

Mathematics, 2016 EC, model exam, Arada

Grade 12 · Mathematics · 2016 E.C.

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1. If x and y are composite natural numbers and z is a prime number, then which one of the following is never true? (1)

- A. $x + y = z$ B. $x + y = 2z$
C. $xz = y$ D. $xy = z^2$

Answer: D

Since z is a prime number, the only factors of z^2 are $1, z, z^2$. If $xy = z^2$ where x and y are composite, they must both be greater than 1, so the only possibility is $x = z$ and $y = z$, which contradicts the assumption that z is prime and x, y are composite.

2. The simplified form of the expression $(\frac{5}{4})^3 (\frac{\sqrt{5}}{2})^{-4} \div (\frac{3\sqrt{5}}{\sqrt{2}})^6$ is: (1)

- A. $\frac{5}{2}$ B. $\frac{2}{5}$
C. $\frac{1}{5}$ D. 5

Answer: B

Simplifying each term: $(\frac{5}{4})^3 = \frac{125}{64}$, $(\frac{\sqrt{5}}{2})^{-4} = (\frac{2}{\sqrt{5}})^4 = \frac{16}{25}$, and $(\frac{3\sqrt{5}}{\sqrt{2}})^6 = \frac{25}{8}$. Multiplying the first two terms gives $\frac{125}{64} \times \frac{16}{25} = \frac{5}{4}$, and dividing by the third term gives $\frac{5}{4} \div \frac{25}{8} = \frac{5}{4} \times \frac{8}{25} = \frac{2}{5}$.

3. To rationalize the denominator $\frac{3+\sqrt{2}}{4-\sqrt{3}}$ which one of the following expression is used as the rationalizing factor? (1)

- A. $\frac{3-\sqrt{2}}{3-\sqrt{2}}$ B. $\frac{3-\sqrt{2}}{4+\sqrt{3}}$
C. $\frac{4+\sqrt{3}}{4+\sqrt{3}}$ D. $\frac{1}{4+\sqrt{3}}$

Answer: C

To rationalize the denominator of a fraction of the form $\frac{a}{b-\sqrt{c}}$, we multiply both the numerator and the denominator by the conjugate of the denominator, which is $b + \sqrt{c}$. Thus, the rationalizing factor is $\frac{4+\sqrt{3}}{4+\sqrt{3}}$.

4. What is the value of x for $2\left(\frac{243}{32}\right)^{2x} = 3\left(\frac{8}{27}\right)^{\frac{2}{3}x-1}$? (1)

- A. $\frac{1}{3}$ B. $\frac{4}{3}$
C. $\frac{2}{5}$ D. $\frac{1}{4}$

Answer: A

Expressing the bases in terms of $\frac{3}{2}$, the equation becomes $2\left(\left(\frac{3}{2}\right)^5\right)^{2x} = 3\left(\left(\frac{3}{2}\right)^{-3}\right)^{\frac{2}{3}x-1}$, which simplifies to $2\left(\frac{3}{2}\right)^{10x} = 3\left(\frac{3}{2}\right)^{-2x+3}$. Dividing both sides yields $\left(\frac{3}{2}\right)^{12x-3} = \frac{3}{2}$, so $12x - 3 = 1$, giving $x = \frac{1}{3}$.

5. Which one of the following systems of linear equation has infinitely many solutions? (1)

A. ✓ $\begin{cases} 3x + y = -1 \\ -x - \frac{1}{3}y = \frac{1}{3} \end{cases}$

B. $\begin{cases} x + y = 6 \\ 2x - 5y = 0 \end{cases}$

C. $\begin{cases} x + 3y = 2 \\ 2x - 4y = 1 \end{cases}$

D. $\begin{cases} x - 2y = 5 \\ -3x + 6y = 1 \end{cases}$

Answer: A

Multiplying the second equation in Option A, $-x - \frac{1}{3}y = \frac{1}{3}$, by -3 gives $3x + y = -1$. This is identical to the first equation, meaning they represent the same line and therefore have infinitely many solutions.

6. What is the solution set of the inequality $|3x + 6| - 2 \geq 4$? (1)

A. $(-\infty, -4) \cup (0, \infty)$

B. $[-4, 0]$

C. ✓ $(-\infty, -4] \cup [0, \infty)$

D. $(-4, 0)$

Answer: C

The inequality $|3x + 6| - 2 \geq 4$ simplifies to $|3x + 6| \geq 6$, which implies $|x + 2| \geq 2$. This means $x + 2 \geq 2$ (giving $x \geq 0$) or $x + 2 \leq -2$ (giving $x \leq -4$), which in interval notation is $(-\infty, -4] \cup [0, \infty)$.

7. The solution set of the inequality $3x^2 + 2x - 1 < 0$ is: (1)

A. $(-\infty, -1) \cup (\frac{1}{3}, \infty)$

B. ✓ $(-1, \frac{1}{3})$

C. $\emptyset$

D. $x : x \in \mathbb{R}$

Answer: B

The quadratic expression can be factored as $3x^2 + 2x - 1 = (3x - 1)(x + 1) < 0$. The roots are $x = -1$ and $x = \frac{1}{3}$, and since the coefficient of x^2 is positive, the inequality is satisfied on the interval between the roots, which is $(-1, \frac{1}{3})$.

8. For any two non-empty sets A and B. If $A \subseteq B$, then which one of the following is always true? (1)

A. $A \cap B = B$

B. $A \cup B = A$

C. ✓ $A \cap B = A$

D. $(A \cup B) \cap A = B$

Answer: C

By definition, if $A \subseteq B$, then every element in A is also in B. Therefore, their intersection $A \cap B$ is exactly the set A.

9. Which one of the following is Not true about the graph of the function $f(x) = 2x^2 - 8x + 3$? (1)

A. It's range is $y : y \geq -5$.

B. It's vertex is $(2, -5)$.

C. It's axis of symmetry is $x = 2$.

D. ✓ It's maximum value is -5 .

Answer: D

The quadratic function $f(x) = 2x^2 - 8x + 3$ opens upward because the coefficient of x^2 is positive ($2 > 0$). Therefore, the vertex at $(2, -5)$ represents the minimum value of the function, not a maximum value.

10. If $f(x) = \frac{2x+4}{x-1}$. Then, which one of the following points is on the graph of $f^{-1}(x)$? (1)

A. ✓ $(1, -5)$

B. $(3, 5)$

C. $(0, -4)$

D. $(2, 8)$

Answer: A

A point (a, b) is on the graph of $f^{-1}(x)$ if and only if (b, a) is on the graph of $f(x)$, meaning $f(b) = a$. Testing $(1, -5)$ gives $f(-5) = \frac{2(-5)+4}{-5-1} = \frac{-6}{-6} = 1$, which confirms that $(1, -5)$ is on the inverse function's graph.

The base area of the equilateral triangle is $A_{\text{base}} = \frac{\sqrt{3}}{4}s^2 = \frac{\sqrt{3}}{4}(8)^2 = 16\sqrt{3} \text{ cm}^2$. The volume of the pyramid is $V = \frac{1}{3}A_{\text{base}}h = \frac{1}{3}(16\sqrt{3})(9) = 48\sqrt{3} \text{ cm}^3$.

16. Let the radii of bases of a frustum of cone are 6 cm and 3 cm . If the slant height of a frustum of a cone is 10 cm , then what is the total surface area of the frustum? (1)
- A. $90\pi\text{ cm}^2$ B. $45\pi\text{ cm}^2$
C. $\checkmark\ 135\pi\text{ cm}^2$ D. $180\pi\text{ cm}^2$

Answer: C

The total surface area of a frustum of a cone is given by $\pi l(R + r) + \pi R^2 + \pi r^2$. Substituting $R = 6, r = 3, l = 10$ gives $\pi(10)(9) + \pi(36) + \pi(9) = 90\pi + 45\pi = 135\pi\text{ cm}^2$.

17. Suppose the mid-point of a line segment is $M(-2, 3)$. If one end point of the segment is $P(4, -8)$, then what is the coordinate of the other end-point? (1)
- A. $(-4, 6)$ B. $(3, -5)$
C. $\checkmark\ (-8, 14)$ D. $(6, 3)$

Answer: C

Using the midpoint formula $M = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$, we have $-2 = \frac{4+x}{2} \implies x = -8$ and $3 = \frac{-8+y}{2} \implies y = 14$. Thus, the other endpoint is $(-8, 14)$.

18. What is the area of the triangle in (sq. unit) formed by the line joining the vertex of the parabola $x^2 = 64y$ to the end points of the latus rectum? (1)
- A. 64 B. 128
C. 256 D. $\checkmark\ 512$

Answer: D

The equation of the parabola is $x^2 = 64y$, which has vertex $(0, 0)$ and focal length $p = 16$. The endpoints of the latus rectum are $(\pm 2p, p) = (\pm 32, 16)$, so the triangle has base 64 and height 16, giving an area of $\frac{1}{2} \times 64 \times 16 = 512$.

19. The equation of an ellipse with center at $(1, 5)$ and vertices at $(10, 5)$ and $(1, 2)$ is: (1)
- A. $(x - 1)^2 + 9(y - 5)^2 = 1$ B. $\checkmark\ (x - 1)^2 + 9(y - 5)^2 = 81$
C. $9(x - 1)^2 + (y - 5)^2 = 1$ D. $9(x - 1)^2 + (y - 5)^2 = 81$

Answer: B

The distance from the center $(1, 5)$ to the horizontal vertex $(10, 5)$ is $a = 9$, and to the vertical vertex $(1, 2)$ is $b = 3$. The equation of the horizontal ellipse is $\frac{(x-1)^2}{81} + \frac{(y-5)^2}{9} = 1$, which simplifies to $(x - 1)^2 + 9(y - 5)^2 = 81$.

20. If the standard deviation of $x_1, x_2, x_3, \dots, x_n$ is 6. Then, what is the variance of $2x_1 + 4, 2x_2 + 4, 2x_3 + 4, 2x_1 + 4$ (1)
- A. 16 B. $\checkmark\ 144$
C. 76 D. 148

Answer: B

For any constant scaling a and shift b , the variance of $aX + b$ is $a^2\text{Var}(X)$. Here, the standard deviation is 6, so the original variance is 36, and the variance of $2X + 4$ is $2^2 \times 36 = 144$.

21. The following is the distribution of the weight of 36 persons. (1)

Weight in kg	40 - 49	50 - 59	60 - 69	70 - 79
Frequency	6	10	17	3

Then, what is the 20th percentile (p_{20}) of the measurement?

- A. ✓ 50.7
B. 50.4
C. 51.4
D. 51.7

Answer: A

The 20th percentile corresponds to the $0.20 \times 36 = 7.2$ -th value, which lies in the class 50 – 59 (boundaries 49.5 – 59.5). Using the percentile formula $P_{20} = L + \left(\frac{7.2 - CF}{f} \right) w = 49.5 + \left(\frac{7.2 - 6}{10} \right) 10 = 50.7$.

22. Let a sample space be $s = \{w_1, w_2, w_3, w_4\}$. Which of the following assignment of probabilities to each outcome are valid? (1)

Assignment	w_1	w_2	w_3	w_4
P	1/4	3/4	1/2	-1/2
Q	1/4	1/4	1/4	1/4
R	0	1	0	0
S	1/2	1/4	1/4	0
T	1/2	3/2	1/4	1/3

- A. Q, R and T
B. ✓ Q, R and S
C. Q, S and T
D. P, S and T

Answer: B

A probability assignment is valid if all probabilities are non-negative and sum to 1. Assignments Q, R, and S satisfy these conditions, whereas P has a negative probability and T sums to more than 1.

23. Given the data (1)

x	2	3	4	6
f	5	4	2	1

Then, what is the mean deviation about the mode of the data set?

- A. ✓ 1
B. $\frac{1}{4}$
C. $\frac{1}{2}$
D. $\frac{1}{6}$

Answer: A

The mode of the data is 2 since it has the highest frequency of 5. The mean deviation about the mode is calculated as $\frac{\sum f_i |x_i - M_o|}{N} = \frac{5|2-2| + 4|3-2| + 2|4-2| + 1|6-2|}{12} = \frac{0+4+4+4}{12} = 1$.

24. The mean of 100 observation was calculated as 40 by a student who took mistake 40 instead of 90 for one observation. Then, what is the correct mean? (1)

- A. 39.5
B. ✓ 40.5
C. 41
D. 41.5

Answer: B

The incorrect sum of the 100 observations is $100 \times 40 = 4000$. Correcting the mistaken value by subtracting 40 and adding 90 gives the correct sum of $4000 - 40 + 90 = 4050$, so the correct mean is $4050/100 = 40.5$.

25. If the mean of the data 6,3,2,9, x , 9 is 5, then what is the mean deviation about the mean? (1)

- A. 4
B. 1
C. 2
D. ✓ 3

Answer: D

Since the mean is 5, the sum of the data is $6 \times 5 = 30$, which means $29 + x = 30 \implies x = 1$. The absolute deviations of the data set $\{6, 3, 2, 9, 1, 9\}$ from the mean 5 are $\{1, 2, 3, 4, 4, 4\}$, whose mean is $18/6 = 3$.

26. The coefficient of mean deviation about the median of the data set (1)

x	1	2	3	4
f	2	4	3	1

is:

- A. 0.7
B. ✓ 0.35
C. 0.6
D. 0.45

Answer: B

For the given data set with $N = 10$, the median is 2. The mean deviation about the median is $\frac{\sum f_i |x_i - M|}{N} = \frac{2(1)+4(0)+3(1)+1(2)}{10} = 0.7$, making the coefficient of mean deviation about the median $\frac{0.7}{2} = 0.35$.

27. The median and the mean deviation about the median of gross incomes of four companies are given below: (1)

Company	md (median)	MD (md)
W	40,000	7,000
X	30,000	6,000
Y	50,000	4,000
Z	60,000	8,000

Then, which company income is most consistent?

- A. W
B. X
C. ✓ Y
D. Z

Answer: C

The consistency of the companies is compared using the coefficient of mean deviation about the median, given by $MD(md)/md$. For company Y, this coefficient is $4,000/50,000 = 0.08$, which is the lowest among all options, indicating it is the most consistent.

28. Which one of the following is always true? (1)

- A. If the mode, median and mean of a distribution are 4,5 and 6 respectively, then the distribution is negatively skewed.
B. If the 1^{st} , 2^{nd} and 3^{rd} quartiles of a distribution are 7,9 and 12 respectively, then the distribution is symmetrical.
C. If the mean, median and standard deviation of a distribution are 4,6 and 2 respectively, then the distribution is skewed to the right.
D. ✓ **If the distribution is negatively skewed, then the mean of the distribution is less than its second quartile.**

Answer: D

In a negatively skewed distribution, the mean is always less than the median. Since the median is equivalent to the second quartile (Q_2), the statement that the mean is less than the second quartile is always true.

29. In Addis Ababa there are 100 secondary school with 100,000 grade 12 students in total. Suppose Addis Ababa Education Bureau has selected 1,000 students from them for survey purposes by collected students' roster from all the schools and combined them into one. After assigning a new roll number to each students, the Bureau selected those numbers of students 40,140,240,340,440, ..., then which sampling method is the Bureau used? (1)
- A. Simple random
B. ✓ **Systematic**
C. Cluster
D. Stratified

Answer: B

Selecting students at regular intervals (such as every 100th student: 40, 140, 240, ...) from an ordered list after a random start is the definition of systematic sampling.

30. Let ABCD is a parallelogram and $\overrightarrow{AC}$ and $\overrightarrow{BD}$ be diagonals, then which one of the following is Not true? (1)
- A. $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$
B. $\overrightarrow{AB} = \overrightarrow{BC} - \overrightarrow{BD}$
C. $\overrightarrow{DC} + \overrightarrow{AD} = \overrightarrow{AC}$
D. ✓ $\overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{DC}$

Answer: D

In parallelogram $ABCD$, the opposite sides are represented by equal vectors, so $\overrightarrow{AB} = \overrightarrow{DC}$. Therefore, $\overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{DC}$ would imply $-\overrightarrow{BC} = 0$, which is false.

31. When $x^3 + 3x^2 - kx + 5$ is divided by $x + 2$, the remainder is 7. Then, what is the value of k? (1)
- A. 1
B. -2
C. ✓ -1
D. 2

Answer: C

By the Remainder Theorem, dividing $P(x) = x^3 + 3x^2 - kx + 5$ by $x + 2$ yields a remainder of $P(-2) = 7$. Solving $(-2)^3 + 3(-2)^2 - k(-2) + 5 = 7$ gives $9 + 2k = 7 \implies k = -1$.

32. Which one of the following is true about polynomial function ? (1)
- A. The domain of polynomial function is the set of positive real number
B. The graph of a polynomial function of degree n-has at most n-turning points.
C. ✓ **The graph of every polynomial function crosses the y-axis.**
D. The graph of a polynomial function of degree n-has at most n-1 zeros.

Answer: C

Because the domain of any polynomial function is the set of all real numbers, the value $f(0)$ is always defined, meaning the graph must cross the y-axis at $(0, f(0))$.

33. Let $f(x) = \left(\frac{3}{2}\right)^x$ and $g(x) = \left(\frac{2}{3}\right)^x$. Then, which one of the following is Not true about $f(x)$ and $g(x)$? (1)
- A. ✓ **If $x > 0$, then $f(x) < g(x)$.**
B. if $x = 0$, then $f(x) = g(x)$.
C. If $x < 0$, then $f(x) < g(x)$.
D. If $x > 0$, then $f(x) > g(x)$.

Answer: A

Since the base of $f(x)$ is $3/2 > 1$ and the base of $g(x)$ is $2/3 < 1$, for any $x > 0$, we have $f(x) > 1$ and $g(x) < 1$. Thus, $f(x) > g(x)$, making statement A false.

34. Which of the following is the solution set of the equation $\log_3 x + \log_{\sqrt{x}} 4x + \log_{\frac{1}{3}} x = 6$? (1)

- A. $\checkmark \{2\}$ B. $\{-2\}$
C. $\emptyset$ D. $\{-2, 2\}$

Answer: A

Using properties of logarithms, $\log_3 x + \log_{1/3} x = \log_3 x - \log_3 x = 0$, simplifying the equation to $\log_{\sqrt{x}}(4x) = 6$. This translates to $(\sqrt{x})^6 = 4x \implies x^3 = 4x$, and since $x > 0$ and $x \neq 1$, we get $x = 2$.

35. If $\tan(\theta - 60^\circ) = \cot 4\theta$. Then, what is the value of θ ? (1)

- A. 60° B. 40°
C. 50° D. $\checkmark 30^\circ$

Answer: D

Using the identity $\cot(4\theta) = \tan(90^\circ - 4\theta)$, we have $\theta - 60^\circ = 90^\circ - 4\theta + 180^\circ k$. Solving for θ gives $\theta = 30^\circ + 36^\circ k$, and for $k = 0$, we find the matching option $\theta = 30^\circ$.

36. 39. Which one of the following is Not true about the graph of $f(x) = \frac{x^3 - x}{x^4 - x^3}$? (1)

- A. The graph has a hole at point $(1, 2)$.
B. The graph intersects its horizontal asymptote at point $(-1, 0)$.
C. The y-axis is the vertical asymptote of the graph.
D. $\checkmark$ **The graph is symmetric with respect to the origin.**

Answer: D

The simplified function is $f(x) = \frac{x+1}{x^2}$ for $x \neq 1, 0$. Since $f(-x) = \frac{1-x}{x^2} \neq -f(x)$, the function is not odd, meaning its graph is not symmetric with respect to the origin.

37. 40. For any real number x and y which one of the following statement is true? (1)

- A. $(\forall_x)(\exists_y)(y^2 + x + 1 = 0)$ B. $(\exists_x \forall_y)(y^2 + x + 1 = 0)$
C. $\checkmark (\forall_y)(\exists_x)(y^2 + x + 1 = 0)$ D. $(\exists_y)(\forall_x)(y^2 + x + 1 = 0)$

Answer: C

For any real number y , we can always find a real number x defined by $x = -y^2 - 1$ such that $y^2 + x + 1 = 0$, making statement C true.

38. 41. If A is a 3×3 matrix and $|A^{-1}| = 3$, then what is the $|6A^T A|$? (1)

- A. 72 B. 36
C. $\checkmark$ 24 D. 18

Answer: C

Since $|A^{-1}| = 3$, we have $|A| = 1/3$. Using determinant properties for a 3×3 matrix, $|6A^T A| = 6^3 |A^T| |A| = 216 |A|^2 = 216 \times (1/9) = 24$.

39. 42. For What value(s) of k does $\begin{cases} x + 2z = 4 \\ 3x + 4y + 4z = 6 \\ kx + 3y - z = 1 \end{cases}$ has a unique solution? (1)

- A. $k = 4$ B. $k = \frac{1}{4}$
C. $k \neq 4$ D. $\checkmark k \neq \frac{1}{4}$

Answer: D

The system has a unique solution if the determinant of the coefficient matrix is non-zero. The determinant is $1(4(-1) - 12) + 2(9 - 4k) = 2 - 8k$, which is non-zero when $k \neq 1/4$.

40. 43. Let Z be a complex number and $a, b \in \mathbb{R}$. Then, which one of the following is Not true? (1)

- A. $z \cdot \bar{z} = |z|^2$
- B. If $Z = a + bi$, then $z + \bar{z} = 2a$
- C. If $z = -bi - a$, then $\bar{z} = bi - a$
- D. ✓ If $z \neq 0$, then it is multiplicative inverse is $\frac{z}{(z)^2}$.

Answer: D

The multiplicative inverse of a non-zero complex number z is $z^{-1} = \frac{\bar{z}}{|z|^2}$, not $\frac{z}{z^2}$ as suggested in option D, making D the incorrect statement.

41. 44. If the translation T takes the point $(3,1)$ to the point $(4,0)$. Then, what is the image of the circle $x^2 + y^2 = 2$ under the translation T ? (1)

- A. ✓ $x^2 - 2x + y^2 + 2y = 0$
- B. $x^2 + 2x + y^2 - 2y = 0$
- C. $x^2 + 2x + y^2 + 2y = 0$
- D. $x^2 - 2x + y^2 - 2y = 0$

Answer: A

The translation vector is $T = (1, -1)$ since $(3 + 1, 1 - 1) = (4, 0)$. Substituting x with $x - 1$ and y with $y + 1$ in $x^2 + y^2 = 2$ gives $(x - 1)^2 + (y + 1)^2 = 2$, which expands to $x^2 - 2x + y^2 + 2y = 0$.

42. 45. What is the amplitude and period of the graph of $f(x) = -4 \sin\left(\frac{1}{4}x\right) \cos\left(\frac{1}{4}x\right)$? (1)

- A. $4, \pi$
- B. ✓ $2, 4\pi$
- C. $4, 2\pi$
- D. $2, 2\pi$

Answer: B

Using the double-angle identity, the function simplifies to $f(x) = -2 \sin(x/2)$. The amplitude is $|-2| = 2$ and the period is $2\pi/(1/2) = 4\pi$.

43. 46. A patrol on a sea sailed from it's station $4\sqrt{2}km$ to the south and changed it's course and sailed $7km$ in the direction of 45° North-west. Then, what is the shortest(straight) distance the boat should travel in order to return to its station? (1)

- A. $5\sqrt{2}km$
- B. ✓ $5km$
- C. $(5 + \sqrt{2})km$
- D. $7km$

Answer: B

The initial position relative to the station $(0, 0)$ is $(0, -4\sqrt{2})$. Sailing $7 km$ North-West adds a displacement of $(-3.5\sqrt{2}, 3.5\sqrt{2})$, leading to the final position $(-3.5\sqrt{2}, -0.5\sqrt{2})$. The shortest distance back is $\sqrt{(-3.5\sqrt{2})^2 + (-0.5\sqrt{2})^2} = \sqrt{24.5 + 0.5} = 5 km$.

44. Which one of the following is the fourth term of the sequence $\left\{\cos\left(\frac{n\pi}{3}\right)\right\}_{n=0}^\infty$? (1)

- A. 0
- B. 1
- C. $-\frac{1}{2}$
- D. ✓ -1

Answer: D

Since the sequence index starts at $n = 0$ (the 1st term), the 4th term corresponds to $n = 3$. Calculating this term gives $\cos(3\pi/3) = \cos(\pi) = -1$.

- [illegible]

Answer: A

With 8 arithmetic means between 26 and -1 , the total number of terms is 10, where $a_1 = 26$ and $a_{10} = -1$. Thus, $26 + 9d = -1 \implies 9d = -27 \implies d = -3$.

- 46.** If the second and sixth terms of a geometric progression are 4 and $\frac{1}{4}$ respectively, then what is the sum of the first eight terms Of the sequence?
- A. ✓ $\frac{255}{16}$
- B. $\frac{127}{8}$
- C. $\frac{63}{4}$
- D. $\frac{129}{32}$

Answer: A

From the given terms, $ar = 4$ and $ar^5 = 1/4$, which gives $r^4 = 1/16 \implies r = 1/2$ and $a = 8$. The sum of the first 8 terms is $S_8 = 8 \frac{1-(1/2)^8}{1-1/2} = 16 \left(1 - \frac{1}{256}\right) = \frac{255}{16}$.

47. What is the sum of the of the infinite series $\frac{1}{2} - \frac{2}{5} + \frac{1}{8} - \frac{2}{25} + \frac{1}{32} - \frac{2}{125} + \cdots$? (1)
- A. $\frac{7}{10}$ B. ☒ $\frac{1}{6}$
- C. $\frac{3}{10}$ D. $\frac{7}{6}$

Answer: B

The series can be split into two infinite geometric series: $S_1 = \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots = \frac{1/2}{1-1/4} = \frac{2}{3}$ and $S_2 = -\left(\frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \cdots\right) = -\frac{2/5}{1-1/5} = -\frac{1}{2}$. Their sum is $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.

- 48.** A contest offers a total of 10 prizes. The first prize is worth Birr 20,000 and each consecutive prize is worth Birr 1500 less than the next higher prize. What is the total value of the prizes in Birr?
- A. 135,000 B. ✓ **132,500**
- C. 142,300 D. 141,500

Answer: B

This is an arithmetic progression with first term $a_1 = 20,000$, common difference $d = -1500$, and $n = 10$ terms. The sum is $S_{10} = \frac{10}{2}[2(20,000) + 9(-1500)] = 132,500$ Birr.

49. A ball is dropped from the height of $16m$ to the horizontal floor. In each bounce it rebounds 60% Of the distance it fell. What maximum possible(total) vertical distance that could be covered by the ball ? (Assume that the ball never comes to rest).
- A. 240m
B. 120m
C. ✓ 64m
D. 80m

Answer: C

The total vertical distance covered by the ball is given by $h + 2 \sum_{n=1}^{\infty} hr^n = h \frac{1+r}{1-r}$. Substituting $h = 16$ and $r = 0.6$, we get $16 \times \frac{1.6}{0.4} = 64$ m.

50. Which one of the following series is convergent?

(1)

- A. $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{-n}$
- B. $\sum_{n=1}^{\infty} (-2)^n$
- C. ✓ $\sum_{n=1}^{\infty} 2^{-n}$
- D. $\sum_{n=1}^{\infty} (\sqrt{2})^n$

Answer: C

The series $\sum_{n=1}^{\infty} 2^{-n}$ is a geometric series with common ratio $r = 1/2$. Since $|r| < 1$, the series is convergent.

51. What is the absolute maximum value of the function $f(x) = x^3 - 12x + 4$ on $[-3, 3]$?

(1)

- A. ✓ 20
- B. 13
- C. 24
- D. 18

Answer: A

The critical points of $f(x)$ are found where $f'(x) = 3x^2 - 12 = 0$, giving $x = \pm 2$. Evaluating $f(x)$ at the critical points and the endpoints of $[-3, 3]$ yields $f(-3) = 13$, $f(-2) = 20$, $f(2) = -12$, and $f(3) = -5$, so the absolute maximum is 20.

52. In which interval that the graph of the function $f(x) = \frac{x}{x^2+1}$ is increasing ?

(1)

- A. $(-\infty, -1] \cup [1, \infty)$
- B. ✓ $[-1, 1]$
- C. $(-\infty, 1]$
- D. $[1, \infty)$

Answer: B

The derivative is $f'(x) = \frac{1-x^2}{(x^2+1)^2}$. The function is increasing when $f'(x) \geq 0$, which occurs when $1 - x^2 \geq 0$, meaning $x \in [-1, 1]$.

53. The radius of a spherical gas balloon is increasing at a rate of 2 cm/sec . At what rate it's surface area increasing when the radius is 3 cm ?

- A. $52\pi\text{ cm}^2/\text{sec}$
- B. ✓ $48\pi\text{ cm}^2/\text{sec}$
- C. $32\pi\text{ cm}^2/\text{sec}$
- D. $16\pi\text{ cm}^2/\text{sec}$

Answer: B

The surface area of a sphere is $A = 4\pi r^2$. Differentiating with respect to time t gives $\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(3)(2) = 48\pi\text{ cm}^2/\text{sec}$.

54. A rectangular field of 1600 m^2 is to be fenced off. Two opposite sides will fencing costing 2 Birr per meter and the remaining sides will use fencing costing 8 Birr per meter. How much is the least cost of fencing it in Birr?

(1)

- A. 320
- B. 420
- C. 560
- D. ✓ 640

Answer: D

Letting the side lengths be x and y with $xy = 1600$, the cost is $C = 4x + 16y = 4x + \frac{25600}{x}$. Taking the derivative and setting it to zero gives $x = 80$, resulting in a minimum cost of $C(80) = 4(80) + 16(20) = 640$ Birr.

55. $\int \left(5x^4 - \frac{1}{\sqrt{x}}\right) dx$ is equal to:

(1)

- A. $\frac{1}{5}x^5 - 2\sqrt{x} + c, c \in \mathbb{R}$
- B. ✓ $x^5 - 2\sqrt{x} + c, c \in \mathbb{R}$
- C. $x^5 - 4\sqrt{x} + c, c \in \mathbb{R}$
- D. $\frac{4}{5}x^5 - 2\sqrt{x} + c, c \in \mathbb{R}$

Answer: B

Using the power rule of integration, we have $\int 5x^4 dx = x^5 + C_1$ and $\int x^{-1/2} dx = 2x^{1/2} + C_2$, so the total antiderivative is $x^5 - 2\sqrt{x} + c$.

56. $\int_1^3 \left(3x + \frac{1}{x^2}\right) dx$ is equal to: (1)
- A. $\frac{35}{3}$ B. $\checkmark \frac{38}{3}$
 C. $\frac{34}{3}$ D. $\frac{37}{3}$

Answer: B

The antiderivative is $F(x) = \frac{3}{2}x^2 - \frac{1}{x}$. Evaluating this from 1 to 3 gives $F(3) - F(1) = \left(\frac{27}{2} - \frac{1}{3}\right) - \left(\frac{3}{2} - 1\right) = \frac{79}{6} - \frac{3}{6} = \frac{38}{3}$.

57. What is the area of the region bounded by the graph of $f(x) = -x^2 + 4x - 3$ and the x-axis from $x = 0$ and $x = 3$ in square units? (1)
- A. $\checkmark \frac{8}{3}$ B. $\frac{7}{3}$
 C. $\frac{10}{3}$ D. $\frac{11}{3}$

Answer: A

The graph of $f(x) = -x^2 + 4x - 3 = -(x-1)(x-3)$ is negative on $[0, 1]$ and positive on $(1, 3]$. Thus, the area is $\int_0^1 (x^2 - 4x + 3)dx + \int_1^3 (-x^2 + 4x - 3)dx = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$.

58. The solution region of the system of inequalities $\begin{cases} 2x - y \leq 3 \\ x > 0 \\ y < 0 \end{cases}$ is found in quadrant _____. (1)
- A. One B. Two
 C. Three D. $\checkmark$ four

Answer: D

The inequalities $x > 0$ and $y < 0$ strictly restrict the solution set to the fourth quadrant, and points in this quadrant like $(1, -0.5)$ also satisfy $2x - y \leq 3$.

59. The solution set of the system of inequalities $\begin{cases} 2x - y \geq 4 \\ x + y \geq 2 \end{cases}$ is (1)
- A. $\checkmark \{(x, y): x \geq 2 \text{ and } -x + 2 \leq y \leq 2x - 4\}$ B. $\{(x, y): x \geq 0 \text{ and } 2 - x \leq y \leq 2x - 4\}$
 C. $\{(x, y): x \geq 2 \text{ and } y \geq 0\}$ D. $\{(x, y): x \geq 0 \text{ and } 2x - 4 \leq y \leq 2 - x\}$

Answer: A

Rewriting the inequalities gives $y \leq 2x - 4$ and $y \geq 2 - x$, which means $2 - x \leq y \leq 2x - 4$. For this interval to be non-empty, we require $2 - x \leq 2x - 4$, which simplifies to $x \geq 2$.

60. The minimum value of the objective function $z = 3x + 5y$ (1)
- Subject to $\begin{cases} x + 3y \geq 3 \\ x + y \geq 2 \\ x \geq 0, y \geq 0 \end{cases}$ is:
- A. 5 B. 6
 C. $\checkmark$ 7 D. 8

Answer: C

The corner points of the feasible region are $(0, 2)$, $(1.5, 0.5)$, and $(3, 0)$. Evaluating $z = 3x + 5y$ at these points gives values of 10, 7, and 9 respectively, so the minimum value is 7.

- 61.** The corner points of the feasible region determined by the following systems of linear inequalities $y \geq x$, $x + 3y \leq 60$, $x + y \geq 10$, $x \geq 0$, $y \geq 0$ are $(0, 20)$, $(15, 15)$, $(0, 10)$ and $(5, 5)$. Let $z = 4ax + 2by$, where $a, b > 0$. Condition on a and b so that the maximum of z occurs at both $(15, 15)$ and $(0, 20)$ is:
- A. ✓ $b = 6a$ B. $a = 6b$
- C. $b = 2a$ D. $a = 2b$

Answer: A

For the maximum to occur at both $(15, 15)$ and $(0, 20)$, their objective function values must be equal, so $4a(15) + 2b(15) = 4a(0) + 2b(20)$, which simplifies to $60a + 30b = 40b$, or $b = 6a$.

62. A manufacturer produces two types of nails, N_1 and N_2 . It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of N_1 . It takes 3 hours of work on machine A and 1 hour on machine B to produce a package of N_2 . He earns a profit of Birr 17.50 per package on N_1 and Birr 7.00 per package on N_2 . How many packages of N_1 and N_2 , respectively should be produced each day so as to maximize his profit, if he operates his machines for at most 12 hours a day?
- A. 4 and 4
B. ✓ 3 and 3
C. 3 and 4
D. 4 and 3

Answer: B

The constraints are $x + 3y \leq 12$ and $3x + y \leq 12$. The corner points of the feasible region are $(0, 0)$, $(4, 0)$, $(0, 4)$, and $(3, 3)$, and evaluating the profit function $P = 17.5x + 7y$ shows it is maximized at $(3, 3)$ with a profit of 73.5 Birr.

END OF THE MARKING KEY

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