

Mathematics, 2015 EC, entrance exam, version 1

Grade 12 · Mathematics · 2015 E.C.

Time allowed: 120 minutes

Questions: 57

Total marks: 57

Name _____ Grade _____ ID No. _____ Date _____

1. If $f(x) = \frac{\sqrt{x+2}}{x+2}$ and $g(x) = \frac{1}{x} - 2$ then what is the value of $f(g(x))$ for all $x > 0$? (1)
- A. $\sqrt{x} - 2$ B. $\sqrt{x} + 2$
C. $\sqrt{x}$ D. $1/\sqrt{x}$
-
2. Which of the following is Not the property of $f(x) = x^{2/5}$? (1)
- A. $f(xy) = f(x)f(y)$
B. decreasing on $(-\infty, 0]$
C. the range is real number
D. the graph is symmetric with respect to the y-axis
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3. Which one of the following function is one to one correspondence function from $A = [0,1]$ to $B = [2,3]$? (1)
- A. $f(x)=x+2$ B. $f(x)=2x+2$
C. $f(x)=x+3$ D. $f(x)=2x+1$
-
4. Suppose $f(x) = \frac{Q(x)}{x(x^2-1)}$ where $Q(x)$ is a quadratic function. Which one of the following is necessarily true about the graph f ? (1)
- A. $x=0$, $x=1$ and $x=-1$ are vertical asymptote of the graph f
B. The graph of f does not intersect with its horizontal asymptote.
C. The vertical asymptote of the graph of f is only $x=-1$, if $Q(x)=x^2-x$
D. The vertical asymptote of the graph of f is only $x=1$, if $Q(x)=2x^2$
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5. What is the solution set of $\frac{2}{x} = \frac{x-2}{x^2-2x} = 1 - \frac{2x-2}{3x-2}$? (Note: The equation has two equals signs, likely a typo; probably it is $\frac{2}{x} = \frac{x-2}{x^2-2x}$ and also equals the other expression. We solve as two equations.) (1)
- A. $\{-2,1\}$ B. $\{1,2\}$
C. $\{-1\}$ D. $\{1\}$
-
6. Given line $4y = 3x + 24$ and another line ℓ parallel to this line at a distance of 4 units from it. A possible equation for ℓ is (1)
- A. $y = (3/4)x$ B. $y = (3/4)x + 1$
C. $y = (3/4)x + 6$ D. $y = (3/4)x - 6$
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7. Suppose an eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is reciprocal to that of the eccentricity of the ellipse $x^2 + 4y^2 = 4$ if the hyperbola passes through a focus of the ellipse, what is the equation of the hyperbola? (1)
- A. $x^2 - 2y^2 = 2$ B. $x^2 - 3y^2 = 3$
C. $x^2 - y^2 = 1$ D. $2x^2 - y^2 = 2$
-
8. Which one of the following is the equation of a circle whose center is on x-axis and radius is 3? (1)
- A. $x^2 + y^2 + 6y = 0$ B. $x^2 + (y-2)^2 = 3$
C. $(x-3)^2 + y^2 = 9$ D. $x^2 + y^2 = 9$

9. Which of the following is NOT true about a parabola $y^2 + 2x - 4y + 6 = 0$? (1)
- A. It opens to the left
B. Its vertex is $(-1, 2)$
C. Its focus is $(3/2, 2)$
D. Its directrix is the line $x = -1/2$
-
10. The distance from the point $P(3,4)$ to the line L with equation: $3x + 4y = 5$ is (1)
- A. 6 units
B. 7 units
C. 4 units
D. 5 units
-
11. What is the locus of all points in the plane in which the difference of the distances from two fixed points is constant? (1)
- A. Ellipse
B. Circle
C. Hyperbola
D. Parabola
-
12. Let p and q be propositions. Which of the following compound statement is a tautology? (1)
- A. $(p \wedge q) \Leftrightarrow p$
B. $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$
C. $(p \vee q) \Leftrightarrow p$
D. $(p \Rightarrow q) \Leftrightarrow q$
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13. If $p \Rightarrow q$ is false, then which of the following compound statement is true? (1)
- A. $p \wedge q$
B. $p \Leftrightarrow q$
C. $p \vee q$
D. $(p \vee q) \Rightarrow q$
-
14. Which of the following arguments is invalid logical argument? (1)
- A. $p \wedge \neg q, p, r \Rightarrow q \vdash \neg r$
B. $p \vee r, p \wedge q \vdash p \Rightarrow \neg q$
C. $p \Rightarrow q, \neg r \Rightarrow \neg q \vdash \neg p \vee r$
D. $\neg p \vee q, \neg q \vdash \neg q$
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15. Which of the following is the inverse of; "If wages increase, then inflation decreases"? (1)
- A. If inflation decreases, then wage increase.
B. If wages decreases, then inflation increases.
C. If wages decreases, then inflation decreases.
D. If wages increase, then inflation increases.
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16. What is the odds in favor of getting exactly two heads when three coins are tossed? (1)
- A. $3/8$
B. $5/8$
C. $5/3$
D. $3/5$
-
17. From a box containing 2 white and 6 green balls, a ball is drawn at random. The probability of not a green ball is: (1)
- A. $1/4$
B. $3/4$
C. $1/3$
D. $2/3$
-
18. The expenditure of 50 families is given below. If the mode of the data is 23.5 what are the values of F_1 and F_2 ? (1)
(Class intervals: 0-9,10-19,20-29,30-39,40-49; frequencies: 6,10, F_1 ,14, F_2 with total 50)
- A. $F_1=17, F_2=13$
B. $F_1=24, F_2=6$
C. $F_1=8, F_2=22$
D. $F_1=22, F_2=8$
-
19. How many three digit number can be formed from the digits 2,5,6,7 and 9 less than 600, if digits is used once only? (1)
- A. 24
B. 40
C. 125
D. 50

20. In a random arrangement of the word "SOCIETY", what is the total number of words in which all the three vowels come together? (1)
- A. 120 B. 720
C. 420 D. 840
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21. Which of the following is the constant term of $(x^2 - \frac{1}{2x^6})^8$? (1)
- A. 28 B. 7
C. -7 D. -28
-
22. From the data (marks distribution), if student in the top 8% are to be awarded a certificate. What is the minimum mark for a certificate? (Data: class intervals 10-14,15-19,20-29,30-39; frequencies 15,23,46,16) (1)
- A. 34.5 B. 30.5
C. 32 D. 33
-
23. A group of 100 items have a mean of 62. If the mean of 40 of these items is 55, what is the mean of the remaining items? (1)
- A. 66.67 B. 69.00
C. 65.00 D. 64.55
-
24. If $[[1-x, 0], [1, 2y]] + [[y-1, 4], [2, 1-3x]] = [[0, 4], [3, 5]]$, then x and y respectively? (1)
- A. -2 and 2 B. 1 and -1
C. 0 and 0 D. -4 and -4
-
25. If A is a square matrix of order 3 and $\det(A)=5$, then what is the value of $\det(A \operatorname{adj}(A))$? (1)
- A. 125 B. 25
C. 5 D. 3
-
26. For any $n \times n$ square matrix A, which of the following is true? (1)
- A. $\det(A) = -\det(A^T)$
B. If k is scalar, $\det(kA) = k^n \det(A)$
C. If B is obtained from A by interchanging two rows, then $\det(A)=\det(B)$
D. If A is invertible, then $\det(A)=\det(A^{-1})$
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27. For what value of x does $((1+i)/(1-i))^x = 1$? (1)
- A. $x=2n+1$ B. $x=2n$
C. $x=4n$ D. $x=4n+1$
-
28. If the real part of $(z+2)/(z-1)$ is 4, then the locus of the point representing z in the complex plane is: (1)
- A. Circle B. Straight line
C. Parabola D. Hyperbola
-
29. If $z = (2-2\sqrt{3}i)/(1+i)$, then which one is the polar representation of z? (1)
- A. $2\sqrt{2}(\cos(-105^\circ)+i \sin(-105^\circ))$ B. $4(\cos(-105^\circ)+i \sin(-105^\circ))$
C. $2\sqrt{2}(\cos 15^\circ+i \sin 15^\circ)$ D. $4(\cos 75^\circ+i \sin 75^\circ)$
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30. If $z_1 = (2-i)/(1+i)$ and $z_2 = (1+i)/(1-i)$, then $z_1 - 2z_2 = ?$ (1)
- A. $1-7i$ B. $(1-7i)/2$
C. $(1+7i)/2$ D. $1-7i$

31. The 15th term of the sequence $\{0, -3, -8, -15, \dots\}$ is (1)
- A. -245
B. -255
C. -224
D. -242
-
32. The sum of the first four terms of an arithmetic progression is 56 and the sum of the last four terms is 112. If its first term is 11, then the number of terms are: (1)
- A. 10
B. 18
C. 12
D. 11
-
33. A ball is dropped from a height of 10m above a flat surface. In each bounce, it rebounds 50% of the distance it fell. What is the maximum possible vertical distance the ball could travel? (1)
- A. 20m
B. 40m
C. 45m
D. 30m
-
34. What is the sum of the series $\sum_{n=1}^{30} (-1)^n (1/n + 1/(n+1))$? (1)
- A. -30/31
B. 29/30
C. -29/30
D. 31/31
-
35. If the third term of a geometric sequence is $4/9$ and the sum of infinite terms of the geometric sequence is 3, then the first term of geometric sequence will be: (1)
- A. $1/2$
B. $2/3$
C. $1/3$
D. 1
-
36. Which one of the following is equal to $\lim_{n \rightarrow \infty} (1^2 + 2^2 + \dots + n^2) / ((n^2 - 1)(n + 1))$? (1)
- A. 0
B. $1/2$
C. $1/3$
D. does not exist
-
37. The value of $\lim_{x \rightarrow 1/2} (2x - 1) / (1 - \sqrt[3]{2x})$ is equal to: (1)
- A. 1
B. 3
C. -3
D. -1.5
-
38. Which of the following statement is NOT True? (1)
- A. A sequence $\{a_n\}$ is a null sequence if $\lim a_n = 0$
B. If $\{a_n\}$ is increasing and bounded above then $\lim a_n = \text{lub of } \{a_n\}$
C. If $\{a_n\}$ is decreasing and bounded below, then $\lim a_n = \text{glb of } \{a_n\}$
D. Every bounded sequence is convergent
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39. Let $f(x) = \{ a \sin x / (x - |x|) \text{ for } x < 0, e^{-x} \text{ for } x \geq 0 \}$. If f is continuous at $x=0$, then $a = ?$ (1)
- A. 4
B. -4
C. $1/2$
D. 2
-
40. Which one of the following is equal to $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x)$? (1)
- A. 0
B. 1
C. ∞
D. does not exist
-
41. Which one of the following is equal to $\lim_{x \rightarrow 0} (2x + \tan x) / (x \sec x)$? (1)
- A. 2
B. 0
C. 1
D. 3

42. $\lim_{x \rightarrow \infty} ((x+1)/x)^{2x+4}$ is equal to: (1)
- A. e B. e^2
C. e^4 D. ∞
-
43. If $f(x) = x^2 + 2 \ln x^2$, then $\lim_{h \rightarrow 0} (f(1+h) - f(1))/h$ is equal to: (1)
- A. 2 B. 6
C. 5 D. 0
-
44. If $f(x) = \pi^3$, then $f'(2)$ is equal to: (1)
- A. $3\pi^2$ B. 0
C. 12 D. 1
-
45. If $y = e^{2x} + x - 3 \cos x$, then d^2y/dx^2 is: (1)
- A. $4e^{2x} + 3 \cos x$ B. $4e^{2x} - 3 \cos x$
C. $e^{2x} + 1 + 3 \sin x$ D. $e^{2x} + 1 - 3 \sin x$
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46. Which of the following function has NO vertical asymptote? (1)
- A. $f(x) = 1/(x-1)$ B. $f(x) = x/(x^2+1)$
C. $f(x) = \tan x$ D. $f(x) = \ln x$
-
47. Which of the following is the equation of the line tangent to the graph of $f(x) = 1/(x+1) + e^{2x} \cos x$ at $x=0$? (1)
- A. $y = x+2$ B. $y = 2x+1$
C. $y = 3x+2$ D. $y = x+1$
-
48. What is the n th derivative of the function $f(x) = e^{3x+1}$? (1)
- A. $3^{n-1} e^{3x+1}$ B. $3^{n+1} e^{3x+1}$
C. $3^n e^{3x+1}$ D. e^{3x+1}
-
49. Which of the following is the solution set of the equation $\sin 2x - \sin x = 0$ in $[0, 2\pi]$? (1)
- A. $\{0, \pi/3, \pi, 5\pi/3, 2\pi\}$ B. $\{0, \pi/3, \pi, 5\pi/3\}$
C. $\{0, \pi/3, 5\pi/3, 2\pi\}$ D. $\{0, \pi/3, 5\pi/3\}$
-
50. If $\sin(\alpha+\theta) = 12/13$ and $\sin \theta = 4/5$, then $\sin \alpha$ is: (1)
- A. $16/65$ B. $1/5$
C. $56/45$ D. $16/45$
-
51. Which of the following is true about $f(x) = -3 \cos(\frac{1}{2}x)$? (1)
- A. The amplitude of f is -3 B. The range of f is $[-1, 1]$
C. 5π is a zero of the function D. $f(-x) = -f(x) \forall x \in \mathbb{R}$
-
52. Your football has landed at the edge of the roof of your school building. When you are 25 meters away from the base of the building, the angle of elevation to your football is 45° . How high off the ground is your football? (1)
- A. 25 meters B. $50/\sqrt{2}$ meters
C. 50 meters D. $25/\sqrt{2}$ meters
-
53. Which one of the following is NOT a correct identity? (1)
- A. $\sin x \cos^2 x - \sin x = -\sin^3 x$ B. $(\cos x \tan x - \sin x)/\cot x = 0$
C. $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$ D. $\sin x/(1+\cos x) = \sec x - \tan x$

54. Which of the following is the image of the point (1,3) when reflected along the line $x - 2y - 5 = 0$? (1)

- A. $(4/3, -11/3)$ B. $(3, -1)$
C. $(-6, -1/2)$ D. $(5, -5)$

55. If $u = -i + 2j$, $v = -3i + j$, $w = -3j$, what is cosine of angle between $u + w$ and v ? (1)

- A. $1/\sqrt{5}$ B. $1/2$
C. $4/\sqrt{20}$ D. 0

56. Let $\mathbf{a} = \mathbf{i} - \mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j}$. What is a unit vector in the direction of $\mathbf{a} - \mathbf{b}$? (1)

- A. $(1/5)i - (2/5)j$ B. $-(1/\sqrt{5})i + (2/\sqrt{5})j$
C. $(1/\sqrt{5})i - (2/\sqrt{5})j$ D. $-(1/5)i + (2/5)j$

57. Let two vectors $\mathbf{a} = -2\mathbf{i} + 2\mathbf{j} + m\mathbf{j}$ (should be $+m\mathbf{k}$?) and $\mathbf{b} = 3\mathbf{i} + 6\mathbf{j}$. For what value of m are they perpendicular? (1)

- A. 1
B. 2
C. 0
D. -1

— END OF EXAMINATION —