Temari.et

Mathematics, 2018 EC, entrance exam, round 1

Grade 12 · Mathematics · 2018 E.C.

MARKING KEY — CONFIDENTIAL

Time allowed: 120 minutes | **Questions:** 58 | **Total marks:** 58

1. Consider the data: 11, 13, 15, 17, 19, 21. Which one of the following is the mean deviation from the mean? (1)

A. 4
B. 3.5
C. 4.5
D. ✓ **3**

Answer: D
The mean of the data is (11+13+15+17+19+21)/6 = 16. The absolute deviations from the mean are 5, 3, 1, 1, 3, and 5, which sum to 18; dividing this sum by 6 yields a mean deviation of 3.

2. Consider the quadrilateral ABCD where A=(0,0), B=(-1,1)), C=(0,2) and D=(2,2). Which one of the following is true? (1)

A. CD is perpendicular to AB.
B. ✓ **BC is parallel to AD.**
C. CD is perpendicular to BE.
D. AB is parallel to CD.

Answer: B
The slope of BC is (2 - 1) / (0 - (-1)) = 1, and the slope of AD is (2 - 0) / (2 - 0) = 1. Since their slopes are equal and they are on distinct lines, BC is parallel to AD.

3. Consider the greatest integer function defined by $f(x) = [x - 1]$. Which of the following sets is the domain of f? (1)

A. The set of rational numbers.
B. The set of integers.
C. ✓ **The set of real numbers.**
D. The set of natural numbers.

Answer: C
The greatest integer function [u] is defined for all real numbers u. Therefore, the domain of f(x) = [x - 1] is the set of all real numbers (R).

4. Which one of the following relations is a function? (1)

A. R={(x,y):x,y\in R and $y^2 = x$}
B. R={(x,y):x,y\in R and |y|=x}
C. ✓ **R={(x,y):x,y\in R and $y = x^2$}**
D. $R = \{(x, y) : x, y \in R \text{ and } y^4 = x\}$

Answer: C
A relation is a function if each input x corresponds to exactly one output y. In y = x^2, each real number x yields exactly one real number y, whereas in the other options, a single positive x-value yields two distinct y-values.

5. Which one of the following is a source of cash loan? (1)

A. ✓ **Non-government organization to alleviate public problems.**
B. Investors involved in home construction for a community.
C. Group of investors committed to drilling out underground water.
D. A money line group investing in health improvement sectors.

Answer: A
Non-governmental organizations (NGOs) are recognized formal institutions that provide micro-credit or cash loans to alleviate public and socio-economic problems in community development.

6. What is the average rate of change of $y = f(x)$ with respect to x on the interval $[a, b]$? (1)

A. $\frac{f(b)+f(a)}{2}$

B. $\frac{b-a}{f(b)-f(a)}$

C. $\frac{f(b)-f(a)}{2}$

D. ✓ $\frac{f(b)-f(a)}{b-a}$

> **Answer:** D
> The average rate of change of a function f over an interval [a, b] is defined as the change in the function values divided by the change in the input, which is given by the formula (f(b) - f(a)) / (b - a).

7. Which one of the following sets is the solution of $\log_3 x + \log_3(x-2) = 1$? (1)

A. {-1,3}

B. ✓ **{3}**

C. {2}

D. {0,2}

> **Answer:** B
> Using logarithmic properties, log_3(x(x-2)) = 1, which implies x(x-2) = 3. Solving x^2 - 2x - 3 = 0 gives x = 3 or x = -1; however, x must be greater than 2 to satisfy the domain of log_3(x-2), so only {3} is the correct solution set.

8. Given the functions $f(x) = ax^2$and $g(x) = ax^2 + c$, for a and $c > 0$. Which of the following statements is true about the graph of f and g? (1)

A. The graph of f is an upward parabola, and the graph of g is obtained from that of f by shifting it downward by cc units.

B. The graph of f is a downward parabola, and the graph of g is obtained from that of f by shifting it to the right by cc units.

C. The graph of f is an upward parabola, and the graph of g is obtained from that of f by shifting it upward by cc units.

D. ✓ **The graph of f is a downward parabola, and the graph of g is obtained from that of f by shifting it upward by cc units.**

> **Answer:** D
> Since a < 0, the graph of f(x) = ax^2 is a downward-opening parabola. Adding a positive constant c shifts the entire graph vertically upward by c units to produce the graph of g(x).

9. Consider the test results of 20 students in a given class. 13 14 14 15 19 18 13 15 16 16 13 14 14 15 19 18 13 15 16 16 17 18 15 13 14 16 14 18 16 16 \nWhich one of the following numbers is the mode of the above data? (1)

A. ✓ **16**

B. 15

C. 14

D. 18

> **Answer:** A
> In the correct dataset of 20 students, the value 16 has the highest frequency, appearing 5 times (compared to 14 which appears 4 times, and others appearing 3 times or less), making 16 the mode.

10. Consider the figure given below:\nWhich one of the following vectors is equal to $\mathrm{AB}^{\rightarrow} + \mathrm{CD}^{\rightarrow} + \mathrm{BC}^{\rightarrow}$? (1)

[Figure: The figure for Question 10 shows four points labeled A, B, C, and D forming a geometric arrangement with vectors. Point A is connected to point B by a line segment with an arrow pointing towards B, representing vector AB. Point C is connected to point D by a line segment with an arrow pointing towards D, representing vector CD. Point C is also connected to point A by a line segment. A semi-transparent watermark text addis_aptitude diagonally crosses the central background of the figure.]

A. $\vec{CD}$

B. $\vec{AC}$

C. ✓ $\vec{AD}$

D. $\vec{BD}$

> **Answer:** C
> Using the properties of vector addition, we can rearrange the terms: AB + CD + BC = AB + BC + CD. Since AB + BC = AC, the sum becomes AC + CD, which equals AD.

11. Which one of the following equations represents a line that passes through the points (2, -1) and (-3, 4)? (1)

A. ✓ $x + y - 1 = 0$

B. $x + y + 1 = 0$

C. $x - y - 1 = 0$

D. $x - y + 1 = 0$

> **Answer:** A
> The slope of the line passing through (2, -1) and (-3, 4) is m = (4 - (-1)) / (-3 - 2) = -1. Using the point-slope form with (2, -1) gives y - (-1) = -1(x - 2), which simplifies to x + y - 1 = 0.

12. What is the coefficient of x^3y^4 in the expansion of $(x - 2y)^7$ (1)

A. 35

B. 70

C. ✓ **560**

D. -280

> **Answer:** C
> According to the Binomial Theorem, the term containing x^3y^4 in the expansion of (x - 2y)^7 is given by C(7, 4) * x^3 * (-2y)^4 = 35 * x^3 * 16y^4 = 560 x^3y^4. Thus, the coefficient is 560.

13. Which one of the following is NOT a power function? (1)

A. f(x)=x^3

B. ✓ **f(x)=3x**

C. f(x)=$4x^{-6}$

D. f(x)=$7x^{-1/3}$

> **Answer:** B
> A power function is of the form f(x) = k*x^p. Option B, which represents f(x) = 3^x, is an exponential function because the variable x is in the exponent, making it not a power function.

14. Given the problem: Maximize Z=3x+y subject to: (1)

$$\begin{cases} x + y \geq 2 \\ y + 2x \leq 4 \\ y \geq 1, x \geq 0 \end{cases}$$

\nWhich one of the following is the maximum value of Z?

A. 2

B. ✓ **5.5**

C. 4

D. 5.8

> **Answer:** B
> The feasible region is bounded by the vertices (0, 2), (0, 4), (1, 1), and (1.5, 1). Evaluating the objective function Z = 3x + y at these vertices yields 2, 4, 4, and 5.5 respectively, meaning the maximum value of Z is 5.5.

15. Let r1 and r2 be the roots of the quadratic equation $2x^2 + 3x - 4 = 0$. Which one of the following is equal to (1)

$$\frac{1}{r_1^2} + \frac{1}{r_2^2}?$$

A. 3/4

B. ✓ **25/16**

C. 3/16

D. 1/4

> **Answer:** B
> From Vieta's formulas, r_1 + r_2 = -3/2 and r_1 * r_2 = -2. The expression simplifies to (r_1^2 + r_2^2) / (r_1 * r_2)^2 = ((r_1 + r_2)^2 - 2*r_1*r_2) / (r_1 * r_2)^2 = (9/4 + 4) / 4 = 25/16.

16. Consider the matrix (1)

$$A = \begin{pmatrix} -1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 1 & 5 \end{pmatrix}$$

\nWhich of the following statements is true about the minor of the entries of A?

A. The cofactor of 3 is 9.

B. ✓ **The cofactor of 4 is -6.**

C. The minor of 4 is 6.

D. The minor of 3 is -9.

Answer: B

The entry 4 is at row 2, column 2. Its minor is det([-1, 1; 1, 5]) = -5 - 1 = -6, and since (-1)^(2+2) = 1, its cofactor is also -6, which matches statement B.

17. In the figure below, let m ($\angle \mathrm{DAC}$)=15° and m(arc BC)=70°. What is the measure of arc CD? (1)

[Figure: A circle with center O is shown. Point A lies outside the circle to the right. A secant line passes through point A, intersects the circle at point D, and continues to point B on the upper left of the circle. A tangent line starts from point A and touches the bottom of the circle at point C. Angle DAC at vertex A is marked as 15 degrees. The arc between B and C contains points D along the secant line.]

A. 42.5°

B. 55°

C. 30°

D. ✓ **40°**

Answer: D

The measure of the angle formed by a secant and a tangent meeting outside a circle is half the difference of the intercepted arcs: m(angle DAC) = (m(arc BC) - m(arc CD)) / 2. Substituting the values gives 15 = (70 - m(arc CD)) / 2, which yields m(arc CD) = 40 degrees.

18. Consider the right-angled triangle $\Delta \mathrm{ABC}$ below, where $\mathrm{AC} = 5\mathrm{cm}$ and $\mathrm{BC} = 4\mathrm{cm}$. \nWhich one of the following is the length of BD? (1)

[Figure: A right-angled triangle labeled ΔABC with the right angle at vertex B. The vertices are labeled A at the top left, B at the bottom left, and C at the bottom right. A line segment is drawn from vertex B perpendicularly to the hypotenuse AC, meeting AC at a point labeled D. There is a right-angle symbol at vertex B and another right-angle symbol where the line segment BD meets the hypotenuse AC at point D.]

A. 5.76cm

B. ✓ **2.4cm**

C. 5cm

D. 7cm

Answer: B

In right triangle ABC, the hypotenuse is AC = 5 cm and leg BC = 4 cm, so the other leg AB = 3 cm. The altitude BD to the hypotenuse can be found using the area of the triangle: BD = (AB * BC) / AC = (3 * 4) / 5 = 2.4 cm.

19. Which one of the following is true about definite integrals? (1)

A. $\int_a^b f(x)dx = 0, a \neq 0$

B. ✓ $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

C. $\int_a^b f(x)dx = \int_a^b g(x)dx$

D. $\int_a^b (f(x) - g(x))dx = \int_a^b (g(x) - f(x))dx$

Answer: B

One of the fundamental properties of definite integrals is linearity, which states that the integral of a sum of two functions is equal to the sum of their individual integrals.

20. Which of the following statements defines a circle? (1)

A. It is the set of points in a plane formed by the intersection of two secants in the plane.

B. It is the set of points in a space formed by the intersection of two secants in the space.

C. It is the set of points in a space each of which is equidistant from a fixed point in the space.

D. ✓ **It is the set of points in a plane each of which is equidistant from a fixed point in the plane.**

> **Answer:** D
> By definition, a circle is the locus (set) of all points in a two-dimensional plane that are equidistant from a fixed point (the center) in that plane.

21. What is the transpose of the matrix (1)

$$M = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 3 \\ 2 & 1 & 5 \end{pmatrix}?$$

A. ✓ $\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 1 & 3 & 5 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \\ 0 & 1 & 5 \end{pmatrix}$

C. $\begin{pmatrix} 2 & 1 & 5 \\ 0 & -1 & 3 \\ 1 & 2 & 1 \end{pmatrix}$

D. $\begin{pmatrix} 1 & 2 & 2 \\ 0 & -1 & 1 \\ 1 & 3 & 5 \end{pmatrix}$

> **Answer:** A
> The transpose of a matrix is obtained by swapping its rows with its columns, so the first row [1, 2, 1] of M becomes the first column of M^T, resulting in option A.

22. Which one of the following sets is the domain of the rational function (1)

$$f(x) = \frac{x^3 + x^2}{x^4 + 2x^2 - 3}?$$

A. R\{1}

B. ✓ **R\{-1,1}**

C. R\{-3,1}

D. R\{-1,1,-3}

> **Answer:** B
> The domain of a rational function excludes values that make the denominator zero. Setting x^4 + 2x^2 - 3 = 0 factors to (x^2 + 3)(x^2 - 1) = 0, which has real solutions only at x = 1 and x = -1, so the domain is R \ {-1, 1}.

23. Which of the following statements is true in borrowing money? (1)

A. The borrower only pays back the principal amount.

B. The lender assumes no risk when giving out a loan.

C. ✓ **Borrowing money involves an agreement to repay the principal amount along with accrued interestz**

D. Collateral is always required for any type of financial borrowing.

> **Answer:** C
> Borrowing money is a financial transaction where the borrower receives a principal amount and legally agrees to pay it back over time along with interest, which compensates the lender for the time value of money and risk.

24. Let f(x)=2x+1/x−1. Which of the following equations is the equation of the tangent line to the graph of f at (2,f(2))? (1)

A. ✓ **3x+y=11**
B. y−3x=−1
C. x+3y=17
D. 3x−y=13

> **Answer:** A
> At x = 2, f(2) = (4+1)/(2-1) = 5. The derivative is f'(x) = -3/(x-1)^2, giving a slope of f'(2) = -3. The equation of the tangent line is y - 5 = -3(x - 2), which simplifies to 3x + y = 11.

25. What is the measure of each interior angle of a regular polygon with 15 sides? (1)

A. 144°
B. 150°
C. ✓ 156°
D. 162°

> **Answer:** C
> The formula for the measure of each interior angle of a regular n-sided polygon is $\frac{(n-2)\times 180^\circ}{n}$. Substituting $n = 15$ gives $\frac{13\times 180^\circ}{15} = 156^\circ$.

26. Let $f(x) = x^3 + 4x$and $g(x) = 3x^3 - x^2$. What is the derivative of (f+g)(x)? (1)

A. ✓ $12x^2 - 2x + 4$
B. $4x^3 - x^2 + 4x$
C. $12x^2 - x + 4$
D. $9x^2 - 2x + 4$

> **Answer:** A
> First, sum the functions: $(f + g)(x) = (x^3 + 4x) + (3x^3 - x^2) = 4x^3 - x^2 + 4x$. Differentiating this expression gives $(f + g)'(x) = 12x^2 - 2x + 4$.

27. Let A={$x \in \mathbb{R} : x^3 - x = 0$}and B={$x \in \mathbb{Z} : -1 \leq x$}. What is the relative complement of B with respect to A (i.e., A\B)? (1)

A. {2}
B. {0,1}
C. ✓ $\emptyset$
D. {−1,0,1}

> **Answer:** C
> Solving $x^3 - x = 0$ yields $x(x - 1)(x + 1) = 0$, so $A = \{-1, 0, 1\}$. The set B consists of integers in $[-1, 3)$, which means $B = \{-1, 0, 1, 2\}$. Since every element of A is also in B, the relative complement $A \setminus B$ is the empty set $\emptyset$.

28. Given the set B={∅,{−2,0}}, what is the power set of B? (1)

A. {∅,{−2,0}}
B. ✓ **{∅,{∅},{{−2,0}},{∅,{−2,0}}}**
C. {{∅},{{−2,0}}}
D. {∅,{−2},{0},{−2,0}}

> **Answer:** B
> The power set of a set with n elements has 2^n elements. Since B has 2 elements, its power set must have $2^2 = 4$ elements, which are $\emptyset$, the singleton sets containing each element, and B itself: $\{\emptyset, \{\emptyset\}, \{\{-2, 0\}\}, \{\emptyset, \{-2, 0\}\}\}$.

29. What is the simplified form of (1)

$$\log_{16}(4) - \log_2(\sqrt{12.5}) + \log_{0.5}(2)?$$

A. $\log_2(5)$
B. 1-$\log_2(5)$
C. ✓ -$\log_2(5)$
D. 0

> **Answer:** C
> Simplify each term: $\log_{16}(4) = \frac{1}{2}$, $\log_2(\sqrt{12.5}) = \log_2((25/2)^{1/2}) = \log_2(5) - \frac{1}{2}$, and $\log_{0.5}(2) = -1$. Combining these gives $\frac{1}{2} - (\log_2(5) - \frac{1}{2}) - 1 = -\log_2(5)$.

30. Which statement is true about the graphs of the exponential functions f(x)=a^x f and g(x)=$(1/a)^x$ where a>1? (1)

A. They are reflections of each other across the x-axis.

B. ✓ **They are reflections of each other across the y-axis.**

C. They never intersect each other.

D. Both functions are monotonically increasing.

> **Answer:** B
> Since $g(x) = (1/a)^x = a^{-x} = f(-x)$, the graph of g is obtained by reflecting the graph of f across the y-axis.

31. Which of the following statements is NOT true about the mean of a given data set? (1)

A. It is highly affected by extreme values (outliers).

B. The sum of the deviations of the items from the mean is always zero.

C. ✓ **It must always be one of the actual data values present in the set.**

D. It uses every value in the data set for its calculation.

> **Answer:** C
> The mean of a data set does not have to be one of the values in the data set (for example, the mean of $\{1, 2\}$ is 1.5, which is not in the set).

32. Which one of the following numbers is an irrational number? (1)

A. 3.14159 (terminating)

B. $\sqrt{16}$

C. 22/7

D. ✓ π

> **Answer:** D
> The number π is a well-known irrational number, whereas terminating decimals, square roots of perfect squares, and ratios of integers are all rational numbers.

33. What is the area of the parallelogram spanned by the vectors $\vec{u}$=i-2j+k and $\vec{v}$=2i+2j-k (1)

A. $\sqrt{14}$

B. ✓ $3\sqrt{5}$

C. 9

D. $\sqrt{35}$

> **Answer:** B
> The area of the parallelogram is the magnitude of the cross product of $\vec{u}$ and $\vec{v}$. Calculating $\vec{u} \times \vec{v}$ yields $3\vec{j} + 6\vec{k}$, whose magnitude is $\sqrt{3^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$.

34. What is the fundamental period of the function f(x)=sin(x)? (1)

A. $\pi/2$

B. π

C. ✓ 2π

D. 4π

> **Answer:** C
> The fundamental period of the sine function, $f(x) = \sin(x)$, is 2π because it is the smallest positive value T for which $\sin(x + T) = \sin(x)$ for all x.

35. What is the simplified form of the expression $(\sqrt{45} + \frac{1}{\sqrt{5}})(\sqrt{5} - \sqrt{20}$? (1)

A. 16

B. ✓ **-16**

C. 8

D. -8

> **Answer:** B
> Simplifying the terms: $\sqrt{45} = 3\sqrt{5}$ and $\sqrt{20} = 2\sqrt{5}$. The expression becomes $(3\sqrt{5} + \frac{1}{\sqrt{5}})(\sqrt{5} - 2\sqrt{5}) = (\frac{16}{\sqrt{5}})(-\sqrt{5}) = -16$.

36. In statistical sampling, if a population is divided into homogeneous groups and a random sample is drawn from each group, this method is known as: (1)

A. Simple random sampling
B. Cluster sampling
C. ✓ **Stratified sampling**
D. Systematic sampling

> **Answer:** C
> Stratified sampling involves dividing the population into homogeneous subgroups (strata) and then taking a random sample from each stratum.

37. A mathematics exam contains 10 multiple-choice questions and 3 short-answer questions. According to the multiplication principle, how many ways can a student select exactly one multiple-choice question and exactly one short-answer question to solve first? (1)

A. 13
B. ✓ **30**
C. 10^3
D. $\binom{13}{2}$

> **Answer:** B
> By the fundamental counting principle, if there are 10 choices for the first task (multiple-choice) and 3 choices for the second task (short-answer), there are $10 \times 3 = 30$ ways to make both selections.

38. A translation vector maps the origin (0,0) to the point (-2,4). Under this same translation, what is the image of the point (3,1)? (1)

A. (5,-3)
B. ✓ **(1,5)**
C. (-6,4)
D. (-5,3)

> **Answer:** B
> The translation vector that maps $(0,0)$ to $(-2,4)$ is $\vec{T} = \langle -2, 4 \rangle$. Applying this translation to $(3,1)$ yields $(3-2, 1+4) = (1,5)$.

39. If the initial point of a vector is A(2,-1,4) and the terminal point is B(4,3,2), what is the position vector $\vec{AB}$? (1)

A. $\langle -2, -4, 2 \rangle$
B. $\langle 6, 2, 6 \rangle$
C. ✓ $\langle 2, 4, -2 \rangle$
D. $\langle 8, -3, 8 \rangle$

> **Answer:** C
> The position vector $\vec{AB}$ is found by subtracting the coordinates of the initial point A from the terminal point B: $\vec{AB} = \langle 4-2, 3-(-1), 2-4 \rangle = \langle 2, 4, -2 \rangle$.

40. Let vector $\vec{u}$ have an initial point at (1,2)and a terminal point at (-3,6). What are the components of the vector $-1/2\vec{u}$? (1)

A. ✓ $\langle 2, -2 \rangle$
B. $\langle -2, 2 \rangle$
C. $\langle 4, -4 \rangle$
D. \langle 1,-1\rangle

> **Answer:** A
> The components of $\vec{u}$ are $\langle -3-1, 6-2 \rangle = \langle -4, 4 \rangle$. Multiplying by $-1/2$ gives $-\frac{1}{2}\vec{u} = \langle 2, -2 \rangle$.

41. Consider the rational function f(x)=x^2-4/x-2. The domain of f is the set of all real numbers EXCEPT: (1)

A. -2
B. ✓ **2**
C. 0
D. The domain is all real numbers.

> **Answer:** B
> A rational function is undefined when its denominator is zero. Here, $x - 2 = 0$ when $x = 2$, so the domain of f is all real numbers except 2.

42. Which of the following properties of matrices and scalar multiplication r is TRUE for any two matrices A and B of the same dimension? (1)

A. r(AB)=(rA)(rB)

B. ✓ **r(A+B)=rA+rB**

C. $(A+B)^T = A^T - B^T$

D. det(A+B)=det(A)+det(B)

> **Answer:** B
> The distributive property of scalar multiplication over matrix addition states that $r(A+B) = rA + rB$ for any scalar r and matrices A and B of the same size.

43. If AA is a 3×3 square matrix, which of the following is always true about its transpose AT? (1)

A. $\det(A^T) = -\det(A)$

B. $\det(A^T) = 1/\det(A)$

C. ✓ $\det(A^T) = \det(A)$

D. $A^T = A^{-1}$

> **Answer:** C
> A fundamental property of determinants is that the determinant of a matrix is equal to the determinant of its transpose, so $\det(A^T) = \det(A)$.

44. Let AA be a 3×3 invertible matrix with determinant |A|=4. What is the determinant of $2(A^{-1})$? (1)

A. 1/2

B. ✓ **2**

C. 8

D. 1/4

> **Answer:** B
> For any $n \times n$ matrix M, $\det(kM) = k^n \det(M)$. Here, A is 3×3, so $\det(2A^{-1}) = 2^3 \det(A^{-1}) = 8 \times \frac{1}{\det(A)} = 8 \times \frac{1}{4} = 2$.

45. What is the solution set for the radical equation (1)

$$\sqrt{x^2 + 3x} = 2?$$

A. ✓ **{1,-4}**

B. {1,4}

C. {-1,4}

D. {-1,-4}

> **Answer:** A
> Squaring both sides of $\sqrt{x^2 + 3x} = 2$ yields $x^2 + 3x = 4$, which factors as $(x+4)(x-1) = 0$. Thus $x = 1$ or $x = -4$, and both satisfy the original equation.

46. Given the exponential functions $f(x) = 2^x$ and $g(x) = 0.5^x$, which statement describes their monotonicity? (1)

A. Both functions are increasing.

B. Both functions are decreasing.

C. f(x) is decreasing and g(x) is increasing.

D. ✓ **f(x)is increasing and g(x)is decreasing.**

> **Answer:** D
> An exponential function b^x is strictly increasing if the base $b > 1$, and strictly decreasing if $0 < b < 1$. Thus, $f(x) = 2^x$ is increasing and $g(x) = 0.5^x$ is decreasing.

47. In an arithmetic progression, the first term is 11 and the common difference is –13. What is the sum of the first 7 terms? (1)

A. -28
B. ✓ **-196**
C. -67
D. 196

> **Answer:** B
> The sum of the first n terms of an arithmetic progression is $S_n = \frac{n}{2}[2a + (n-1)d]$. For $n = 7, a = 11, d = -13$, we get $S_7 = \frac{7}{2}[22 + 6(-13)] = \frac{7}{2}[-56] = -196$.

48. In a geometric sequence, the first term is $g_1 = 2$ and the fifth term is $g_5 = 8$ (assume the common ratio is positive). What is the value of the 13th term (g_{13})? (1)

A. 32
B. 64
C. ✓ **128**
D. 256

> **Answer:** C
> Since $g_5 = g_1 r^4$, we have $8 = 2r^4 \implies r^4 = 4$. The 13th term is $g_{13} = g_1 r^{12} = 2(r^4)^3 = 2(4)^3 = 128$.

49. The infinite repeating decimal 0.373737... can be expressed as which rational fraction in lowest terms? (1)

A. 37/100
B. ✓ **37/99**
C. 37/90
D. 1/3

> **Answer:** B
> Let $x = 0.373737...$, then $100x = 37.373737...$. Subtracting the first equation from the second gives $99x = 37$, which simplifies to $x = 37/99$ in lowest terms.

50. According to the properties of integration, which of the following is equivalent to $\int (3f(x) - g(x))dx$? (1)

A. $3\int f(x)dx \cdot \int g(x)dx$
B. ✓ $\int 3f(x)dx - \int g(x)dx$
C. $3(\int f(x)dx - \int g(x)dx)$
D. $\int f(x)dx - 3\int g(x)dx$

> **Answer:** B
> By the difference and constant multiple rules of integration, the integral of a difference is the difference of the integrals, making $\int (3f(x) - g(x))dx$ equivalent to $\int 3f(x)dx - \int g(x)dx$.

51. Three vertices of a parallelogram ABCD are A(0,0), B(5,0), and C(7,4). If AD is parallel to BC, what are the coordinates of vertex D? (1)

A. ✓ **(2,4)**
B. (-2,4)
C. (12,4)
D. (2,-4)

> **Answer:** A
> For ABCD to be a parallelogram with AD parallel to BC, the vector $\vec{BC}$ must equal $\vec{AD}$. Since $\vec{BC} = C - B = (7-5, 4-0) = (2,4)$, we have $D = A + (2,4) = (0,0) + (2,4) = (2,4)$.

52. If α and β are complementary angles ($\alpha + \beta = 90°$), which of the following trigonometric relations is always true? (1)

A. $\sin(\alpha) = \sin(\beta)$
B. $\tan(\alpha) = \tan(\beta)$
C. ✓ $\sin(\alpha) = \cos(\beta)$
D. $\sec(\alpha) = \cos(\beta)$

> **Answer:** C
> For any complementary angles where $\alpha + \beta = 90°$, the cofunction identity states that the sine of an angle is equal to the cosine of its complement, so $\sin(\alpha) = \cos(\beta)$ is always true.

53. In descriptive statistics, the third decile (D_3) corresponds to which percentile? (1)

A. 3rd percentile

B. ✓ **30th percentile**

C. 33rd percentile

D. 70th percentile

> **Answer:** B
> Deciles divide data into 10 equal parts, so each decile represents a multiple of 10% of the data. Thus, the third decile (D_3) corresponds exactly to the 30th percentile (P_{30}).

54. Which one of the following statements defines a matrix? (1)

A. ✓ **A matrix is a rectangular array of numbers.**

B. A matrix is a vertical array of letters.

C. A matrix is a horizontal array of letters.

D. A matrix is a circular array of numbers.

> **Answer:** A
> In mathematics, a matrix is formally defined as a rectangular array of numbers (or elements) arranged in rows and columns.

55. Which one of the following is the decomposition of the rational expression $3x^2 + 5x - 1/(x+2)(x+1)^2$ as a sum of partial fractions? (1)

A. $\frac{2}{x+2} + \frac{3}{x+1} + \frac{1}{(x+1)^2}$

B. $\frac{3}{x+2} - \frac{3}{(x+1)^2}$

C. ✓ $\frac{1}{x+2} + \frac{2}{x+1} - \frac{3}{(x+1)^2}$

D. $\frac{5}{x+2} + \frac{4}{(x+1)^2}$

> **Answer:** C
> Setting $\frac{3x^2+5x-1}{(x+2)(x+1)^2} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ and solving for the coefficients yields $A = 1, B = 2$, and $C = -3$, which matches option C.

56. A farmer having a total capital of Birr 400,000 bought oxen and sheep. An ox costs Birr 60,000 and a sheep Birr 5,000. He sold each ox at Birr 65,000 and each sheep at Birr 6,000. Which one of the following is a representation of the problem in order to maximize his profit, assuming uniform prices where x is the number of oxen and y is the number of sheep? (1)

A. Z(x,y)=6000x+5000y subject to $x + y \leq 35, 12x + y \leq 80, x \geq 0, y \geq 0$

B. Z(x,y)=60000x+5000y subject to $x + y \leq 35, 12x + y \leq 80, x \geq 0, y \geq 0$

C. Z(x,y)=5000x+1000ysubject to $x + y \leq 35, 12x + y \leq 800, x \geq 0, y \geq 0$

D. ✓ **Z(x,y)=5000x+1000ysubject to** $x + y \leq 35, 12x + y \leq 80, x \geq 0, y \geq 0$

> **Answer:** D
> The profit is $65000 - 60000 = 5000$ per ox and $6000 - 5000 = 1000$ per sheep, so $Z(x, y) = 5000x + 1000y$. The capital constraint is $60000x + 5000y \leq 400000$, which simplifies to $12x + y \leq 80$.

57. Which one of the following statements is true about the given numbers and points on the number line? (1)

A. There is a one-to-one correspondence between the set of natural numbers and points on the number line.

B. There is a one-to-one correspondence between the set of rational numbers and points on the number line.

C. There is a one-to-one correspondence between the set of irrational numbers and points on the number line.

D. ✓ **There is a one-to-one correspondence between the set of real numbers and points on the number line.**

> **Answer:** D
> According to the completeness property of the real numbers, there is a one-to-one correspondence between the set of real numbers and the points on a geometric number line.

58. A spherical ball has diameter 30cm. What is its surface area (in cm^2)? (1)

A. 3600π

B. 1000π

C. ✓ **900π**

D. 600π

> **Answer:** C
> The radius of the sphere is half the diameter, so $r = 15\ \text{cm}$. The surface area formula is $A = 4\pi r^2 = 4\pi(15)^2 = 900\pi\ \text{cm}^2$.

— END OF MARKING KEY —