2$ and $x = 2$

**D.** $x = 0$ and $x = 2$

**Answer:** B

First find where the derivative is zero: $f'(x) = x^3 - 4x = x(x-2)(x+2)$, so the candidates are $x = -2$, $x = 0$ and $x = 2$. Now test each with the second derivative $f''(x) = 3x^2 - 4$. At $x = 0$ it gives $-4$, which is negative, so the curve bends downward there and $x = 0$ is a local maximum. At $x = \pm 2$ it gives $8$, which is positive, so those two are local minima, and that is why option C is wrong.

**25.** In the set of complex numbers, which one of the following is the solution set of $Z^3 - iZ^2 + 2Z = 0$? (1)

**A.** $\{0\}$

**B.** $\{0, -i\}$

**C.** ✓ $\{0, -i, 2i\}$

**D.** $\{0, i, -2i\}$

**Answer:** C

Take out the common factor: $Z(Z^2 - iZ + 2) = 0$, so either $Z = 0$ or $Z^2 - iZ + 2 = 0$. The quadratic formula on the second factor gives $Z = \frac{i \pm \sqrt{i^2 - 8}}{2} = \frac{i \pm \sqrt{-9}}{2} = \frac{i \pm 3i}{2}$, which is $2i$ or $-i$. So the solution set is $\{0, -i, 2i\}$. Option D has both signs turned around, and you can test it: putting $Z = i$ into the equation gives $-i + i + 2i = 2i$, which is not zero.

**26.** The population of certain country is currently 80 million with growth rate of 2% per year. Given: $(0.02)^9 = 5.12 \times 10^{-16}$, $(1.02)^9 = 1.19$, $(0.02)^{10} = 1.024 \times 10^{-17}$, $(1.02)^{10} = 1.22$, which one of the following is the best approximation of the population after 10 years? (1)

*[Figure: A boxed 'Given:' panel sits between the first and second sentences of the stem, holding four values laid out in two rows of two columns. Left column: $(0.02)^9 = 5.12 \times 10^{-16}$ on the first row and $(0.02)^{10} = 1.024 \times 10^{-17}$ on the second row, each followed by a comma. Right column: $(1.02)^9 = 1.19$ on the first row and $(1.02)^{10} = 1.22$ on the second row. All four values are reproduced verbatim in the stem.]*

**A.** 81.9 million

**B.** 86.8 million

**C.** 95.2 million

**D.** ✓ **97.6 million**

**Answer:** D

Growth of 2% a year multiplies the population by 1.02 every year, so after 10 years it is $80 \times (1.02)^{10}$ million. The table hands you $(1.02)^{10} = 1.22$, so the population is $80 \times 1.22 = 97.6$ million. The powers of 0.02 in the table are there to distract you: 2% growth is a factor of 1.02, not 0.02. Option C, 95.2, uses $(1.02)^9 = 1.19$, which is the population after 9 years, one year short of what was asked.

**27.** The volume V of a melting ice cube after t seconds is $V = 2000 - 40t + 0.2t^2$ (in $\text{cm}^3$). How fast is the volume changing when t=40 seconds? (1)

**A.** $24 \text{ cm}^3/\text{sec}$

**B.** $15 \text{ cm}^3/\text{sec}$

**C.** $-15 \text{ cm}^3/\text{sec}$

**D.** ✓ $-24 \text{ cm}^3/\text{sec}$

**Answer:** D

How fast the volume is changing is the derivative of $V$ with respect to $t$. Differentiating $V = 2000 - 40t + 0.2t^2$ gives $\frac{dV}{dt} = -40 + 0.4t$. At $t = 40$ seconds this is $-40 + 16 = -24 \text{ cm}^3/\text{sec}$. Option A has the right size but no minus sign, and the cube is melting, so its volume has to be falling and the answer has to be negative.

**28.** Which one of the following is equal to $\lim_{n\to\infty} \dfrac{1 - n - 3n^2}{6n^2 + 1}$? (1)

**A.** 1/6

**B.** ✓ **-1/2**

**C.** -1/6

**D.** $-\infty$

**Answer:** B

When $n$ grows without bound only the highest power of $n$ matters, so divide the top and the bottom by $n^2$. That turns the fraction into $\frac{\frac{1}{n^2} - \frac{1}{n} - 3}{6 + \frac{1}{n^2}}$. Every term with $n$ in the denominator goes to 0, leaving $\frac{-3}{6} = -\frac{1}{2}$. Option A, $\frac{1}{6}$, comes from comparing the constant terms 1 and 6 instead of the $n^2$ terms, and those constants are the parts that stop mattering.

**29.** What is the solution set of $\dfrac{2}{x} - \dfrac{x-2}{x^2-2x} = 1 - \dfrac{2x-2}{3x-2}$ ? (1)

**A.** $\{1, -2\}$
**B.** $\{1, 2\}$
**C.** $\{-1\}$
**D.** ✓ $\{1\}$

**Answer:** D
Simplify each side before solving. On the left, $\frac{x-2}{x^2-2x} = \frac{x-2}{x(x-2)} = \frac{1}{x}$, so the left side is $\frac{2}{x} - \frac{1}{x} = \frac{1}{x}$. On the right, $1 - \frac{2x-2}{3x-2} = \frac{3x-2-2x+2}{3x-2} = \frac{x}{3x-2}$. Cross multiplying gives $3x-2 = x^2$, so $x^2 - 3x + 2 = 0$ and $x = 1$ or $x = 2$. Option B keeps both, but $x = 2$ makes $x^2 - 2x$ zero in the original equation, so it is outside the domain and the solution set is $\{1\}$.

**30.** Let $f(x) = \begin{cases} 3^x + k, & \text{if } x \le 0 \\ 3\dfrac{\sin(2x)}{x}, & \text{if } x > 0 \end{cases}$ if f is continuous at x=0, then what is the value of k? (1)

**A.** 6
**B.** ✓ **5**
**C.** 2
**D.** 0

**Answer:** B
For $f$ to be continuous at 0 the two one sided limits must agree. From the right, $\lim_{x\to0^+} \frac{3\sin 2x}{x} = 3 \times 2 = 6$, using $\lim_{u\to0} \frac{\sin u}{u} = 1$ with $u = 2x$. From the left, $\lim_{x\to0^-}(3^x + k) = 3^0 + k = 1 + k$. Setting $1 + k = 6$ gives $k = 5$. Option A, 6, forgets that $3^0 = 1$ and treats the left hand value as $k$ on its own.

**31.** Which one of the following is equal to $\int (1+x)3^x\,dx$ ? (1)

**A.** $(1+x)3^x - 3x + c$
**B.** $(1+x)3^x + (\log_3 e)3^x + c$
**C.** ✓ $(1+x)3^x \log_3 e - (\log_3 e)^2 3^x + c$
**D.** $(1+x)3^x \log_3 e - 3^x(\log_3 e) + c$

**Answer:** C
Integrate by parts with $u = 1 + x$ and $dv = 3^x\,dx$. Since $\int 3^x\,dx = \frac{3^x}{\ln 3} = 3^x \log_3 e$, we have $v = 3^x \log_3 e$. The formula then gives $\int(1+x)3^x\,dx = (1+x)3^x \log_3 e - \int 3^x \log_3 e\,dx$, and the remaining integral brings in a second factor, ending at $(1+x)3^x \log_3 e - (\log_3 e)^2 3^x + c$. Option D carries only one factor of $\log_3 e$ in the second term, but $\int 3^x\,dx$ supplies one of its own on top of the one already outside.

**32.** A committee consisting of 3 students is to be selected from 10 candidates among which 4 are girls. What is the probability that at least one girl is selected? (1)

**A.** ✓ **5/6**
**B.** 2/3
**C.** 1/3
**D.** 1/6

**Answer:** A
It is easier to count the opposite case first. There are 6 boys, so the committees with no girl at all number $\binom{6}{3} = 20$, out of $\binom{10}{3} = 120$ possible committees. That makes $P(\text{no girl}) = \frac{20}{120} = \frac{1}{6}$, so $P(\text{at least one girl}) = 1 - \frac{1}{6} = \frac{5}{6}$. Option D, $\frac{1}{6}$, is the middle step, the chance of getting no girl at all, and the question asks for the opposite of it.

**33.** A group of six students take their seats at random in a round table for a discussion. What is the probability that two specific students do NOT sit together? (1)

**A.** ✓ **3/5**
**B.** 2/3
**C.** 2/5
**D.** 1/3

**Answer:** A
Around a round table only the positions relative to each other count, so 6 students can be seated in $(6-1)! = 120$ ways. Now tie the two named students together as a single block: that leaves 5 items to arrange in $(5-1)! = 24$ ways, and the pair can swap places inside the block, giving $24 \times 2 = 48$ seatings with them together. So $P(\text{together}) = \frac{48}{120} = \frac{2}{5}$, and the answer is $1 - \frac{2}{5} = \frac{3}{5}$. Option C, $\frac{2}{5}$, is the chance that they do sit together, which is what the word NOT in the question turns around.

**34.** The mark that students scored in an examination is grouped in class intervals as shown in the following table. What is the median of the mark? (1)

*[Figure: A two-column frequency table sits between the two sentences of the stem. Header row: 'Class Interval (Mark)' and 'Number of Students'. Rows, in order: 55 - 64 with 8 students; 65 - 74 with 12 students; 75 - 84 with 20 students; 85 - 94 with 6 students; 95 - 100 with 4 students.]*

**A.** 25.0

**B.** 75.5

**C.** ✓ **77.0**

**D.** 79.5

**Answer:** C

There are $8+12+20+6+4=50$ students, so the median sits at position 25. Counting up, 8 students are below 65 and 20 are below 75, so the 25th student falls inside the class 75 to 84. Use $\text{median} = L + \frac{\frac{n}{2}-cf}{f} \times w$ with lower boundary $L = 74.5, cf = 20, f = 20$ and width $w = 10$: $74.5 + \frac{5}{20}(10) = 74.5 + 2.5 = 77$. Option A, 25.0, is the position of the median student in the list, not the mark that student scored.

**35.** A box seen below is to have a square base, an open top and volume of 32 cubic units. If x is the length of each side of its base and y is its height, how many units should x and y be in order to make the box with the smallest amount of material? (1)

*[Figure: A scanned line drawing of a rectangular box in three-quarter perspective with a square base and no lid. A short horizontal leader line at the top right carries a downward-pointing arrow into the open mouth of the box and is labelled 'Open - top'. The vertical left edge of the box is labelled y and the bottom front edge is labelled x. The front and right faces are shaded/stippled; the top rim is drawn as an open rectangle.]*

**A.** ✓ $x = 4,\ y = 2$

**B.** $x = 2,\ y = 8$

**C.** $x = \sqrt{8},\ y = 4$

**D.** $x = \sqrt{2},\ y = 16$

**Answer:** A

The volume fixes one variable in terms of the other: $x^2y = 32$, so $y = \frac{32}{x^2}$. With no lid, the material is the base plus the four sides, $S = x^2 + 4xy = x^2 + \frac{128}{x}$. Setting $S' = 2x - \frac{128}{x^2} = 0$ gives $x^3 = 64$, so $x = 4$ and $y = \frac{32}{16} = 2$. Option B describes a box that also holds 32 cubic units, but it needs $2^2 + \frac{128}{2} = 68$ square units of material against 48 for the answer, so it is a possible box and not the cheapest one.

**36.** Which one of the following is equal to $\int \frac{1}{x^2+x}\,dx$ ? (1)

**A.** $\ln|x^2+x| + c$

**B.** $2\ln|x+1| + \ln|x| + c$

**C.** ✓ $\ln|x| - \ln|x+1| + c$

**D.** $\ln|x| + \ln|x+1| + c$

**Answer:** C

Factor the denominator and split the fraction: $\frac{1}{x^2+x} = \frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$, which you can check by putting the right side back over a common denominator. Integrating the two pieces gives $\ln|x| - \ln|x+1| + c$. Option A treats $x^2+x$ as a single block and writes $\ln|x^2+x|$. The rule $\int \frac{1}{u}\,du = \ln|u|$ needs the top to be the derivative of the bottom, and here the derivative of $x^2+x$ is $2x+1$, not 1.

**37.** Consider the following argument: �If he does not love her, she will not marry him. He loves her. Therefore, she will marry him.� If �p? he loves her and q ? she will marry him, which one of the following is the correct representation of the argument and its validty? (1)

**A.** $\neg p \Rightarrow \neg q,\ p \vdash q$: valid argument

**B.** ✓ $\neg p \Rightarrow \neg q,\ p \vdash q$**; invalid argument**

**C.** $p \Rightarrow q,\ p \vdash q$; valid argument

**D.** $p \Rightarrow q,\ p \vdash q$; valid argument

**Answer:** B

With $p$ for 'he loves her' and $q$ for 'she will marry him', the first premise is $\neg p \Rightarrow \neg q$, the second is $p$, and the conclusion drawn is $q$. That shape is called denying the antecedent: knowing $p$ is true tells you nothing about a conditional whose starting point is $\neg p$. A counterexample settles it. If he loves her but she still refuses, then $\neg p \Rightarrow \neg q$ is true because its starting point is false, $p$ is true, and yet $q$ is false. So the symbols are right but the argument is invalid, and that is the one difference between B and A.

**38.** What is the value of $\int_0^{\frac{\pi}{2}} 2x \cos x \, dx$ ? (1)

**A.** ✓ $\pi - 2$

**B.** $\pi/2 + 1$

**C.** $\pi + 2$

**D.** $\pi/2 - 1$

**Answer:** A

Integrate by parts with $u = 2x$ and $dv = \cos x \, dx$, so $du = 2\, dx$ and $v = \sin x$. Then $\int_0^{\frac{\pi}{2}} 2x \cos x \, dx = [2x \sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2 \sin x \, dx = \pi + [2 \cos x]_0^{\frac{\pi}{2}} = \pi + (0 - 2) = \pi - 2$. Option D, $\frac{\pi}{2} - 1$, is exactly half of the answer, which is what you get by leaving the factor of 2 out of the working.

**39.** A box contains 5 white, 6 red and 4 black balls of all identical size. If 3 balls are randomly taken out of the box after the other, what is the probability that the first ball is white and both the second and third balls are red? (1)

**A.** 2/15

**B.** 3/15

**C.** 4/75

**D.** ✓ **5/91**

**Answer:** D

The balls are not put back, so the pool shrinks after every draw. At the start there are 15 balls, so $P(\text{white first}) = \frac{5}{15}$. Then 14 balls remain, 6 of them red, so $P(\text{red second}) = \frac{6}{14}$, and after that 13 remain with 5 red, so $P(\text{red third}) = \frac{5}{13}$. Multiplying, $\frac{5}{15} \times \frac{6}{14} \times \frac{5}{13} = \frac{150}{2730} = \frac{5}{91}$. Option C, $\frac{4}{75}$, keeps 15 balls in the box for every draw, which would be right only if each ball were returned before the next one was taken.

**40.** Which one of the following is the equation of the line tangent to the graph of $f(x) = 1/(x+1) + \cos x$ at $(0, f(0))$? (1)

**A.** $x + y = 1$

**B.** $x - y = -2$

**C.** ✓ $x + y = 2$

**D.** $x + 4y = 2$

**Answer:** C

A tangent line needs a point and a slope. At $x = 0$, $f(0) = \frac{1}{0+1} + \cos 0 = 1 + 1 = 2$, so the point is $(0, 2)$. Differentiating, $f'(x) = -\frac{1}{(x+1)^2} - \sin x$, so $f'(0) = -1 - 0 = -1$. The line is $y - 2 = -1(x - 0)$, that is $y = 2 - x$, or $x + y = 2$. Option A, $x + y = 1$, has the correct slope but passes through $(0, 1)$, so it misses the point on the curve.

**41.** Which one of the following is equal to $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x^2 - 1}$ ? (1)

**A.** $\infty$

**B.** 0

**C.** -1/4

**D.** ✓ **1/4**

**Answer:** D

Putting $x = 1$ straight in gives $\frac{0}{0}$, so rewrite before substituting. Multiply the top and the bottom by $\sqrt{x} + 1$: $\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x^2-1)(\sqrt{x}+1)} = \frac{x-1}{(x-1)(x+1)(\sqrt{x}+1)} = \frac{1}{(x+1)(\sqrt{x}+1)}$. Now $x = 1$ is safe and gives $\frac{1}{2 \times 2} = \frac{1}{4}$. Option B, 0, comes from watching only the top, which does head to 0, while the bottom heads to 0 as well and the two effects cancel.

**42.** Which one of the following is equal to $\lim_{x \to 1} \dfrac{1-x}{1-\frac{1}{x^2}}$ ? (1)

**A.** 1

**B.** 0

**C.** ✓ **-1/2**

**D.** Doesn�t exist

**Answer:** C
Clear the small fraction sitting inside the denominator. Since $1-\frac{1}{x^2}=\frac{x^2-1}{x^2}$, the whole expression becomes $\frac{(1-x)x^2}{x^2-1}=\frac{(1-x)x^2}{(x-1)(x+1)}$. Because $1-x=-(x-1)$, this reduces to $\frac{-x^2}{x+1}$ for every $x \neq 1$, and at $x=1$ that is $-\frac{1}{2}$. Option D says the limit does not exist, which is the reading you get from the $\frac{0}{0}$ form before the cancelling is done.

**43.** Which one the following is equal to $\int \dfrac{\ln(xe^x)}{x}\,dx$ (1)

**A.** $\ln|x|+\dfrac{1}{2}e^x+c$

**B.** ✓ $\dfrac{1}{2}(\ln x)^2+x+c$

**C.** $\ln|x|+e^{2x}+c$

**D.** $-\dfrac{1}{x^2}+\ln x)^2+c$

**Answer:** B
Break the logarithm apart first: $\ln(xe^x)=\ln x+\ln e^x=\ln x+x$. The integrand is then $\frac{\ln x+x}{x}=\frac{\ln x}{x}+1$. The first piece yields to the substitution $u=\ln x$ with $du=\frac{dx}{x}$, giving $\frac{u^2}{2}=\frac{(\ln x)^2}{2}$, and the second piece integrates to $x$. The answer is $\frac{1}{2}(\ln x)^2+x+c$. Option A carries $\ln|x|$, which is the integral of $\frac{1}{x}$ rather than of $\frac{\ln x}{x}$.

**44.** Suppose f is differentiable on $(-\infty,\infty)$ and the graph of its derivative is as shown below. Which one of the following is true about f ? (1)

*[Figure: A scanned sketch of a cubic-looking curve on x-y axes, labelled $y=f'(x)$ beside its upper-left branch. The x-axis carries marked dots and labels at -3, -2, -1, 1, 2 and 3; the y-axis is drawn as an upward arrow and the x-axis as a rightward arrow. The curve comes down steeply from the upper left, crosses the x-axis at $x=-2$, continues down to a local minimum at about $x=-1$ (a dashed vertical segment runs from the tick at -1 down to that minimum), then rises, crossing the x-axis at the origin $x=0$, reaches a local maximum above $x=1$ (a dashed vertical segment joins the tick at 1 to that peak), then falls, crossing the x-axis at $x=2$ and continuing steeply down to the lower right past $x=3$. No y-axis values are marked.]*

**A.** f is decreasing on $(-\infty,-1)\cup[1,\infty)$

**B.** f has a local minimum at $x=-2$

**C.** f is concave down on $[0,\infty)$

**D.** ✓ **f is concave up on** $(-1,1)$

**Answer:** D
The curve drawn is $f'$, so read the slope of $f$ from its height and the concavity of $f$ from whether it is rising or falling. Between $x=-1$ and $x=1$ the sketch climbs from its low point up to its peak, so $f'$ is increasing there, which means $f''>0$ and $f$ is concave up on $(-1,1)$. Option B claims a local minimum at $x=-2$, but $f'$ crosses zero there going from positive to negative, so $f$ stops rising and starts falling, which makes $x=-2$ a local maximum.

**45.** Which one of the following is true about the derivative of $f(x)=x|x|$? (1)

**A.** f is not differentiable at $x=0$

**B.** ✓ $f'(x)=2|x|$**, for every** $x\in(-\infty,\infty)$

**C.** $f'(x)=2x$, for every $x\in(-\infty,\infty)$

**D.** $f'(x)=|x|+x$, for every x ?$(-\infty,\infty)$

**Answer:** B
Split the function by the sign of $x$. For $x>0$, $f(x)=x\cdot x=x^2$, so $f'(x)=2x=2|x|$. For $x<0$, $f(x)=x(-x)=-x^2$, so $f'(x)=-2x=2|x|$ again. At $x=0$ go back to the definition: $\lim_{h\to 0}\frac{h|h|-0}{h}=\lim_{h\to 0}|h|=0$, which is also what $2|x|$ gives at 0. So $f'(x)=2|x|$ for every real $x$. Option A assumes the corner in $|x|$ must survive in the product, but multiplying by $x$ smooths that corner away.

**46.** What is the area of the region between the graphs of $y = x^2$ and $y = -x + 2$, where $0 \le x \le 2$ ? (1)

**A.** ✓ **3**

**B.** 2

**C.** 3/2

**D.** 2/3

**Answer:** A
The graphs meet where $x^2 = -x + 2$, that is $x^2 + x - 2 = 0$, which gives $x = 1$ inside the interval. On $[0, 1]$ the line is above the parabola and on $[1, 2]$ the parabola is above the line, so the area has to be found in two pieces. The first is $\int_0^1 (2 - x - x^2)\, dx = 2 - \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$, the second is $\int_1^2 (x^2 + x - 2)\, dx = \frac{11}{6}$, and together they give 3. Option D, $\frac{2}{3}$, is what one single integral from 0 to 2 produces, where the part after the crossing point subtracts from the part before it instead of adding to it.

**47.** For what value of b does the parabola $p(x) = ax^2 + x + b$ pass through the points (-1, 5) and (2, -1)? (1)

**A.** ✓ **9**

**B.** 3

**C.** -3

**D.** -15

**Answer:** A
Put each point into $p(x) = ax^2 + x + b$. From $(-1, 5)$: $a - 1 + b = 5$, so $a + b = 6$. From $(2, -1)$: $4a + 2 + b = -1$, so $4a + b = -3$. Subtracting the first equation from the second gives $3a = -9$, so $a = -3$, and then $b = 6 - (-3) = 9$. Option C, $-3$, is the value of $a$, while the question asks for $b$.

**48.** Which one of the following is equal to $\sec(\pi/2 - x)\sin^3 x + \cos 2x$? (1)

**A.** $2\cos x$

**B.** $2\sin x$

**C.** ✓ $\cos 2x$

**D.** $\sin 2x$

**Answer:** C
Use the cofunction identity $\cos\left(\frac{\pi}{2} - x\right) = \sin x$, so $\sec\left(\frac{\pi}{2} - x\right) = \frac{1}{\sin x}$. The first term becomes $\frac{\sin^3 x}{\sin x} = \sin^2 x$. Now bring in the double angle form $\cos 2x = 1 - 2\sin^2 x$: the whole expression is $\sin^2 x + 1 - 2\sin^2 x = 1 - \sin^2 x$, which is $\cos^2 x$. Option A, $2\cos x$, comes from cancelling the sines carelessly and then reading $\cos 2x$ as $2\cos x$, which is not an identity.

**49.** Suppose P(1, 2, 1) and Q(1, 0, 2) are points in space and $\vec{A} = \overrightarrow{PQ}$. If $\vec{B}$ is parallel to $\overrightarrow{PQ}$ and $\vec{A} \cdot \vec{B} = -10$,, then which one of the following is true? (1)

**A.** $\vec{A}$ and $\vec{B}$ has the same direction

**B.** $||\vec{B}|| = 10||\vec{A}||$

**C.** $||\vec{B}|| = \dfrac{1}{10}||\vec{A}||$

**D.** ✓ $||\vec{B}|| = 2||\vec{A}||$

**Answer:** D
First build the vector: $\vec{A} = \overrightarrow{PQ} = (1 - 1,\ 0 - 2,\ 2 - 1) = (0, -2, 1)$, so $||\vec{A}||^2 = 0 + 4 + 1 = 5$. Because $\vec{B}$ is parallel to $\vec{A}$, write $\vec{B} = k\vec{A}$ for some real $k$. Then $\vec{A} \cdot \vec{B} = k||\vec{A}||^2 = 5k = -10$, so $k = -2$ and $||\vec{B}|| = |k|\, ||\vec{A}|| = 2||\vec{A}||$. Option A says the two vectors point the same way, but $k$ came out negative, so $\vec{B}$ points the opposite way to $\vec{A}$.

**50.** What is the solution of $\cos^2 x + 0.5\sin 2x = 1$ in the interval $[0, 2\pi)$? (1)

**A.** ✓ $\{0,\ \pi/4,\ \pi,\ 5\pi/4\}$

**B.** $\{0,\ \pi/4,\ 3\pi/4,\ \pi\}$

**C.** $\{0,\ \pi\}$

**D.** $\{0,\ \pi/4,\ \pi\}$

**Answer:** A
Replace $\sin 2x$ with $2\sin x\cos x$, so the equation becomes $\cos^2 x + \sin x\cos x = 1$. Since $1 - \cos^2 x = \sin^2 x$, this rearranges to $\sin x\cos x = \sin^2 x$, that is $\sin x(\cos x - \sin x) = 0$. From $\sin x = 0$ come $x = 0$ and $x = \pi$; from $\cos x = \sin x$ comes $\tan x = 1$, giving $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ inside $[0, 2\pi)$. Option B offers $\frac{3\pi}{4}$, where $\tan x = -1$, so sine and cosine have opposite signs there and the bracket does not vanish.

**51.** If $\vec{u} = (-3, x)$ and $\vec{v} = (x, y-2)$ are vectors, what is the value of y so that $\vec{u} + \vec{v} = 3\vec{u} - \dfrac{1}{2}\vec{v}$ ? (1)

**A.** 2/3
**B.** ✓ **-10/3**
**C.** -4
**D.** -22/3

**Answer:** B
Gather the vectors on each side. From $\vec{u} + \vec{v} = 3\vec{u} - \frac{1}{2}\vec{v}$ we get $\frac{3}{2}\vec{v} = 2\vec{u}$, so $\vec{v} = \frac{4}{3}\vec{u}$. Comparing first components, $x = \frac{4}{3}(-3) = -4$. Comparing second components, $y - 2 = \frac{4}{3}x = \frac{4}{3}(-4) = -\frac{16}{3}$, so $y = 2 - \frac{16}{3} = -\frac{10}{3}$. Option C, $-4$, is the value of $x$ found along the way, and the question asks for $y$.

**52.** Which one of the following points is closer to the sphere $x^2 + y^2 + z^2 - 2x + 6z + 9 = 0$ ? (1)

**A.** (1, 0, 0)
**B.** (0, 0, 0)
**C.** (0, -1, 0)
**D.** ✓ **(0, 0, -1)**

**Answer:** D
Complete the squares to see the sphere: $x^2 - 2x + y^2 + z^2 + 6z + 9 = 0$ becomes $(x-1)^2 + y^2 + (z+3)^2 = 1$, a sphere centred at $(1, 0, -3)$ with radius 1. The nearest point is the one whose distance from the centre is smallest. Those distances are 3 for $(1, 0, 0)$, $\sqrt{10}$ for the origin, $\sqrt{11}$ for $(0, -1, 0)$ and $\sqrt{5}$ for $(0, 0, -1)$. Since $\sqrt{5} \approx 2.24$ is the smallest, $(0, 0, -1)$ is closest. Option A shares the $x$ value of the centre and looks close because of it, but it sits a full 3 units away.

**53.** What is the cot(arc sinx) if $0 < x < 1$? (1)

**A.** $\dfrac{x}{\sqrt{1-x^2}}$
**B.** ✓ $\dfrac{1}{x}\sqrt{1-x^2}$
**C.** $\sqrt{1-x^2}$
**D.** $\dfrac{1}{\sqrt{1-x^2}}$

**Answer:** B
Let $\theta = \arcsin x$, so $\sin\theta = x$, and with $0 < x < 1$ the angle sits in the first quadrant. Draw a right triangle with opposite side $x$ and hypotenuse 1; Pythagoras makes the adjacent side $\sqrt{1-x^2}$. Then $\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\sqrt{1-x^2}}{x}$. Option A is that same fraction upside down, which is the tangent of the angle rather than its cotangent.

**54.** Suppose $\vec{A} = 2\vec{j} - \vec{k}$ and $\vec{B} = 5\vec{i} + 15\vec{k}$, where $\vec{i}, \vec{j}$ and $\vec{k}$ are the standard unit vectors in the directions of positive x, y and z axis, respectively. Which one of the following is the unit vector in the direction of $\vec{A} + \dfrac{1}{5}\vec{B}$? (1)

**A.** $\dfrac{3}{5}\vec{i} + \dfrac{4}{5}\vec{k}$
**B.** ✓ $\dfrac{1}{3}\vec{i} + \dfrac{2}{3}\vec{j} + \dfrac{2}{3}\vec{k}$
**C.** $\dfrac{4}{5}j - \dfrac{3}{5}k$
**D.** $\dfrac{2}{3}i - \dfrac{1}{3}j + \dfrac{2}{3}k$

**Answer:** B
Write both vectors as triples: $\vec{A} = (0, 2, -1)$ and $\frac{1}{5}\vec{B} = (1, 0, 3)$. Adding them gives $\vec{A} + \frac{1}{5}\vec{B} = (1, 2, 2)$, whose length is $\sqrt{1+4+4} = 3$. Dividing by that length gives the unit vector $\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$. Option C also has length 1, but it carries no $\vec{i}$ part, and the sum does have one, brought in by $\vec{B}$.

**55.** A line given by a vector equation r(t)=(0, 3) + t(1, 1) is tangent to a circle at point (0,3). If the radius of the circle is ?2 which one of the following is the center of the circle? (1)

**A.** (1,4)
**B.** (1,-4)
**C.** (-1,2)
**D.** ✓ **(1,2)**

**Answer:** D
A tangent meets a circle at a right angle to the radius, so the centre lies on the line through $(0, 3)$ perpendicular to the direction $(1, 1)$. One perpendicular direction is $(1, -1)$, and its length is $\sqrt{2}$, which is exactly the radius. So the centre is $(0, 3) + (1, -1) = (1, 2)$ or $(0, 3) - (1, -1) = (-1, 4)$, and only $(1, 2)$ is offered. Option A, $(1, 4)$, is what you reach by stepping along the line's own direction $(1, 1)$, which would put the centre on the tangent line itself.

**56.** Suppose the following statements are the premises of an argument. �He was lazy or he did not like the classroom. If he was lazy, he could not pass the exam. He passed the exam.� Which one of the following can be a conclusion that makes the argument valid? (1)

**A.** He did like the classroom.

**B.** ✓ **He did not like the classroom.**

**C.** If he was not lazy, he did like the classroom.

**D.** He was not lazy and he did like the classroom.

> **Answer:** B
> Work through the premises in order. He passed the exam, and the second premise says being lazy would have stopped him passing, so he was not lazy. The first premise says he was lazy or he did not like the classroom, and with the first half now ruled out the second half has to be the true one, so he did not like the classroom. Option A states the opposite of that. The premises have already forced laziness out, and it is precisely that which makes the classroom half of the first premise the surviving one.

**57.** What is the image of the ellipse whose equation is $2(x+2)^2+(y-1)^2=2$ under a translation that takes (2, 1) to (4, 0) followed by a rotation of $90^0$? (1)

**A.** ✓ $x^2+2y^2=2$

**B.** $2x^2+y^2=2$

**C.** $2(x$ � $4)^2+y^2=2$

**D.** $(x$ � $4)^2+2y^2=2$

> **Answer:** A
> The ellipse $2(x+2)^2+(y-1)^2=2$ is centred at $(-2,1)$. The translation sends $(2,1)$ to $(4,0)$, so it moves every point 2 units right and 1 unit down, and the centre lands on the origin, leaving $2x^2+y^2=2$. A rotation of $90^\circ$ about the origin exchanges the roles of the two axes, so the coefficient 2 moves from $x^2$ across to $y^2$ and the image is $x^2+2y^2=2$. Option B is the ellipse after the translation only, with the rotation left out.

**58.** Let $\vec{a}=2\vec{i}+(x-1)\vec{j}+\vec{k}$ and $\vec{c}=\vec{i}-\vec{j}+y\vec{k}$ be vectors. If $\vec{a}\cdot\vec{c}=0$ and $||\vec{a}||=3$ which one of the following is a possible value of y? (1)

**A.** ✓ **-4**

**B.** -1

**C.** 3

**D.** 4

> **Answer:** A
> Two conditions have to hold together. From $\vec{a}\cdot\vec{c}=0$: $2(1)+(x-1)(-1)+1(y)=0$, which tidies up to $y=x-3$. From $||\vec{a}||=3$: $4+(x-1)^2+1=9$, so $(x-1)^2=4$ and $x=3$ or $x=-1$. Those give $y=0$ or $y=-4$, and only $-4$ appears among the options. Option D, 4, has the right size with the wrong sign, which is what you get by reading the first condition as $y=3-x$.

**59.** If $\vec{A}$ is perpendicular to $\vec{B}$, what is the cosine of the angle between $\vec{A}$ and $\vec{A}-\vec{B}$ ? (1)

**A.** $\dfrac{||\vec{A}-\vec{B}||}{||\vec{A}||}$

**B.** ✓ $\dfrac{||\vec{A}||}{||\vec{A}-\vec{B}||}$

**C.** $\dfrac{||\vec{A}-\vec{B}||}{||\vec{B}||}$

**D.** $\dfrac{||\vec{B}||}{||\vec{A}-\vec{B}||}$

> **Answer:** B
> Use the dot product formula $\cos\theta=\frac{\vec{A}\cdot(\vec{A}-\vec{B})}{||\vec{A}||\,||\vec{A}-\vec{B}||}$. Expanding the numerator, $\vec{A}\cdot\vec{A}-\vec{A}\cdot\vec{B}=||\vec{A}||^2-0$, because perpendicular vectors have a dot product of zero. So $\cos\theta=\frac{||\vec{A}||^2}{||\vec{A}||\,||\vec{A}-\vec{B}||}=\frac{||\vec{A}||}{||\vec{A}-\vec{B}||}$. Option A is that fraction upside down, and since $||\vec{A}-\vec{B}||\geq||\vec{A}||$ here, it would hand you a cosine bigger than 1.

**60.** Which one of the following is necessarily true? (1)

**A.** If $||\vec{A}|| = ||\vec{B}||$, then $\vec{A} = \vec{B}$

**B.** $||k\vec{A}|| = K||\vec{A}||$, for any real number k

**C.** if $\vec{A}$ is parallel to $\vec{B}$, then $\vec{A} \cdot \vec{B} = 0$

**D.** ✓ **if $\vec{u}$ is a unit vector in the direction of $\vec{A}$, then $\vec{A} \cdot \vec{u} = ||\vec{A}||$**

**Answer:** D

A unit vector in the direction of $\vec{A}$ is $\vec{u} = \frac{\vec{A}}{||\vec{A}||}$, so $\vec{A} \cdot \vec{u} = \frac{\vec{A} \cdot \vec{A}}{||\vec{A}||} = \frac{||\vec{A}||^2}{||\vec{A}||} = ||\vec{A}||$, and that holds for every non-zero $\vec{A}$. Option B fails as soon as $k$ is negative: the correct rule is $||k\vec{A}|| = |k|\ ||\vec{A}||$, so with $k = -1$ the left side is $||\vec{A}||$ while the right side would be $-||\vec{A}||$, and no length can be negative.

— END OF MARKING KEY —