

Mathematics, 2008 EC, entrance exam

Grade 12 · Mathematics · 2008 E.C.

Time allowed: 120 minutes

Questions: 57

Total marks: 57

Name _____ Grade _____ ID No. _____ Date _____

1. Which one of the following is a one to one correspondence function from $A=[0, 1]$ to $B=[1, 2]$? (1)
- A. $f(x) = x$ B. $f(x) = 1/3x^3 + 1$
C. $f(x) = 2x + 1$ D. $f(x) = x^2 + 1$

2. What is the solution set of $\frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}} = 3x^2 - \frac{x}{1 + \frac{1}{x}}$? ? (1)
- A. $\{-1, 1/3\}$ B. $\{1/3\}$
C. $\{3, -1/3\}$ D. $\{-1/3\}$

3. Suppose $\frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}} = 3x^2 - \frac{x}{1 + \frac{1}{x}}$? , where $Q(x)$ is a quadratic function. Which one of the following is necessarily true about the graph of f . (1)
- A. $x = 0$, $x = 1$ and $x = -1$ are vertical asymptotes of graph of f .
B. the graph of f doesn't intersect with its horizontal asymptote.
C. the vertical asymptotes of the graph of f is only $x = -1$
D. the vertical asymptotes of graph of f are $x = 1$ and $x = -1$ if $Q(x) = 2x^2$.

4. Which one of the following is equation of a circle whose center is on y -axis and radius is 3? (1)
- A. $x^2 + y^2 + 6y = 0$ B. $(x - 2)^2 + y^2 = 9$
C. $x^2 + (y - 2)^2 = 9$ D. $x^2 - 2x + y^2 = 8$

5. The planet Mercury's orbit around the sun is an ellipse with eccentricity 0.206, length of the major axis 1.16×10^8 km and the sun at one focus. What is the max distance from Mercury to the sun? (1)
- A. 6.99×10^8 km B. 6.99×10^7 km
C. 9.66×10^7 km D. 9.66×10^8 km

6. The graph of a hyperbola and the lines of its asymptotes are given as shown in the following figure. Which one of the following is an equation of the hyperbola? (1)

[Figure: A scanned hand-drawn Cartesian plane. The horizontal axis is drawn as a solid arrow to the right and labelled x ; the vertical axis is a solid arrow upward and labelled y . Two solid curves form a hyperbola that opens vertically (up and down): the upper branch has its vertex on the positive y -axis roughly two units above the origin and sweeps up to the left and to the right; the lower branch has its vertex on the y -axis just below the origin and sweeps down to the left and to the right. Two dashed straight lines are drawn as the asymptotes; they cross each other on the y -axis at a point about one unit above the origin (the centre of the hyperbola), and each branch approaches them. The dashed line of positive slope, running from lower left to upper right, carries the printed label $y = x + 1$ near its upper-right end. The dashed line of negative slope, running from upper left to lower right, carries the printed label $y = -x + 1$ near its lower-right end. No numeric tick marks are shown on either axis.]

- A. $y^2 - 2y - x^2 = 0$ B. $y^2 - 3y - x^2 = 0$
C. $x^2 - (y - 1)^2 = 1$ D. $(x - 1)^2 - y^2 = 1$

7. If the truth value of a proposition p is False, then which one of the following compound proposition has a truth value True? (1)
- A. $\neg p \wedge p$ B. $\neg p \Rightarrow p$
 C. $\neg(\neg p \vee p)$ D. $p \Rightarrow \neg p$
-
8. Which one of the following compound propositions is a tautology? (1)
- A. $(q \vee \neg q) \Rightarrow p$ B. $p \Rightarrow (p \vee \neg q)$
 C. $p \vee (q \wedge \neg q)$ D. $p \Rightarrow (q \wedge \neg q)$
-
9. The following is sample frequency distribution of data with variable x . | X | 3 | 5 | 6 | 7 | | Frequency | 2 | 5 | 2 | 1 | (1)
 What are the mean ($\bar{x}$) and variance (δ^2) of the data?
 [Figure: A bordered two-row frequency table sitting directly under the stem. The first row is headed X in the leftmost cell and then carries the four values 3, 5, 6 and 7 across four columns. The second row is headed Frequency in the leftmost cell and carries 2, 5, 2 and 1 in the matching columns. So $x = 3$ occurs twice, $x = 5$ occurs five times, $x = 6$ occurs twice and $x = 7$ occurs once, for a total frequency of 10.]
- A. $\bar{x} = 5, \delta^2 = 0.7$ B. $\bar{x} = 6, \delta^2 = 1.4$
 C. $\bar{x} = 6, \delta^2 = 0.7$ D. $\bar{x} = 5, \delta^2 = 1.4$
-
10. A box contains 10 items of which 3 are defective. If 2 items are randomly taken out of the box, what is the probability that both items are not defective? (1)
- A. 7/10 B. 4/7
 C. 7/15 D. 49/100
-
11. Items produced by accompany are subjected to two kinds of defects D1 and D2. Out of the total product, 5% have the defect D1, 10% have defect D2, and 2% have both defects. What is the probability that a randomly selected item has neither defect D1 nor defect D2? (1)
- A. 0.13 B. 0.5
 C. 0.98 D. 0.87
-
12. If $A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$, then $(AB)^{-1}$ is equal to: (1)
- A. $\begin{pmatrix} 4 & -3 \\ 4 & -5 \end{pmatrix}$ B. $\begin{pmatrix} -2 & 5 \\ 2 & -4 \end{pmatrix}$
 C. $\begin{pmatrix} -3 & 11 \\ 1 & -3 \end{pmatrix}$ D. $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$
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13. Let $A = \begin{pmatrix} 0 & \alpha & \beta \\ 2 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$, and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. If $\det(A)=3$, then what is the solution set of the system $AX = b$? (1)
- A. $\{(6, -2, -8)^T\}$ B. $\left\{\left(0, \frac{1}{\alpha}, \frac{5}{\beta}\right)^T\right\}$
 C. $\{(-3, 1, 4)^T\}$ D. $\emptyset$
-
14. In set of complex numbers, what is the solution set of $x^2 + 4x + 5 = 0$? (1)
- A. $\{\emptyset\}$ B. $\{1 - 2i, 1 + 2i\}$
 C. $\{2 - i, 2 + i\}$ D. $\{-2 - i, -2 + i\}$

15. If $z = (1 + i)^{10}$, then which one of the following is equal to z ? (1)
- A. $1 + 32i$ B. $32i$
C. $10i$ D. $1 + 10i$
-
16. If $\{A_n\}_{n=1}^{\infty}$ is an arithmetic sequence such that its 1st term $A_1 = -5$ and its 5th term $A_5 = 15$, then its 11th term A_{11} is equal to: (1)
- A. 40 B. 50
C. 45 D. 55
-
17. What is the sum of all multiples of 4 that are between 30 and 301? (1)
- A. 12,882 B. 11,288
C. 6,288 D. 6,882
-
18. The left hand side limit $\lim_{x \rightarrow 0^-} \frac{xe^x - |x|}{x}$ is equal to: (1)
- A. 0 B. 2
C. 1 D. does not exist
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19. Which one of the following is equal to $\lim_{x \rightarrow 0} \frac{2x + \tan x}{x \sec x}$? (1)
- A. 2 B. 0
C. 1 D. 3
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20. Which one of the following is true about the horizontal asymptote(s) of the graph of $y = \frac{|x| + 2}{x}$? (1)
- A. $y = 2$ is the only horizontal asymptote of the graph.
B. $y = 1$ & $y = -1$ are the horizontal asymptote of the graph.
C. $y = 2$ & $y = -2$ are the horizontal asymptote of the graph.
D. $y = 1$ is the only horizontal asymptote of the graph.
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21. If $f(x) = e^{3x} \cos x - \frac{x + \pi}{x^2 + 2}$, then $f'(0)$ is equal to: (1)
- A. $3 - \pi/2$ B. $3/2$
C. $7/4$ D. $5/2$
-
22. If $f(x) = \ln \sqrt{x^2 - 5}$, then which of the following is equal to $f'(x)$? (1)
- A. $\frac{x}{x^2 - 5}$ B. $\frac{-x}{\sqrt{x^2 - 5}}$
C. $\frac{2x}{x^2 - 5}$ D. $\frac{-x}{x^2 - 5}$
-
23. Let f be twice differentiable function on $\mathbb{R}$. which one of the following is necessarily true? (1)
- A. if $f'(c) = 0$, at some $c \in \mathbb{R}$, then f has relative extreme at $x = c$.
B. if $f'(x)$ is increasing, then graph of $y = f(x)$ is concave upward.
C. if $f'(x) = 0$ for all $x \in \mathbb{R}$, then $f(x) = 0$ for all $x \in \mathbb{R}$
D. if $f(x)$ is increasing, then $f''(x) = 0$ for all $x \in \mathbb{R}$

24. Suppose $f(x)$ is differentiable on $(-\infty, \infty)$ and the graph of its derivative, $y = f'(x)$ is as shown in the following figure. (1)

[Figure: A scanned hand-drawn Cartesian plane showing one continuous solid curve labelled $y = f'(x)$ (the label is written along the steeply rising right-hand arm of the curve). The horizontal axis is a solid arrow to the right labelled x ; the vertical axis is a solid arrow upward labelled y . Reading the curve left to right: it comes up steeply from the bottom-left of the frame as a nearly straight line, crosses the x -axis from below to above at the tick labelled -2 , keeps rising and rounds off to a local maximum on the y -axis (at about $x = 0$), then falls and comes down to touch the x -axis at the tick labelled 2 without crossing it (the curve is tangent to the axis there), and immediately after that turns and climbs steeply upward off the top of the frame. Tick labels printed under the x -axis, left to right, read -2 , a short dash (a faint -1), 1 and 2 . No values are marked on the y -axis.]

- A. $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$ B. $f(x)$ has a local extreme value at $x = 2$
C. $f(x)$ has a local minimum value at $x = -2$ D. $f(x)$ has a local maximum value at $x = 0$

25. A closed cylindrical can is to be made to hold 1000 cm^3 of oil. What are the dimensions (radius r and height h) that will minimize the total surface area of the can? (1)

- A. $r = \frac{\sqrt[3]{\sqrt{50}}}{\pi}, h = 2\frac{\sqrt[3]{\sqrt{50}}}{\pi}$ B. $r = \frac{\sqrt[3]{\sqrt{500}}}{\pi}, h = 2\frac{\sqrt[3]{\sqrt{500}}}{\pi}$
C. $r = \sqrt[3]{\frac{50}{\pi}}, h = 2\sqrt[3]{\frac{50}{\pi}}$ D. $r = \sqrt[3]{\frac{500}{\pi}}, h = 2\sqrt[3]{\frac{500}{\pi}}$

26. Which one of the following is equal to $\int \frac{(\ln x)^2 + x^2 \cos x}{x} dx$? (1)

- A. $\frac{1}{x^2} \ln x + x \sin x - \cos x + c$ B. $\frac{1}{3} (\ln x)^3 + x \sin x - \cos x + c$
C. $\frac{1}{3} (\ln x)^3 + x \sin x + \cos x + c$ D. $\frac{1}{x^2} \ln x + x \sin x + \cos x + c$

27. The volume of the solid generated when the region bounded between graph of $y = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 4, & 2 \leq x \leq 3 \end{cases}$ and x -axis is rotated about the x -axis: (1)

- A. $32\pi/5$ B. $112\pi/5$
C. $112\pi/3$ D. $64\pi/5$

28. If $f: A \rightarrow B$ and $g: B \rightarrow C$ are functions, then which one of the following is true about the composition function? (1)

- A. domain of $g \circ f \subseteq \text{domain of } f$ B. range of $g \circ f \not\subseteq \text{range of } g$
C. domain of $g \circ f \not\subseteq \text{domain of } f$ D. range of $g \circ f \subseteq \text{range of } f$

29. If the point $(3, -2)$ is on the graph of $y = f(x)$, then which point is on the graph of $y = f^{-1}(x)$? (1)

- A. $(1/3, -2)$ B. $(3, -1)$
C. $(-2, 3)$ D. $(3, -1/2)$

30. The equation of the line that passes through $(2, -1)$ and perpendicular to the line $3x + 4y = 6$ is: (1)

- A. $-4x + 3y = 5$ B. $4x - 3y = 5$
C. $4x + 3y = 11$ D. $-4x + 3y = -11$

31. If $Z = \frac{2 - 4i}{1 + i}$, then the modulus of the conjugate of Z , $|\bar{Z}|$ is: (1)

- A. $\sqrt{10}$ B. $3\sqrt{2}$
C. $2\sqrt{3}$ D. $2\sqrt{2}$

32. The n^{th} term of the sequence: 1, -4, 9, -16,... is: (1)

A. $a_n = (-2)^n$

B. $a_n = (-1)^n n^2$

C. $a_n = (-1)2^n n^2$

D. $a_n = (-1)^{n-1} n^2$

33. Which one of the following is true about the horizontal asymptote(s) of the graph of $y = \frac{|x| + 2}{x}$ (1)

A. $y = 2$ is the only horizontal asymptote of the graph.

B. $y = 1$ and $y = -1$ are the horizontal asymptotes of the graph.

C. $y = 2$ and $y = -2$ are the horizontal asymptote of the graph.

D. $y = 1$ is the only horizontal asymptote of the graph.

34. For any $n \times n$ square matrix A , which of the following is true? (1)

A. $\det(A) = -\det(A^T)$, where A^T is the transpose of A

B. If k is a scalar, then $\det(kA) = k^n \det(A)$

C. If B is a matrix obtained from A by interchanging of two rows of A , then $\det(B) = \det(A)$

D. If A is invertible, then $\det(A) = \det(A^{(-1)})$

35. The solution of the system of linear equation of $\begin{cases} x - 3y - 2z = 6 \\ 2x - 4y - 3z = 8 \\ -3x + 6y + 8z = -5 \end{cases}$ is: (1)

A. $x = -1, y = -3, z = -2$

B. $x = -1, y = -3, z = 2$

C. $x = 1, y = -3, z = 2$

D. $x = 1, y = 3, z = -2$

36. In how many more ways can 4 people be arranged in a row than if they were arranged in a circle? (1)

A. 1

B. 6

C. 18

D. 12

37. The value of $\int_0^1 (x+1)e^{(x^2+2x)} dx$ is: (1)

A. $\frac{e^3}{2}$

B. $\frac{e^4 - e}{2}$

C. $\frac{e^3 - 1}{2}$

D. $e^3 - 1$

38. $\int_0^3 (x+1)^{\frac{1}{2}} dx = \dots\dots\dots$. (1)

A. $21/2$

B. $14/3$

C. 7

D. $16/3$

39. The sum of $\sum_{n=0}^{\infty} 5 \left(\frac{2}{3}\right)^n$ is $\dots\dots\dots$. (1)

A. 0

B. 15

C. $10/3$

D. 5

40. In which interval the sequence $\left\{ \frac{(-1)^n}{3n} \right\}_{n=1}^{\infty}$ is bounded? (1)
- A. $\left[\frac{-1}{9}, \frac{1}{12} \right]$ B. $\left[\frac{-1}{3}, \frac{1}{6} \right]$
 C. $\left[\frac{-1}{6}, \frac{1}{3} \right]$ D. $\left[\frac{-1}{12}, \frac{1}{9} \right]$
-
41. Which one of the following is true about the function given by the formula $f(x) = \begin{cases} \frac{x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$? (1)
- A. f is continuous except at $x = 0$ B. f has an infinite discontinuity at $x = 0$
 C. f is continuous everywhere D. f has $x = 0$ as a vertical asymptote
-
42. $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$ is equal to :? (1)
- A. 0 B. 1
 C. ∞ D. -1
-
43. $\frac{d}{dx} (\ln e^{2x})$ is equal to :? (1)
- A. $\frac{1}{e^{2x}}$ B. $\frac{2}{e^{2x}}$
 C. $2x$ D. 2
-
44. If $f(x) = 2 + |x - 3|$ for all x , then the value of the derivative $f'(x)$ at $x = 3$ (1)
- A. -1 B. Does not exist
 C. 1 D. 2
-
45. The graph of $y = 5x^4 - x^5$ has a point of inflection at: (1)
- A. (3, 162) only B. (4, 256) only
 C. (0, 0) only D. (0, 0) and (3, 162)
-
46. Let the angle between $\vec{V} = -2\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{PQ}$ be 60° , where P and Q are points in space. If $\vec{V} \cdot \vec{PQ} = 2$, (1)
 then what is the distance between P and Q?
- A. $3/4$ B. $4/5$
 C. $4/3$ D. $5/4$
-
47. Consider the following assertion of a person and his proof "If x and y are equal positive integers, then $x + y = y$." (1)
 Proof: the following steps and reasons are used to prove the assertion. Step | Reason
 1. $x = y$ | Given hypothesis
 2. $x^2 = xy$ | Multiply both sides of (1) by x
 3. $x^2 - y^2 = xy - y^2$ | Subtract y^2 from both sides of (2)
 4. $(x - y)(x + y) = (x - y)y$ | Factor both sides of (3)
 5. $x + y = y$ | Divide both sides of (4) by $x - y$
 Step 5 completes the proof.
 Which one of the following is true about this proof?
- A. It is a correct direct proof of the assertion.
 B. It follows the technique of a proof by contradiction because the steps lead to a contradiction.
 C. The proof is invalid because Step 4 does not lead to Step 5.
 D. The proof is invalid because Step 4 does not follow from Step 3.
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48. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction? (1)
- A. $2^n > 10^n$ for every integer n such that $n \geq 6$. B. $r^2 > 0$ for every real number r such that $r \geq 1$.
 C. $n^2 + 10n > 2n^2$ for every natural number $n \geq 1$. D. $2^n > 8n$ for every integer n such that $n \geq 3$.

