

Mathematics, 2017 EC, social science, entrance exam, round 1

Grade 12 · Mathematics · 2017 E.C.

MARKING KEY — CONFIDENTIAL

Time allowed: 120 minutes

Questions: 59

Total marks: 59

1. Let $f(x)$ be a polynomial of degree at least 1, $g(x)$ be a non-zero polynomial and $c \in \mathbb{R}$. Which of the following states the Factor Theorem? (1)
- A. ✓ $f(c) = 0$ if and only if $x - c$ is a factor of $f(x)$.
- B. For distinct real numbers a and b , if $f(a)f(b) < 0$, then $f(x)$ has at least one zero between a and b .
- C. There exist polynomials $q(x)$ and $r(x)$ such that $f(x) = g(x)q(x) + r(x)$.
- D. If $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

Answer: A

The Factor Theorem states that for any polynomial $f(x)$ of degree at least 1, $x - c$ is a factor of $f(x)$ if and only if $f(c) = 0$.

2. Let $A = \{1, 2\}$ and $B = \{1, 3\}$ be sets. Then which one of the following Cartesian products is correct? (1)
- A. $B \times A = \{(1, 1), (3, 1), (1, 2), (3, 2), (2, 3)\}$.
- B. $B \times B = \{(1, 1), (1, 3), (3, 3), (2, 3)\}$.
- C. $A \times B = \{(1, 1), (1, 3), (2, 1), (2, 3), (3, 2)\}$.
- D. ✓ $A \times A = \{(1, 1), (2, 2), (2, 1), (1, 2)\}$.

Answer: D

For $A = \{1, 2\}$, the Cartesian product $A \times A$ consists of all ordered pairs (x, y) where both x and y are in A , yielding $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$. The set in option D contains exactly these elements.

3. Consider the matrix $\begin{pmatrix} 2 & -3 & 1 \\ 0 & -1 & \pi \\ 3 & 1 & 4 \end{pmatrix}$. What is the cofactor of entry π ? (1)
- A. 7
- B. ✓ -11
- C. -11π
- D. -7π

Answer: B

The entry π is at row 2, column 3, so its cofactor is $(-1)^{2+3} \det \begin{pmatrix} 2 & -3 \\ 3 & 1 \end{pmatrix} = -(2 \cdot 1 - (-3) \cdot 3) = -(2 + 9) = -11$.

4. When converting the recurring decimal $0.\overline{25}$ into a fraction using convergent geometric series, then what are the first term and the common ratio respectively? (1)
- A. ✓ 0.25 and 0.01
- B. 25 and 0.01
- C. 0.25 and 0.1
- D. 2.5 and 0.1

Answer: A

The recurring decimal $0.\overline{25}$ can be written as the infinite series $0.25 + 0.0025 + 0.000025 + \dots$, which is a geometric series with the first term $a_1 = 0.25$ and common ratio $r = 0.01$.

5. What are the maximum and minimum values respectively of $z = 3x + y$ subject to constraints (1)
- $$\begin{cases} 3x + 2y \leq 20 \\ 2x - y \geq 4 \\ x \geq 0, y \geq 0 \end{cases} ?$$
- A. ✓ 20 and 6
B. 40 and 6
C. 16 and 6
D. 18 and 8

Answer: A

The vertices of the feasible region defined by the constraints are $(2, 0)$, $(20/3, 0)$, and $(4, 4)$. Evaluating $z = 3x + y$ at these vertices gives 6, 20, and 16 respectively, making the maximum value 20 and the minimum value 6.

6. What is the domain of the function $f(x) = \sqrt{3 - x^2}$? (1)
- A. $[\sqrt{3}, \infty)$
B. ✓ $[-\sqrt{3}, \sqrt{3}]$
C. $[-3, 3]$
D. $(-\infty, \sqrt{3}] \cup [\sqrt{3}, \infty)$

Answer: B

For the square root function $f(x) = \sqrt{3 - x^2}$ to be defined on real numbers, the radicand must be non-negative: $3 - x^2 \geq 0 \implies x^2 \leq 3$, which gives the interval $[-\sqrt{3}, \sqrt{3}]$.

7. If $x = 0.00471$, then which of the following gives the mantissa of $\log x$? (1)
- A. -3
B. 3
C. ✓ $\log 4.71$
D. $\log 0.471$

Answer: C

Writing $x = 0.00471$ in scientific notation gives 4.71×10^{-3} , so $\log_{10} x = \log_{10}(4.71) - 3$. The integer part -3 is the characteristic and the fractional part $\log_{10}(4.71)$ is the mantissa.

8. Dibabe was hired in an elementary school with a monthly salary of Birr 30,000. The agreement was to raise her salary every year by Birr 3,000. What is the total amount of money that Dibabe will earn at the end of the 8th year in Birr? (1)
- A. 2,964,000
B. ✓ 3,888,000
C. 2,976,000
D. 324,000

Answer: B

Dibabe's monthly salary in year 1 is Birr 30,000, which gives an annual salary of Birr 360,000. With an annual increase of Birr 3,000 per month, the annual salary increases by Birr 36,000 each year, forming an arithmetic progression with $a_1 = 360,000$ and $d = 36,000$, resulting in a sum over 8 years of $S_8 = 4(2(360,000) + 7(36,000)) = 3,888,000$ Birr.

9. For what value of x is the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 2x+1 & 3 & 0 \\ 1 & -1 & x \end{pmatrix}$ invertible? (1)
- A. $\{x \in \mathbb{R} : x \neq -1\}$
B. ✓ $\{x \in \mathbb{R} : x \neq 4\}$
C. $\{x \in \mathbb{R} : x \neq 0\}$
D. $\{x \in \mathbb{R} : x \neq 3\}$

Answer: B

A matrix is invertible if and only if its determinant is non-zero. Expanding the determinant of the given matrix along the first row yields $\det(A) = 1(3x) + 1(-(2x+1) - 3) = x - 4$, which is non-zero if and only if $x \neq 4$.

10. What is the surface area (in cm^2) of leather material required to make a spherical shape ball of radius 25cm? (1)
- A. 625π B. ✓ 2500π
C. 100π D. 1250 sphere

Answer: B

The surface area A of a sphere of radius r is given by the formula $A = 4\pi r^2$. Substituting $r = 25 \text{ cm}$ yields $A = 4\pi(25)^2 = 2500\pi \text{ cm}^2$.

11. Mr. Bekele finances two investments, I_1 and I_2 . The profits of I_1 and I_2 are 8% and 10%, respectively. The total investment is Birr one million with at least Birr 50,000 in I_1 and at most Birr 300,000 in I_2 . Which of the following is the Mathematical representation of the problem if Mr. Bekele wishes to maximize his profit? (Let x and y be the amount of money invested on I_1 and I_2 in Birr, respectively)

A.
$$\begin{cases} \text{Maximize } z = 0.08x + 0.1y \\ \text{s.t. } x + y \geq 1,000,000 \\ x \leq 50,000 \\ y \geq 300,000 \\ x, y \geq 0 \end{cases}$$

B.
$$\begin{cases} \text{Maximize } z = 0.08x + 0.1y \\ \text{s.t. } x + y \geq 1,000,000 \\ x \geq 50,000 \\ y \leq 300,000 \\ x, y \geq 0 \end{cases}$$

C.
$$\begin{cases} \text{Maximize } z = 0.08x + 0.1y \\ \text{s.t. } x + y \leq 1,000,000 \\ x \leq 50,000 \\ y \geq 300,000 \\ x, y \geq 0 \end{cases}$$

D. ✓
$$\begin{cases} \text{Maximize } z = 0.08x + 0.1y \\ \text{s.t. } x + y \leq 1,000,000 \\ x \geq 50,000 \\ y \leq 300,000 \\ x, y \geq 0 \end{cases}$$

Answer: D

The objective function representing the total profit is $z = 0.08x + 0.1y$. The constraints are $x + y \leq 1,000,000$ for the total investment, $x \geq 50,000$ for the minimum in I_1 , $y \leq 300,000$ for the maximum in I_2 , and $x, y \geq 0$ for non-negativity, which matches Option D.

12. What is the result when $3\sqrt{2} - 7\sqrt{6}$ is subtracted from $2\sqrt{2} + 4\sqrt{3}$? (1)
- A. $\sqrt{2} + (4 - 7\sqrt{2})\sqrt{3}$ B. $-\sqrt{2} + 11\sqrt{2}\sqrt{3}$
C. ✓ $-\sqrt{2} + (4 + 7\sqrt{2})\sqrt{3}$ D. $\sqrt{2} - \sqrt{3}(7\sqrt{2} + 4)$

Answer: C

Subtracting $3\sqrt{2} - 7\sqrt{6}$ from $2\sqrt{2} + 4\sqrt{3}$ gives $(2\sqrt{2} + 4\sqrt{3}) - (3\sqrt{2} - 7\sqrt{6}) = -\sqrt{2} + 4\sqrt{3} + 7\sqrt{6}$. Factoring out $\sqrt{3}$ from the last two terms gives $-\sqrt{2} + (4 + 7\sqrt{2})\sqrt{3}$, which is Option C.

13. Which one of the following is qualitative data? (1)
- A. The time Derartu took to win Atlanta Olympic.
B. Age of President Trump.
C. The volume of oil in a container.
D. ✓ **The type of Coffee produced in Keffa.**

Answer: D

Qualitative data represents categories or attributes rather than numerical measurements. The type of Coffee produced in Keffa is categorical (qualitative), whereas time, age, and volume are all numerical (quantitative) variables.

14. Consider the functions $f(x) = 5^x$ and $g(x) = \log_5 x$. Which of the following is correct about the relations of domains and ranges of f and g ? (1)

A. ✓ $\text{Domain}(f) = \text{Range}(g)$ B. $\text{Domain}(f) \subseteq \text{Domain}(g)$
C. $\text{Domain}(f) = \text{Domain}(g)$ D. $\text{Range}(f) = \text{Range}(g)$

Answer: A

The functions $f(x) = 5^x$ and $g(x) = \log_5 x$ are inverses of each other. Therefore, the domain of f is equal to the range of g (both are the set of all real numbers, $\mathbb{R}$).

15. In the figure shown below, $\overline{AC} = \overline{DE}$, $\angle BAC = \angle DEF$, $\angle ACB = \angle FDE$ and $\overline{AC} \equiv \overline{DE}$. Then which one of the following is true? (1)

A. $\triangle ABC \equiv \triangle FED$ B. ✓ $\triangle ABC \equiv \triangle EFD$
C. $\triangle ABC \equiv \triangle DEF$ D. $\triangle ABC \equiv \triangle DFE$

Answer: B

By identifying the corresponding vertices from the given congruent angles, we find that $A \leftrightarrow E$, $C \leftrightarrow D$, and therefore $B \leftrightarrow F$. Written in corresponding order, the triangle congruence is $\triangle ABC \equiv \triangle EFD$.

16. Let f be a function and let c be an element of domain of f at which the gradient of f is zero. Which one of the following statements is true about $f'(c)$? (1)

A. ✓ $f'(c) = 0$ B. $f'(c) \neq 0$
C. $f'(c) > 0$ D. $f'(c)$

Answer: A

The gradient of a function f at a point c in its domain is given by the derivative $f'(c)$. Therefore, if the gradient at c is zero, then $f'(c) = 0$.

17. What is the solution set of the equation $|2x - 3| - 1 = 0$? (1)

A. $[\frac{3}{2}, \infty)$ B. $\{2\}$
C. ✓ $\{1, 2\}$ D. $\{\}$

Answer: C

Adding 1 to both sides gives $|2x - 3| = 1$. This splits into two equations: $2x - 3 = 1$ (so $x = 2$) and $2x - 3 = -1$ (so $x = 1$). The solution set is $\{1, 2\}$.

18. An equilateral triangle is inscribed in a circle with radius $\sqrt{3}$ cm. Then what is the perimeter (in cm) of the triangle? (1)

A. $\frac{9}{2}$ B. $\frac{9\sqrt{3}}{8}$
C. ✓ 9 D. $3\sqrt{3}$

Answer: C

For an equilateral triangle inscribed in a circle, the circumradius R is related to the side length 'a' by $R = a/\sqrt{3}$. Substituting $R = \sqrt{3}$ gives $a = 3$, making the perimeter $3a = 9$ cm.

19. Which of the following statements defines frustum of a pyramid? (1)
- A. It is part of a pyramid excluded the base of the pyramid and horizontal cross-section made by the plane slant to the base.
 - B. **It is part of a pyramid included between the base of the pyramid and a horizontal cross-section made by the plane parallel to the base.**
 - C. It is part of a cone included between the base of the cone and a plane parallel to the base.
 - D. It is a surface in space all points of which are equidistant from a point.

Answer: B

A frustum of a pyramid is defined geometrically as the portion of the pyramid lying between its base and a plane slicing through it parallel to the base.

20. If $\mathbf{u} = (1, -1)$ and $\mathbf{v} = (2, -4)$ are vectors, then $\mathbf{u} - \mathbf{v}$ is equal to: (1)
- A. $(-1, 3)$
 - B. $(-3, 5)$
 - C. $(3, -5)$
 - D. $(1, -3)$

Answer: A

To subtract vector $\mathbf{v}$ from vector $\mathbf{u}$, we subtract their corresponding components: $(1 - 2, -1 - (-4)) = (-1, 3)$.

21. The gradient of a function $f(x) = \frac{1}{x^2}$ at $x = a, a \neq 0$ is equal to: (1)
- A. $-\frac{2}{a^4}$
 - B. $-\frac{2}{a^3}$
 - C. $\frac{2}{a^2}$
 - D. $\frac{2}{a^3}$

Answer: B

The gradient (derivative) of $f(x) = 1/x^2 = x^{-2}$ is $f'(x) = -2x^{-3} = -2/x^3$. Evaluating this at $x = a$ gives $-2/a^3$.

22. Which of the following is NOT a valid reason for a person to save money? (1)
- A. To buy shares from private banks
 - B. **To give loan to other people and collect profit**
 - C. For down payment of a house
 - D. To support life after retirement

Answer: B

Standard financial literacy teaches that saving is for emergencies, future purchases (like a down payment), retirement, and formal investing (like buying shares). Giving personal loans to collect profit is an informal, risky lending practice rather than a recognized reason for personal saving.

23. Consider the following data. 12, 21, 16, 15, 10, 11, 12, 18, 9, 10, 13, 14, 15, 13, 19 Which one of the following is true? (1)
- A. $Q_3 = 16$
 - B. $Q_2 = 13.5$
 - C. $Q_1 = 4$
 - D. $Q_1 = 12$

Answer: A

Sorting the 15 data points gives: 9, 10, 10, 11, 12, 12, 13, 13, 14, 15, 15, 16, 18, 19, 21. The third quartile (Q_3) is the value at the $3(n+1)/4 = 12$ th position, which is 16.

24. A line ℓ_1 in the x, y -plane has equation $x - y + 3 = 0$. A second line ℓ_2 which is perpendicular to ℓ_1 contains the point $(2, 3)$. Which one of the following gives equation of ℓ_2 ? (1)

- A. $3y + 2x + 3 = 0$ B. $y - x + 5 = 0$
C. ✓ $y + x - 5 = 0$ D. $y - x - 3 = 0$

Answer: C

Line ℓ_1 can be rewritten as $y = x + 3$, which has a slope of 1. A perpendicular line ℓ_2 must have a slope of -1. Using the point-slope form with $(2, 3)$ gives $y - 3 = -1(x - 2)$, which simplifies to $y + x - 5 = 0$.

25. When the Division Algorithm is applied to the fraction $\frac{283}{20}$, then which one of the following is the corresponding representation? (1)

- A. $283 = 10 \times 20 + 83$ B. $283 = 1 \times 283 + 0$
C. $283 = 7 \times 20 + 143$ D. ✓ $283 = 14 \times 20 + 3$

Answer: D

The Division Algorithm states $a = bq + r$, where $0 \leq r < b$. For 283 divided by 20, the quotient q is 14 and the remainder r is 3, yielding $283 = 14 \times 20 + 3$.

26. Consider the data 7, 12, 21, x , 14. What is the value of x if its range is 23? (1)

- A. ✓ 30 B. 33.5
C. 2 D. 23

Answer: A

The range is the difference between the maximum and minimum values. If $x = 30$, the maximum is 30 and the minimum is 7, giving a range of $30 - 7 = 23$, which matches the requirement.

27. The following is raw data of 15 students in a test: 9, 6, 6, 10, 6, 10, 9, 7, 7, 5, 9, 7, 10, 7, 7. Which one of the following is frequency distribution of the given data? (1)

- A.

v	5	6	7	9	10
f	5	4	3	2	1

 B. ✓

v	5	6	7	9	10
f	1	3	5	3	3

C.

v	5	6	7	8	9	10
f	1	3	5	1	3	2

 D.

v	1	3	5	3	3
f	5	6	7	9	10

Answer: B

Counting the occurrences of each number in the dataset gives: one 5, three 6s, five 7s, three 9s, and three 10s. This corresponds exactly to the frequency table in option B.

28. Let $R_1(x) = \frac{x^3 - 9x}{x^2 - 2}$, $x \neq \pm\sqrt{2}$, and $R_2(x) = \frac{x^2 + 3x}{x - \sqrt{2}}$, $x \neq \sqrt{2}$ be rational functions. Then what is the simplified form of $\frac{R_1(x)}{R_2(x)}$? (1)

- A. 1 B. $x - 3$
C. $\frac{x - \sqrt{2}}{x + 3}$, $x \neq -3, 0$ D. ✓ $\frac{x - 3}{x + \sqrt{2}}$, $x \neq \pm\sqrt{2}, 0$

Answer: D

Dividing $R_1(x)$ by $R_2(x)$ means multiplying $R_1(x)$ by the reciprocal of $R_2(x)$. Factoring yields $[x(x-3)(x+3) / (x-\sqrt{2})(x+\sqrt{2})] \cdot [(x-\sqrt{2}) / x(x+3)]$. Canceling common terms x , $(x+3)$, and $(x-\sqrt{2})$ leaves $(x-3)/(x+\sqrt{2})$.

29. Which one of the following is a scalar quantity? (1)
- A. Displacement of an object to change its position.
 - B. The average velocity of airplane to travel.
 - C. Force applied on an object to change its place.
 - D. ✓ **The total time taken to complete surgery.**

Answer: D

Time is a scalar quantity because it is fully described by a magnitude (duration) and has no direction. Displacement, velocity, and force are all vector quantities.

30. Let $a > 0$ and $a \neq 1$. Then which one of the following is NOT true about the graphs of exponential functions $f(x) = a^x$ and $g(x) = \left(\frac{1}{a}\right)^x$? (1)
- A. They intersect each other at $(0, 1)$.
 - B. ✓ **They are increasing function.**
 - C. They have the same domain.
 - D. They have the same range.

Answer: B

For any base $a > 0$ and $a \neq 1$, one of a or $1/a$ is greater than 1 while the other is less than 1. This means one of the functions is exponentially increasing and the other is decreasing, so they cannot both be increasing functions.

31. A spherical fuel container with radius 3m is filled with fuel. What is the amount (in m^3) of fuel in the container? (1)
- A. $\frac{12}{3}\pi$
 - B. 9π
 - C. ✓ **36π**
 - D. 18π

Answer: C

The volume of a sphere is given by the formula $V = (4/3)\pi r^3$. Substituting the given radius $r = 3\text{m}$ yields $V = (4/3) * \pi * 27 = 36\pi$ cubic meters.

32. A circle with radius 3 cm increases its radius at a rate of 0.2 cm/sec. What is the area of the circle at the 5th second? (1)
- A. $9.64\pi \text{ cm}^2$
 - B. $4\pi \text{ cm}^2$
 - C. ✓ **$16\pi \text{ cm}^2$**
 - D. $9\pi \text{ cm}^2$

Answer: C

The radius of the circle after 5 seconds is $r = 3 + (0.2 \times 5) = 4 \text{ cm}$. The area of the circle is then $A = \pi r^2 = \pi(4)^2 = 16\pi \text{ cm}^2$.

33. A school has 6,000 students. The ratio of female students to the total number of students is 2:3. Which one of the following is true about the students in the school? (1)
- A. ✓ **The number of female students is twice the number of male students.**
 - B. The ratio of male students to female students is 1:3.
 - C. The number of male students is greater than that of female students.
 - D. About 33% of the students in the school are female.

Answer: A

Since the ratio of female students to total students is 2:3, females make up $2/3$ of the 6,000 students, which is 4,000. The remaining 2,000 are male students, meaning the number of female students is exactly twice the number of male students.

34. Let S be a set given by $\{x \in \mathbb{N} : 1 < x \leq 100\}$. Then which one of the following statement is true? (1)
- A. ✓ S is equal to $\{2, 3, 4, \dots, 99, 100\}$. B. S is equivalent to $\{1, 2, 3, \dots, 100\}$.
 C. S is equal to $\{2, 3, 4, \dots, 99\}$. D. S is equivalent to $\{101, 102, \dots, 198\}$.

Answer: A

The set $S = \{x \in \mathbb{N} : 1 < x \leq 100\}$ consists of all natural numbers strictly greater than 1 and up to 100 inclusive. This is precisely the set $\{2, 3, 4, \dots, 99, 100\}$.

35. What is the inverse of the matrix $\begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$? (1)
- A. $\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ B. $\frac{-1}{6} \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$
 C. ✓ $\begin{pmatrix} -\frac{1}{2} & \frac{1}{6} \\ 0 & \frac{1}{3} \end{pmatrix}$ D. $\begin{pmatrix} -2 & 0 \\ 1 & 3 \end{pmatrix}$

Answer: C

The determinant of the given matrix is $(-2)(3) - (1)(0) = -6$. The inverse is found by swapping the main diagonal elements, negating the off-diagonal elements, and dividing by the determinant, giving a top row of $-3/6, 1/6$ and a bottom row of $0, -2/6$, which simplifies to option C.

36. Which one of the following is equal to $\sum_{k=1}^4 (2k - 1)$? (1)
- A. 20 B. 30
 C. ✓ 16 D. 32

Answer: C

The expression represents the sum of the first four odd natural numbers: $(2(1)-1) + (2(2)-1) + (2(3)-1) + (2(4)-1) = 1 + 3 + 5 + 7 = 16$.

37. Given the polynomial function $p(x) = 5x^2 - x^3 - \sqrt{2}$. Then what are the degree, leading coefficient and constant term of the polynomial function $p(x)$ respectively? (1)
- A. 3, 5, $-\sqrt{2}$ B. ✓ 3, -1 , $-\sqrt{2}$
 C. 2, 5, $-\sqrt{2}$ D. 3, 1, $\sqrt{2}$

Answer: B

Writing the polynomial in standard descending order gives $p(x) = -x^3 + 5x^2 - \sqrt{2}$. The highest power of x is 3 (degree), its coefficient is -1 (leading coefficient), and the term without x is $-\sqrt{2}$ (constant term).

38. Let S be a set given by $\{x \in \mathbb{N} : 1 < x \leq 100\}$. Then which one of the following statement is true? (1)
- A. ✓ S is equal to $\{2, 3, 4, \dots, 99, 100\}$. B. S is equivalent to $\{1, 2, 3, \dots, 100\}$.
 C. S is equal to $\{2, 3, 4, \dots, 99\}$. D. S is equivalent to $\{101, 102, \dots, 198\}$.

Answer: A

The set $S = \{x \in \mathbb{N} : 1 < x \leq 100\}$ contains all natural numbers from 2 to 100, making it equal to $\{2, 3, 4, \dots, 100\}$.

39. What is the inverse of the matrix $\begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$? (1)
- A. $\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ B. $\frac{-1}{6} \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$
 C. ✓ $\begin{pmatrix} -\frac{1}{2} & \frac{1}{6} \\ 0 & \frac{1}{3} \end{pmatrix}$ D. $\begin{pmatrix} -2 & 0 \\ 1 & 3 \end{pmatrix}$

Answer: C

The matrix determinant is -6 . Dividing the adjugate matrix $[3, -1; 0, -2]$ by -6 yields the inverse matrix $[-1/2, 1/6; 0, 1/3]$.

40. Which one of the following is equal to $\sum_{k=1}^4 (2k - 1)$? (1)
- A. 20
B. 30
C. ✓ 16
D. 32

Answer: C

The sum of the first n odd positive integers is n^2 . For $n = 4$, the sum is $\sum_{k=1}^4 (2k - 1) = 1 + 3 + 5 + 7 = 16$.

41. Given the polynomial function $p(x) = 5x^2 - x^3 - \sqrt{2}$. Then what are the degree, leading coefficient and constant term of the polynomial function $p(x)$ respectively? (1)
- A. 3, 5, $-\sqrt{2}$
B. ✓ 3, -1 , $-\sqrt{2}$
C. 2, 5, $-\sqrt{2}$
D. 3, 1, $\sqrt{2}$

Answer: B

Rewriting the polynomial in standard form gives $p(x) = -x^3 + 5x^2 - \sqrt{2}$. The highest power of x is 3 (degree), the coefficient of x^3 is -1 (leading coefficient), and the constant term is $-\sqrt{2}$.

42. A person, whose height is 2 m, is standing away from a 16 m tall building. The angle of depression from the top of the building to the head of the person is 30° . How far is the person from the building in meters? (1)
- A. ✓ $14\sqrt{3}$
B. $\frac{16}{\sqrt{3}}$
C. $16\sqrt{3}$
D. $\frac{14}{\sqrt{3}}$

Answer: A

The vertical distance from the top of the building to the person's head is $16 \text{ m} - 2 \text{ m} = 14 \text{ m}$. Using the angle of depression of 30° , we have $\tan(30^\circ) = \frac{14}{d}$, which gives $d = \frac{14}{\tan(30^\circ)} = 14\sqrt{3} \text{ m}$.

43. If a sampling interval is calculated by dividing the size of the population by the size of the sample, then which probability sampling technique is used? (1)
- A. Stratified Sampling
B. ✓ Systematic Sampling
C. Simple Random Sampling
D. Cluster Sampling

Answer: B

In systematic sampling, the sampling interval k is calculated by dividing the population size N by the sample size n (i.e., $k = N/n$), and then selecting every k -th element from a random starting point.

44. Which one of the following systems of equations possesses no solution? (1)
- A. $\begin{cases} -x + 3y = -2 \\ -2x + 6y = -4 \end{cases}$
B. ✓ $\begin{cases} -2x + y = 2 \\ 4x - 2y = 4 \end{cases}$
C. $\begin{cases} x - 3y = 0 \\ 2x + y = -7 \end{cases}$
D. $\begin{cases} x + y = \sqrt{2} \\ -x + y = \pi \end{cases}$

Answer: B

Multiplying the first equation of option B by -2 gives $4x - 2y = -4$, which contradicts the second equation $4x - 2y = 4$. Thus, the system has no solution.

45. Let $A(-1, 2)$ and $B(0, 5)$ be initial and terminal points of vector $\overrightarrow{AB}$. What is the position vector of $\overrightarrow{AB}$? (1)
- A. $-i + 7j$ B. $5j$
 C. $-i - 3j$ D. $i + 3j$

Answer: D

The position vector of $\overrightarrow{AB}$ is found by subtracting the coordinates of the initial point A from the terminal point B , which gives $(0 - (-1))i + (5 - 2)j = i + 3j$.

46. The system of linear equations $\begin{cases} x + z = 0 \\ 2x + y = 1 \\ x + 2y + z = 0 \end{cases}$ in matrix form is equal to: (1)
- A. $\begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ B. $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$
 C. $\checkmark \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ D. $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Answer: C

By representing the coefficients of x, y, z in each equation as rows of the coefficient matrix, we get the first row $(1, 0, 1)$, second row $(2, 1, 0)$, and third row $(1, 2, 1)$, which corresponds to option C.

47. Which one of the following is NOT true about the coefficient of variation? (1)
- A. $\checkmark$ **It is used to evaluate the variation of a data.**
 B. It is the ratio of standard deviation to the mean.
 C. It is a measure of relative dispersion that is based on standard deviation.
 D. It is a relative measure used to measure the degree of consistency.

Answer: A

The coefficient of variation (CV) is a relative measure of dispersion used to compare the variation or consistency of two or more datasets (often with different units or means), rather than to evaluate the variation of a single dataset.

48. Which of the following describe combination of objects? (1)
- A. The number of ways of arrangement of n objects in a specific order using r objects at a time.
 B. The number of ways of arranging the seat of n people in a circle.
 C. $\checkmark$ **The number of ways of choosing r objects from n objects disregarding the order of selection.**
 D. The number of ways of arranging n objects taking n at a time considering the order of the objects.

Answer: C

A combination is a selection of items where the order of selection does not matter, which is correctly described as choosing r objects from n objects while disregarding the order of selection.

49. Which one of the following is true about measures of dispersion? (1)
- A. It gives representative of the whole data.
 B. It locates the central value of the data.
 C. $\checkmark$ **It measures the scatter of the data.**
 D. It gives the middle value of the data.

Answer: C

Measures of dispersion (such as range, variance, and standard deviation) describe how spread out or scattered the data values are, whereas measures of central tendency locate the middle or representative values.

50. The solution set of $\sqrt[3]{64^{x-1}} = \left(\frac{1}{16}\right)^{2x+3}$ is equal to: (1)
- A. $\left\{-\frac{1}{2}\right\}$ B. $\left\{\frac{1}{3}\right\}$
C. $\checkmark \{-1\}$ D. $\{1\}$

Answer: C

Expressing both sides with base 4, we get $\sqrt[3]{(4^3)^{x-1}} = 4^{x-1}$ and $(1/16)^{2x+3} = (4^{-2})^{2x+3} = 4^{-4x-6}$. Equating the exponents yields $x - 1 = -4x - 6$, which simplifies to $5x = -5$, or $x = -1$.

51. In how many ways can the name for vertices of a triangle be given in different order by using letters A, B, C, D, E? (1)
- A. 20 B. 120
C. 10 D. $\checkmark$ 60

Answer: D

To name the vertices of a triangle using 3 letters in a specific order from 5 available letters, we calculate the number of permutations of 5 objects taken 3 at a time: $P(5, 3) = \frac{5!}{(5-3)!} = 5 \times 4 \times 3 = 60$.

52. If $\sin(-53^\circ) = \beta$, then which one of the following is correct? (1)
- A. $\sin(37^\circ) = \beta$ B. $\sin(127^\circ) = \beta$
C. $\sin(-37^\circ) = \beta$ D. $\checkmark \sin(307^\circ) = \beta$

Answer: D

Since the sine function has a period of 360° , we have $\sin(307^\circ) = \sin(307^\circ - 360^\circ) = \sin(-53^\circ) = \beta$.

53. What is the solution set of the inequality $1 - \frac{3}{x} \leq \frac{4}{x^2}$? (1)
- A. $\checkmark [-1, 0) \cup (0, 4]$ B. $[-1, 4]$
C. $(-\infty, -1) \cup (4, \infty)$ D. $(-\infty, -1] \cup [4, \infty)$

Answer: A

The inequality can be rewritten as $1 - \frac{3}{x} - \frac{4}{x^2} \leq 0$. Multiplying by x^2 (for $x \neq 0$) gives $x^2 - 3x - 4 \leq 0$, which factors as $(x - 4)(x + 1) \leq 0$, yielding $[-1, 4]$. Excluding $x = 0$ because it is in the denominator gives the solution set $[-1, 0) \cup (0, 4]$.

54. The slope of a line passing through $P(1, -1)$ and $Q(0, 1)$ is equal to: (1)
- A. 2 B. -1
C. 3 D. $\checkmark$ -2

Answer: D

The slope m of the line passing through $P(1, -1)$ and $Q(0, 1)$ is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{0 - 1} = \frac{2}{-1} = -2$.

55. Given the function $f(x) = 2 \operatorname{sgn} x + 1$. Then what are the domain and the range of $f(x)$ respectively? (1)
- A. $\mathbb{R}$ and $\{-3, 1, 3\}$ B. $\mathbb{R}$ and $\{-3, -1, 3\}$
C. $\mathbb{R}$ and $\{-2, 0, 2\}$ D. $\checkmark \mathbb{R}$ and $\{-1, 1, 3\}$

Answer: D

The signum function $\operatorname{sgn} x$ is defined for all real numbers, so the domain is $\mathbb{R}$. Its range is $\{-1, 0, 1\}$, so the range of $f(x) = 2 \operatorname{sgn} x + 1$ is $\{2(-1) + 1, 2(0) + 1, 2(1) + 1\} = \{-1, 1, 3\}$.

56. What is the rationalizing factor of the number $\frac{5\sqrt{3}}{\sqrt{3}-\sqrt{2}}$? (1)
- A. $\frac{1}{\sqrt{3}+\sqrt{2}}$ B. $\frac{\sqrt{3}}{\sqrt{3}+\sqrt{2}}$
C. $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ D. ✓ $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Answer: D

To rationalize the denominator of the fraction, we multiply both the numerator and the denominator by the conjugate of the denominator, which is $\sqrt{3} + \sqrt{2}$. Thus, the rationalizing factor is $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

57. An entrance examination for a certain job was given to 35 applicants and their results are summarized in the table below. Results Number of applicants 41-50 3 51-60 14 61-70 10 71-80 5 81-90 2 91-100 1 Then which one of the following is NOT true? (1)
- A. The upper class limit of the second class is 60.
B. ✓ **The class midpoint of the second class is 55.**
C. 61-70 is the class interval of the third class.
D. The lower class limit of the fifth class is 81.

Answer: B

The second class interval is 51-60. The class midpoint is calculated as $\frac{51+60}{2} = 55.5$, which means option B is NOT true.

58. What is the equation of the tangent line to the graph of $f(x) = 2x^3 - 3x^2 + 1$ at $x = 2$? (1)
- A. $y = 120x - 235$ B. $y = 6x - 7$
C. $y = 2x + 5$ D. ✓ $y = 12x - 19$

Answer: D

At $x = 2$, we have $f(2) = 5$. The derivative is $f'(x) = 6x^2 - 6x$, so the slope at $x = 2$ is $f'(2) = 12$. The equation of the tangent line is $y - 5 = 12(x - 2)$, which simplifies to $y = 12x - 19$.

59. An entrance examination for a certain job was given to 35 applicants and their results are summarized in the table below. Results Number of applicants 41-50 3 51-60 14 61-70 10 71-80 5 81-90 2 91-100 1 Then which one of the following is NOT true? (1)
- A. The upper class limit of the second class is 60.
B. ✓ **The class midpoint of the second class is 55.**
C. 61-70 is the class interval of the third class.
D. The lower class limit of the fifth class is 81.

Answer: B

This question is identical to question 58. The class midpoint of the second class (51-60) is 55.5 rather than 55, making option B the incorrect statement.

— END OF MARKING KEY —