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Mathematics, 2016 EC, natural science, entrance exam, round 1, Tigray

Grade 12 · Mathematics · 2016 E.C.

Time allowed: 120 minutes | **Questions:** 52 | **Total marks:** 52

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1. Let p and q be propositions. Which one of the following propositions is equivalent to the proposition $\neg p \vee (p \wedge q)$? (1)

A. $\neg p \vee \neg q$
B. $p \Rightarrow q$
C. $\neg p \wedge q$
D. $\neg p \Rightarrow q$

2. Which one of the following is NOT true about the function $f(x) = \tan^{-1} x$? (1)

A. The line $x = -\frac{\pi}{2}$ is a vertical asymptote to the graph of f.
B. The graph of f is the image of the graph of $y = \tan x$ after it is reflected with respect to the line $y = x$.
C. The graph of f intersects the x-axis at $(0, 0)$.
D. The line $y = \frac{\pi}{2}$ is a horizontal asymptote to the graph of f.

3. Which one of the following statements is true? (1)

A. If x and y are integers and x divides y, then y divides x.
B. If x is an even integer and y is an odd integer, then $x^2 + y^2$ is odd.
C. If x and y are even integers, then $x + y$ is an odd integer.
D. If x and y are positive integers, then $\frac{x}{y}$ is a positive integer.

4. Let p, q, and r be propositions. If p is True , q is False , and r is False , which one of the following propositions has truth value True ? (1)

A. $(p \wedge \neg q) \Leftrightarrow (p \vee r)$
B. $(q \wedge r) \Leftrightarrow (p \wedge \neg r)$
C. $p \Rightarrow (q \vee r)$
D. $(p \vee q) \Rightarrow r$

5. Which one of the following is the equation of the tangent line to the graph of $f(x) = 2x^3 - 4x^2 + 5x + 3$ at the point $(0, 3)$? (1)

A. $3x + 5y - 3 = 0$
B. $3x - 5y - 3 = 0$
C. $5x - y + 3 = 0$
D. $5x + y + 3 = 0$

6. Let $v = \vec{AB}$ and $w = \vec{CD}$ be vectors in the plane, where $A(1, -1), B(2, -2), C(-1, 3)$, and $D(4, -3)$. Which one of the following is equal to $v - w$? (1)

A. $5i - 6j$
B. $i - 15j$
C. $-4i + 5j$
D. $4i - 5j$

7. Which one of the following is true about the graph of the function $f(x) = 4\sin\left(\frac{2}{3}x\right)$? (1)

A. The amplitude of the graph of f is 8.
B. The maximum value of the function is 4.
C. The range of the function is $\left[-\frac{2}{3}, \frac{2}{3}\right]$.
D. The period of the graph of f is $\frac{2}{3}$.

8. Let $z_1 = 1 + 2i$ and $z_2 = 2 - 2i$ be complex numbers. Which one of the following is equal to $\frac{z_1}{z_2}$? (1)

A. $\frac{1}{4} - \frac{3}{4}i$
B. $-\frac{1}{4} + \frac{3}{4}i$
C. $\frac{1}{8} + \frac{1}{4}i$
D. $\frac{1}{8} - \frac{1}{4}i$

9. Let $z_1 = 3 - 4i$ and $z_2 = -4 + 3i$ be complex numbers. Which one of the following is equal to $z_1 - z_2$? (1)

A. $7 - 7i$
B. $-7 + 7i$
C. $1 - i$
D. $-1 + i$

10. Let $f(x) = x^2 + 2x - 5$. What is the meaning of the value $\frac{f(3)-f(1)}{3-1}$? It is the slope of the tangent line to the graph of f at: (1)

A. $(2.5, f(2.5))$
B. $(3, f(3))$
C. $(1, f(1))$
D. $(2, f(2))$

11. If θ is the principal argument of the complex number $z = 4 + 3i$, which one of the following is equal to $\tan\theta$? (1)

A. $\frac{3}{5}$
B. $\frac{4}{5}$
C. $\frac{3}{4}$
D. $\frac{4}{3}$

12. Let p, q, r, and s be propositions. Which one of the following defines that the argument form $p, q, r \vdash s$ is a valid argument? (1)

A. If $p \wedge (q \wedge r)$ is true, then s is true.
B. If $p \wedge (q \vee r)$ is true, then s is true.
C. If $p \vee (q \wedge r)$ is true, then s is true.
D. If $p \vee (q \vee r)$ is true, then s is true.

13. Let A and B be two nonempty sets. Which one of the following is true about an onto function $f : A \to B$? (1)

A. For any $x \in A$, there exists $y \in B$ such that $f(y) = x$.
B. For $x, y \in A$, if $f(x) = f(y)$, then $x - y = 2$.
C. The range of f is a proper subset of B.
D. For any $y \in B$, there exists $x \in A$ such that $f(x) = y$.

14. Which one of the following is the standard form of the equation of a parabola with vertex $V(1, -4)$ and focus $F(1, 4)$? (1)

A. $(x-1)^2 = 16(y+4)$
B. $(x-1)^2 = 16(y-4)$
C. $(x-1)^2 = 32(y+4)$
D. $(x-1)^2 = 32(y-4)$

15. Which of the following is the value of the tangent of the angle between the lines $\ell_1 : 3x + 2y = 3$ and $\ell_2 : 2x + 3y = 1$ measured counterclockwise from ℓ_1 to ℓ_2? (1)

A. $\frac{5}{12}$
B. $-\frac{5}{12}$
C. $\frac{5}{6}$
D. $-\frac{5}{6}$

16. Let $p(n)$: For any natural number n, $n^2 + 3n + 33$ is divisible by 3. We can show that $p(1)$ is false. What kind of proof have we used? (1)

A. Proof by contrapositive
B. Proof by counterexample
C. Proof by contradiction
D. Proof by direct proof

17. Which one of the following is true about a right-angled triangle $\triangle ABC$ with a right angle at B? (1)

A. $\cot(\angle ACB) = \frac{BC}{AB}$
B. $\sin(\angle BAC) = \frac{AC}{BC}$
C. $\csc(\angle BAC) = \frac{AB}{AC}$
D. $\csc(\angle BCA) = \frac{BC}{AC}$

18. Which one of the following is true about a rigid motion? (1)

A. Reflection is an example of a rigid motion.

B. A rotation by 180 degrees is the identity rotation.

C. A rigid motion carries an isosceles triangle to a scalene triangle.

D. Every rigid motion preserves the orientation of plane figures.

19. Consider the sequence defined by $a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Which one of the following is equal to a_{15}? (1)

A. 377

B. 233

C. 987

D. 610

20. A radioactive substance of 1024 grams loses half of its weight per year. What is the weight of the substance that will be left at the end of the ninth year? (1)

A. 2 grams

B. $\frac{1}{2}$ grams

C. 4 grams

D. $\frac{1}{4}$ grams

21. What is the value of (1)

$$\lim_{x \to 0} \frac{\sqrt{4-x} - 2}{x^2 - x}?$$

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. $\frac{1}{8}$

D. $-\frac{1}{8}$

22. Which one of the following is the distance between the points $P(1, -1, 2)$ and $Q(3, 1, -1)$ in space? (1)

A. 7 units

B. 17 units

C. $\sqrt{7}$ units

D. $\sqrt{17}$ units

23. Which one of the following numbers satisfies Rolle's Theorem for the function $f(x) = \frac{1}{4}x^4 - 2x^2$ on the closed interval $[-3, 3]$? (1)

A. 3

B. 4

C. 1

D. 2

24. Let $f(x) = x^3 + 2x^2 - 3$. Which one of the following is true about f? (1)

A. f is increasing on $(0, \infty)$.

B. f is decreasing on $\left(\frac{4}{3}, \infty\right)$.

C. f is increasing on $\left(-\frac{4}{3}, 0\right)$.

D. f is decreasing on $\left(-\infty, -\frac{4}{3}\right)$.

25. Which one of the following is equal to $\int \cos(3x+5)\, dx$? (1)

A. $\frac{\sin(3x+5)}{3} + c$

B. $\frac{\sin(3x+5)}{5} + c$

C. $5\sin(3x+5) + c$

D. $3\sin(3x+5) + c$

26. Which one of the following is equal to $\int 3x^2(x^3+2)^4\, dx$? (1)

A. $\frac{x^3(x^3+2)^5}{15} + c$

B. $\frac{(x^3+2)^5}{5} + c$

C. $15x^3(x^3+2)^5 + c$

D. $5x^3(x^3+2)^5 + c$

27. Let f and g be continuous functions on an interval I. If $\int f(x)\,dx = F(x)$ and $\int g(x)\,dx = G(x)$ on I, which one of the following expressions is true? (1)

A. $\int (f+g)(x)\,dx = (F+G)(x)$

B. $\int (fg)(x)\,dx = (FG)(x)$

C. $\int \left(\frac{f}{g}\right)(x)\,dx = \frac{F(x)}{G(x)}$

D. $\int (2f)(x)\,dx = \frac{F(x)}{2}$

28. What is the sum of the first 7 terms of a sequence defined by $G_n = 10^{n-4}$ for $n = 1, 2, 3, \ldots$? (1)

A. 1111.001

B. 1111.111

C. 1110.111

D. 1110.111

29. Which one of the following is the greatest lower bound of the sequence $\left\{\frac{2n-3}{3n+2}\right\}$? (1)

A. $-\frac{3}{5}$

B. $-\frac{1}{2}$

C. $-\frac{3}{2}$

D. $-\frac{1}{5}$

30. Let $v = \vec{AB}$ and $w = \vec{CD}$ be vectors in the plane where $A(-1,2), B(2,1), C(3,1)$, and $D(4,-3)$. Which one of the following is equal to the scalar (or dot) product of $\vec{v}$ and $\vec{w}$? (1)

A. 7

B. 8

C. 5

D. 3

31. Let $M = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 3 & 4 \\ -1 & -1 & 2 \end{pmatrix}$. Which one of the following is the $\det(M)$? (1)

A. -10

B. 0

C. 30

D. 10

32. Let A, B, and C be three exhaustive and mutually exclusive events of a random experiment, and S be the sample space of the given experiment. Which one of the following is true? (1)

A. $A \cap B \cap C = S$

B. $A \cap B = S \cup C$

C. $A \cup B \cup C = S$

D. $A \cup B = S \cup C$

33. Which one of the following is the contrapositive of the statement "If x is an even natural number, then x is divisible by 2"? (1)

A. If x is a natural number that is not divisible by 2, then x is not even.

B. If x is an even natural number, x is NOT divisible by 2.

C. If x is a natural number divisible by 2, then x is even.

D. If x is not an even natural number, then x is not divisible by 2.

34. Let $P(n,r)$ be the number of permutations of n objects taking r at a time, and $C(n,r)$ be the number of ways of selecting r objects from a set of n objects without considering the order of selection. Which of the following is true? (1)

A. $C(10,4) = P(10,4) \cdot 4!$

B. $P(10,4) = P(10,4) \cdot 4!$

C. $C(10,5) = P(10,5) \cdot 5!$

D. $P(10,5) = C(10,5) \cdot 5!$

35. Let $f(x) = 2\sin\left(\frac{1}{2}x\right)$. Which one of the following is equal to $f^{(5)}(x)$? (1)

A. $-\frac{1}{16}\sin\left(\frac{1}{2}x\right)$

B. $-\frac{1}{16}\cos\left(\frac{1}{2}x\right)$

C. $\frac{1}{16}\sin\left(\frac{1}{2}x\right)$

D. $\frac{1}{16}\cos\left(\frac{1}{2}x\right)$

36. Which one of the following is the derivative of the function $f(x) = \frac{x^2e^x}{x+1}$? (1)

A. $\dfrac{(2x+2x^2+x^3)e^x}{(x+1)^2}$

B. $\dfrac{(2x+2x^2+x^3)e^x}{x^2+1}$

C. $\dfrac{(2x+x^2)e^x}{(x+1)^2}$

D. $\dfrac{(2x+x^2)e^x}{x^2+1}$

37. What is the derivative of the function $f(x) = \ln(\sqrt{x^2+1})$? (1)

A. $\frac{2x}{2\sqrt{x^2+1}}$

B. $\frac{2x+1}{2x^2+1}$

C. $\frac{x}{x^2+1}$

D. $\frac{x}{\sqrt{x^2+1}}$

38. Let a and b be real numbers and $a < b$. Let f be a continuous function on $[a, b]$ and F be an antiderivative of f. Which one of the following is true? (1)

A. $\int_a^b f(x)\,dx = F(a) - F(b)$

B. $\int_a^b f(x)\,dx = F(b) - F(a)$

C. $\int_a^b F(x)\,dx = f(b) - f(a)$

D. $\int_a^b F(x)\,dx = f(a) - f(b)$

39. Which one of the following is true in a standard coordinate space? (1)

A. The intersection of the x-axis and the x-axis is the yz-plane.

B. The intersection of the xz-plane and the xy-plane is the xz-plane.

C. The x-axis and the y-axis determine the xy-plane.

D. The y-axis and the z-axis determine the xz-plane.

40. Suppose S and T can complete a job in two hours if they work together. T needs 3 more hours than S to complete the job. If T works alone, in how many hours can he complete the job? (1)

A. 3 hours

B. 6 hours

C. 9 hours

D. 12 hours

41. Which one of the following points is the intersection point of the line $l : 3x - 2y + 4 = 0$ and the circle $c : x^2 + y^2 - 2x - 4y - 5 = 0$? (1)

A. $(-4, -4)$

B. $(2, 5)$

C. $(4, 8)$

D. $(-2, -1)$

42. Which one of the following is NOT true about the graph of the function $f(x) = \frac{x^2-4}{x^4+5x^2+6}$? (1)

A. The graph of f is symmetric with respect to the y-axis.

B. The graph of f is symmetric with respect to the origin.

C. The x-intercepts of the graph of f are $(-2, 0)$ and $(2, 0)$.

D. The y-intercepts of the graph of f are $(0, -\frac{2}{3})$.

43. Consider the ellipse defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{16} = 1$. Which one of the following is true about the ellipse? (1)

A. The eccentricity of the ellipse is $\frac{3}{5}$.

B. The length of the latus rectum of the ellipse is 9 units.

C. The length of its major axis is 50 units.

D. The length of its minor axis is 32 units.

44. Which one of the following is the limit of the sequence $\left\{\frac{2n^2-3n+4}{2-5n-6n^2}\right\}$? (1)

A. $\frac{1}{3}$
B. $\frac{2}{3}$
C. $-\frac{1}{3}$
D. $-\frac{2}{3}$

45. Let $\vec{C} = 3i - 2j$ and $\vec{P} = 6i + 2j$ be the position vectors of the center C of a circle and a point P on the circle respectively. Which one of the following is an equation of the circle? (1)

A. $x^2 + y^2 + 6x - 4y - 12 = 0$
B. $x^2 + y^2 + 6x - 4y + 12 = 0$
C. $x^2 + y^2 - 6x + 4y - 12 = 0$
D. $x^2 + y^2 - 6x + 4y + 12 = 0$

46. Let P be a point in the coordinate space. If P is located 3 units in the direction of the negative x-axis from the origin, 1 unit in the direction of the positive y-axis from the origin, and 2 units in the direction of the positive z-axis from the origin, which one of the following ordered triple of numbers represents point P? (1)

A. $(3, -1, 2)$
B. $(3, 1, -2)$
C. $(3, -1, -2)$
D. $(-3, 1, 2)$

47. Let A and B be two nonempty sets and $f : A \to B$ be a function. Which one of the following is a sufficient condition so that f has an inverse function? (1)

A. f has to be a 1-1 correspondence.
B. The range of f is B.
C. f is a one-to-one function.
D. f is an onto function.

48. Let f be a function defined by: (1)

$$f(x) = \begin{cases} 3 - bx^2, & \text{for } x \geq 1 \\ ax, & \text{for } -1 < x < 1 \\ 2x + b, & \text{for } x \leq -1 \end{cases}$$

What are the values of a and b so that f is continuous on the whole real line?

A. $a = -3$ and $b = 6$
B. $a = 3$ and $b = -6$
C. $a = 3$ and $b = 6$
D. $a = -3$ and $b = -6$

49. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Which one of the following is true about minors and cofactors of entries of A? (1)

A. The minor of the entry 5 is 12.
B. The cofactor of the entry 7 is 3.
C. The minor of the element 9 is 3.
D. The cofactor of the element 6 is 6.

50. Consider the following system of linear equations: (1)

$$\begin{cases} x + y = 0 \\ y + z = 1 \\ x + z = 0 \end{cases}$$

Which one of the following is the solution set of the given system of linear equations?

A. $\left\{ \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right\}$
B. $\left\{ \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right\}$
C. $\left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \right\}$
D. $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$

51. The angle of elevation from the top of a building of height 15 meters to the top of building A is 30°. If the distance between the two buildings is 60 meters, what is the height of building A? (1)

A. $20\sqrt{3}$ meters

B. $(20\sqrt{3}+15)$ meters

C. $60\sqrt{3}$ meters

D. $(60\sqrt{3}+15)$ meters

52. Let f and g be two continuous functions on the closed interval $[a,b]$, k be a constant, and c be a real number between a and b. Which one of the following is NOT true about the functions f and g? (1)

A. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

B. $\int_a^b f(x)\,dx = \int_c^b f(x)\,dx - \int_a^c f(x)\,dx$

C. $\int_a^b (kf)(x)\,dx = k\int_a^b f(x)\,dx$

D. $\int_a^b (f+g)(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

END OF THE EXAMINATION

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