

6. The graph of a hyperbola and the lines of its asymptotes are given as shown in the following figure. Which one of the following is an equation of the hyperbola? (1)

[Figure: A scanned hand-drawn Cartesian plane. The horizontal axis is drawn as a solid arrow to the right and labelled x ; the vertical axis is a solid arrow upward and labelled y . Two solid curves form a hyperbola that opens vertically (up and down): the upper branch has its vertex on the positive y -axis roughly two units above the origin and sweeps up to the left and to the right; the lower branch has its vertex on the y -axis just below the origin and sweeps down to the left and to the right. Two dashed straight lines are drawn as the asymptotes; they cross each other on the y -axis at a point about one unit above the origin (the centre of the hyperbola), and each branch approaches them. The dashed line of positive slope, running from lower left to upper right, carries the printed label $y = x + 1$ near its upper-right end. The dashed line of negative slope, running from upper left to lower right, carries the printed label $y = -x + 1$ near its lower-right end. No numeric tick marks are shown on either axis.]

- A. $y^2 - 2y - x^2 = 0$ B. $y^2 - 3y - x^2 = 0$
C. $x^2 - (y - 1)^2 = 1$ D. $(x - 1)^2 - y^2 = 1$

Answer: A

The asymptotes intersect at the center $(0, 1)$ and have slopes ± 1 . Since the hyperbola opens vertically, its equation is $(y - 1)^2 - x^2 = 1$, which expands to $y^2 - 2y - x^2 = 0$.

7. If the truth value of a proposition p is False, then which one of the following compound proposition has a truth value True? (1)

- A. $\neg p \wedge p$ B. $\neg p \Rightarrow p$
C. $\neg(\neg p \vee p)$ D. $\checkmark p \Rightarrow \neg p$

Answer: D

When p is False, $\neg p$ is True. The conditional statement $p \Rightarrow \neg p$ has a False hypothesis, which makes the implication vacuously True.

8. Which one of the following compound propositions is a tautology? (1)

- A. $(q \vee \neg q) \Rightarrow p$ B. $\checkmark p \Rightarrow (p \vee \neg q)$
C. $p \vee (q \wedge \neg q)$ D. $p \Rightarrow (q \wedge \neg q)$

Answer: B

By the rule of addition, p implies $p \vee \neg q$ regardless of the truth values of p and q , making $p \Rightarrow (p \vee \neg q)$ a tautology.

9. The following is sample frequency distribution of data with variable x . | X | 3 | 5 | 6 | 7 | | Frequency | 2 | 5 | 2 | 1 | (1)
What are the mean ($\bar{x}$) and variance (δ^2) of the data?

[Figure: A bordered two-row frequency table sitting directly under the stem. The first row is headed X in the leftmost cell and then carries the four values 3, 5, 6 and 7 across four columns. The second row is headed Frequency in the leftmost cell and carries 2, 5, 2 and 1 in the matching columns. So $x = 3$ occurs twice, $x = 5$ occurs five times, $x = 6$ occurs twice and $x = 7$ occurs once, for a total frequency of 10.]

- A. $\bar{x} = 5, \delta^2 = 0.7$ B. $\bar{x} = 6, \delta^2 = 1.4$
C. $\bar{x} = 6, \delta^2 = 0.7$ D. $\checkmark \bar{x} = 5, \delta^2 = 1.4$

Answer: D

The mean is $(3 \cdot 2 + 5 \cdot 5 + 6 \cdot 2 + 7 \cdot 1) / 10 = 50 / 10 = 5$. The variance is $\sum f(x - \bar{x})^2 / N = [2(4) + 5(0) + 2(1) + 1(4)] / 10 = 14 / 10 = 1.4$.

10. A box contains 10 items of which 3 are defective. If 2 items are randomly taken out of the box, what is the probability that both items are not defective? (1)
- A. $7/10$ B. $4/7$
 C. ✓ $7/15$ D. $49/100$

Answer: C

The number of ways to choose 2 non-defective items out of 7 is $C(7, 2) = 21$, and the total ways to choose 2 items from 10 is $C(10, 2) = 45$. Thus, the probability is $21/45 = 7/15$.

11. Items produced by accompany are subjected to two kinds of defects D1 and D2. Out of the total product, 5% have the defect D1, 10% have defect D2, and 2% have both defects. What is the probability that a randomly selected item has neither defect D1 nor defect D2? (1)
- A. 0.13 B. 0.5
 C. 0.98 D. ✓ 0.87

Answer: D

The probability of having at least one defect is $P(D1 \cup D2) = P(D1) + P(D2) - P(D1 \cap D2) = 0.05 + 0.10 - 0.02 = 0.13$. The probability of having neither defect is $1 - 0.13 = 0.87$.

12. If $A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$, then $(AB)^{-1}$ is equal to: (1)
- A. $\begin{pmatrix} 4 & -3 \\ 4 & -5 \end{pmatrix}$ B. ✓ $\begin{pmatrix} -2 & 5 \\ 2 & -4 \end{pmatrix}$
 C. $\begin{pmatrix} -3 & 11 \\ 1 & -3 \end{pmatrix}$ D. $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$

Answer: B

Using the property $(AB)^{-1} = B^{-1}A^{-1}$, we have $\det(A) = 6 - 7 = -1$, so $A^{-1} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$. Multiplying B^{-1} by A^{-1} yields $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 2 & -4 \end{bmatrix}$.

13. Let $A = \begin{pmatrix} 0 & \alpha & \beta \\ 2 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$, and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. If $\det(A)=3$, then what is the solution set of the system $AX = b$? (1)
- A. ✓ $\{(6, -2, -8)^T\}$ B. $\left\{ \left(0, \frac{1}{\alpha}, \frac{5}{\beta} \right)^T \right\}$
 C. $\{(-3, 1, 4)^T\}$ D. $\emptyset$

Answer: A

From $\det(A) = -\alpha - 4\beta = 3$, we have $\alpha + 4\beta = -3$. Solving the homogeneous part gives $x = -3y$ and $z = 4y$, and the first equation $y(\alpha + 4\beta) = 6$ yields $y = -2$, giving $X = (6, -2, -8)^T$.

14. In set of complex numbers, what is the solution set of $x^2 + 4x + 5 = 0$? (1)
- A. $\{\emptyset\}$ B. $\{1 - 2i, 1 + 2i\}$
 C. $\{2 - i, 2 + i\}$ D. ✓ $\{-2 - i, -2 + i\}$

Answer: D

Using the quadratic formula, $x = \frac{-4 \pm \sqrt{(16 - 20)}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$.

15. If $z = (1 + i)^{10}$, then which one of the following is equal to z ? (1)
- A. $1 + 32i$ B. ✓ $32i$
C. $10i$ D. $1 + 10i$

Answer: B

Notice that $(1 + i)^2 = 1 + 2i - 1 = 2i$. Therefore, $(1 + i)^{10} = ((1 + i)^2)^5 = (2i)^5 = 32i^5 = 32i$.

16. If $\{A_n\}_{n=1}^{\infty}$ is an arithmetic sequence such that its 1st term $A_1 = -5$ and its 5th term $A_5 = 15$, then its 11th term A_{11} is equal to: (1)
- A. 40 B. 50
C. ✓ 45 D. 55

Answer: C

The common difference is $d = (A_5 - A_1)/4 = (15 - (-5))/4 = 5$. Therefore, $A_{11} = A_1 + 10d = -5 + 10(5) = 45$.

17. What is the sum of all multiples of 4 that are between 30 and 301? (1)
- A. 12,882 B. ✓ 11,288
C. 6,288 D. 6,882

Answer: B

The multiples of 4 strictly between 30 and 301 range from 32 (4×8) to 300 (4×75), so there are $75 - 8 + 1 = 68$ terms. The sum is $68/2 \times (32 + 300) = 34 \times 332 = 11,288$.

18. The left hand side limit $\lim_{x \rightarrow 0^-} \frac{xe^x - |x|}{x}$ is equal to: (1)
- A. 0 B. ✓ 2
C. 1 D. does not exist

Answer: B

For $x < 0$, $|x| = -x$, so $(xe^x - |x|)/x = (xe^x + x)/x = e^x + 1$. As $x \rightarrow 0^-$, $e^x + 1 \rightarrow e^0 + 1 = 2$.

19. Which one of the following is equal to $\lim_{x \rightarrow 0} \frac{2x + \tan x}{x \sec x}$? (1)
- A. 2 B. 0
C. 1 D. ✓ 3

Answer: D

Dividing the numerator and denominator by x gives $(2 + (\tan x)/x) / \sec x$. As $x \rightarrow 0$, $(\tan x)/x \rightarrow 1$ and $\sec x \rightarrow 1$, so the limit is $(2 + 1)/1 = 3$.

20. Which one of the following is true about the horizontal asymptote(s) of the graph of $y = \frac{|x| + 2}{x}$? (1)
- A. $y = 2$ is the only horizontal asymptote of the graph.
B. ✓ $y = 1$ & $y = -1$ are the horizontal asymptote of the graph.
C. $y = 2$ & $y = -2$ are the horizontal asymptote of the graph.
D. $y = 1$ is the only horizontal asymptote of the graph.

Answer: B

As x approaches positive infinity, $|x| = x$ and the expression approaches 1, giving $y = 1$. As x approaches negative infinity, $|x| = -x$ and the expression approaches -1, giving $y = -1$.

21. If $f(x) = e^{3x} \cos x - \frac{x + \pi}{x^2 + 2}$, then $f'(0)$ is equal to: (1)

A. $3 - \pi/2$ B. $3/2$
C. $7/4$ D. $\checkmark$ $5/2$

Answer: D

Using the product and quotient rules, the derivative of the first term at $x = 0$ is $3(1)(1) - (1)(0) = 3$, and the derivative of the second term at $x = 0$ is $(1(2) - \pi(0))/(2^2) = 1/2$. Subtracting gives $f'(0) = 3 - 1/2 = 5/2$.

22. If $f(x) = \ln \sqrt{x^2 - 5}$, then which of the following is equal to $f'(x)$? (1)

A. $\checkmark$ $\frac{x}{x^2 - 5}$ B. $\frac{-x}{\sqrt{x^2 - 5}}$
C. $\frac{2x}{x^2 - 5}$ D. $\frac{-x}{x^2 - 5}$

Answer: A

Using logarithmic properties, $f(x) = (1/2) \ln(x^2 - 5)$. Applying the chain rule yields $f'(x) = (1/2) * (2x / (x^2 - 5)) = x / (x^2 - 5)$.

23. Let f be twice differentiable function on $\mathbb{R}$. which one of the following is necessarily true? (1)

A. if $f'(c) = 0$, at some $c \in \mathbb{R}$, then f has relative extreme at $x = c$.
B. $\checkmark$ **if $f'(x)$ is increasing, then graph of $y = f(x)$ is concave upward.**
C. if $f'(x) = 0$ for all $x \in \mathbb{R}$, then $f(x) = 0$ for all $x \in \mathbb{R}$
D. if $f(x)$ is increasing, then $f''(x) = 0$ for all $x \in \mathbb{R}$

Answer: B

By definition, a function is concave upward on an interval if its first derivative $f'(x)$ is an increasing function on that interval.

24. Suppose $f(x)$ is differentiable on $(-\infty, \infty)$ and the graph of its derivative, $y = f'(x)$ is as shown in the following figure. (1)

[Figure: A scanned hand-drawn Cartesian plane showing one continuous solid curve labelled $y = f'(x)$ (the label is written along the steeply rising right-hand arm of the curve). The horizontal axis is a solid arrow to the right labelled x ; the vertical axis is a solid arrow upward labelled y . Reading the curve left to right: it comes up steeply from the bottom-left of the frame as a nearly straight line, crosses the x -axis from below to above at the tick labelled -2 , keeps rising and rounds off to a local maximum on the y -axis (at about $x = 0$), then falls and comes down to touch the x -axis at the tick labelled 2 without crossing it (the curve is tangent to the axis there), and immediately after that turns and climbs steeply upward off the top of the frame. Tick labels printed under the x -axis, left to right, read -2 , a short dash (a faint -1), 1 and 2 . No values are marked on the y -axis.]

A. $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$ B. $f(x)$ has a local extreme value at $x = 2$
C. $\checkmark$ **$f(x)$ has a local minimum value at $x = -2$** D. $f(x)$ has a local maximum value at $x = 0$

Answer: C

The graph shows that $f'(x)$ crosses the x -axis from negative to positive at $x = -2$, which by the first derivative test means f has a local minimum at $x = -2$.

25. A closed cylindrical can is to be made to hold 1000 cm³ of oil. What are the dimensions (radius r and height h) that will minimize the total surface area of the can? (1)

- A. $r = \frac{\sqrt[3]{\sqrt{50}}}{\pi}, h = 2\frac{\sqrt[3]{\sqrt{50}}}{\pi}$ B. $r = \frac{\sqrt[3]{\sqrt{500}}}{\pi}, h = 2\frac{\sqrt[3]{\sqrt{500}}}{\pi}$
 C. $r = \sqrt[3]{\frac{50}{\pi}}, h = 2\sqrt[3]{\frac{50}{\pi}}$ D. ✓ $r = \sqrt[3]{\frac{500}{\pi}}, h = 2\sqrt[3]{\frac{500}{\pi}}$

Answer: D

For a closed cylinder of volume $V = \pi r^2 h = 1000$, the surface area $A = 2\pi r^2 + 2000/r$ is minimized when $dA/dr = 4\pi r - 2000/r^2 = 0$, giving $r = (500/\pi)^{1/3}$ and $h = 2r = 2(500/\pi)^{1/3}$.

26. Which one of the following is equal to $\int \frac{(\ln x)^2 + x^2 \cos x}{x} dx$? (1)

- A. $1/x^2 \ln x + x \sin x - \cos x + c$ B. $1/3 (\ln x)^3 + x \sin x - \cos x + c$
 C. ✓ $1/3 (\ln x)^3 + x \sin x + \cos x + c$ D. $1/x^2 \ln x + x \sin x + \cos x + c$

Answer: C

Splitting the integrand gives $\int (\ln x)^2 / x dx + \int x \cos x dx$. The first integral evaluates to $(1/3)(\ln x)^3$ via substitution, and the second evaluates to $x \sin x + \cos x$ by parts, plus an arbitrary constant c .

27. The volume of the solid generated when the region bounded between graph of $y = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 4, & 2 \leq x \leq 3 \end{cases}$ and x -axis is rotated about the x -axis: (1)

- A. $32\pi/5$ B. ✓ $112\pi/5$
 C. $112\pi/3$ D. $64\pi/5$

Answer: B

The volume is given by $V = \pi \int_0^2 (x^2)^2 dx + \pi \int_2^3 4^2 dx = \pi [x^5/5]_0^2 + \pi [16x]_2^3 = \pi(32/5 + 16) = 112\pi/5$.

28. If $f: A \rightarrow B$ and $g: B \rightarrow C$ are functions, then which one of the following is true about the composition function? (1)

- A. ✓ **domain of $g \circ f \subseteq$ domain of f** B. **range of $g \circ f \not\subseteq$ range of g**
 C. **domain of $g \circ f \not\subseteq$ domain of f** D. **range of $g \circ f$ range of f**

Answer: A

By definition, the domain of the composite function $g \circ f$ consists of elements x in the domain of f such that $f(x)$ is in the domain of g , which is always a subset of the domain of f .

29. If the point $(3, -2)$ is on the graph of $y = f(x)$, then which point is on the graph of $y = f^{-1}(x)$? (1)

- A. $(1/3, -2)$ B. $(3, -1)$
 C. ✓ **$(-2, 3)$** D. $(3, -1/2)$

Answer: C

If a point (a, b) lies on the graph of $y = f(x)$, then the reflected point (b, a) lies on the graph of its inverse $y = f^{-1}(x)$, so $(3, -2)$ becomes $(-2, 3)$.

30. The equation of the line that passes through $(2, -1)$ and perpendicular to the line $3x + 4y = 6$ is: (1)

- A. $-4x + 3y = 5$ B. $4x - 3y = 5$
 C. $4x + 3y = 11$ D. ✓ **$-4x + 3y = -11$**

Answer: D

The given line $3x + 4y = 6$ has slope $-3/4$, so a perpendicular line has slope $4/3$. Using point-slope form with $(2, -1)$ yields $y + 1 = \frac{4}{3}(x - 2)$, which simplifies to $-4x + 3y = -11$.

31. If $Z = \frac{2-4i}{1+i}$, then the modulus of the conjugate of Z , $|\bar{Z}|$ is: (1)
- A. $\checkmark \sqrt{10}$ B. $3\sqrt{2}$
 C. $2\sqrt{3}$ D. $2\sqrt{2}$

Answer: A

The modulus of the conjugate satisfies $|\bar{Z}| = |Z| = \frac{|2-4i|}{|1+i|} = \frac{\sqrt{2^2+(-4)^2}}{\sqrt{1^2+1^2}} = \frac{\sqrt{20}}{\sqrt{2}} = \sqrt{10}$.

32. The n^{th} term of the sequence: 1, -4, 9, -16,... is: (1)
- A. $a_n = (-2)^n$ B. $a_n = (-1)^n n^2$
 C. $a_n = (-1)2^n n^2$ D. $\checkmark a_n = (-1)^{n-1} n^2$

Answer: D

The sequence alternates signs starting with positive, and the magnitudes are squares of consecutive positive integers n^2 , giving the formula $a_n = (-1)^{n-1} n^2$.

33. Which one of the following is true about the horizontal asymptote(s) of the graph of $y = \frac{|x|+2}{x}$ (1)
- A. $y = 2$ is the only horizontal asymptote of the graph.
 B. $\checkmark y = 1$ and $y = -1$ are the horizontal asymptotes of the graph.
 C. $y = 2$ and $y = -2$ are the horizontal asymptote of the graph.
 D. $y = 1$ is the only horizontal asymptote of the graph.

Answer: B

As $x \rightarrow \infty$, $|x| = x$ giving $\lim_{x \rightarrow \infty} \frac{x+2}{x} = 1$, and as $x \rightarrow -\infty$, $|x| = -x$ giving $\lim_{x \rightarrow -\infty} \frac{-x+2}{x} = -1$, meaning $y = 1$ and $y = -1$ are the horizontal asymptotes.

34. For any $n \times n$ square matrix A , which of the following is true? (1)
- A. $\det(A) = -\det(A^T)$, where A^T is the transpose of A
 B. $\checkmark$ If k is a scalar, then $\det(kA) = k^n \det(A)$
 C. If B is a matrix obtained from A by interchanging of two rows of A , then $\det(B) = \det(A)$
 D. If A is invertible, then $\det(A) = \det(A^{(-1)})$

Answer: B

By the determinant properties of an $n \times n$ matrix, scaling every row by k results in $\det(kA) = k^n \det(A)$.

35. The solution of the system of linear equation of $\begin{cases} x - 3y - 2z = 6 \\ 2x - 4y - 3z = 8 \\ -3x + 6y + 8z = -5 \end{cases}$ is: (1)
- A. $x = -1, y = -3, z = -2$ B. $x = -1, y = -3, z = 2$
 C. $\checkmark x = 1, y = -3, z = 2$ D. $x = 1, y = 3, z = -2$

Answer: C

Substituting $x = 1, y = -3, z = 2$ into all three equations satisfies each one: $1 - 3(-3) - 2(2) = 6$, $2(1) - 4(-3) - 3(2) = 8$, and $-3(1) + 6(-3) + 8(2) = -5$.

36. In how many more ways can 4 people be arranged in a row than if they were arranged in a circle? (1)
- A. 1
B. 6
C. ✓ 18
D. 12

Answer: C

Four people can be arranged in a row in $4! = 24$ ways and in a circle in $(4 - 1)! = 3! = 6$ ways, which is $24 - 6 = 18$ more ways.

37. The value of $\int_0^1 (x + 1)e^{(x^2+2x)} dx$ is: (1)
- A. $\frac{e^3}{2}$
B. $\frac{e^4 - e}{2}$
C. ✓ $\frac{e^3 - 1}{2}$
D. $e^3 - 1$

Answer: C

Using substitution with $u = x^2 + 2x$, we have $du = 2(x + 1) dx$. The integral becomes $\frac{1}{2} \int_0^3 e^u du = \frac{e^3 - 1}{2}$.

38. $\int_0^3 (x + 1)^{\frac{1}{2}} dx = \text{-----}$. (1)
- A. $21/2$
B. ✓ $14/3$
C. 7
D. $16/3$

Answer: B

The antiderivative of $(x + 1)^{1/2}$ is $\frac{2}{3}(x + 1)^{3/2}$. Evaluating from 0 to 3 gives $\frac{2}{3}(4^{3/2} - 1^{3/2}) = \frac{2}{3}(8 - 1) = \frac{14}{3}$.

39. The sum of $\sum_{n=0}^{\infty} 5 \left(\frac{2}{3}\right)^n$ is ----- (1)
- A. 0
B. ✓ 15
C. $10/3$
D. 5

Answer: B

This is an infinite geometric series with first term $a = 5$ and common ratio $r = 2/3$. The sum is $S = \frac{a}{1-r} = \frac{5}{1-2/3} = 15$.

40. In which interval the sequence $\left\{ \frac{(-1)^n}{3n} \right\}_{n=1}^{\infty}$ is bounded? (1)
- A. $\left[\frac{-1}{9}, \frac{1}{12} \right]$
B. ✓ $\left[\frac{-1}{3}, \frac{1}{6} \right]$
C. $\left[\frac{-1}{6}, \frac{1}{3} \right]$
D. $\left[\frac{-1}{12}, \frac{1}{9} \right]$

Answer: B

The terms alternate in sign with decreasing magnitude: $a_1 = -1/3, a_2 = 1/6, a_3 = -1/9$, etc. The minimum term is $-1/3$ and the maximum term is $1/6$, so all terms lie within $[-1/3, 1/6]$.

41. Which one of the following is true about the function given by the formula $f(x) = \begin{cases} \frac{x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$? (1)

A. f is continuous except at $x = 0$ B. f has an infinite discontinuity at $x = 0$
C. ✓ **f is continuous everywhere** D. f has $x = 0$ as a vertical asymptote

Answer: C

For $x \neq 0$, $f(x) = \frac{x^2}{x} = x$. Since $\lim_{x \rightarrow 0} f(x) = 0 = f(0)$, the function is continuous at $x = 0$ as well, making it continuous everywhere on $\mathbb{R}$.

42. $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$ is equal to :? (1)

A. 0 B. ✓ 1
C. ∞ D. -1

Answer: B

Substituting $t = 1/x$, as $x \rightarrow \infty$, $t \rightarrow 0^+$. The limit becomes $\lim_{t \rightarrow 0^+} \frac{\sin t}{t} = 1$.

43. $\frac{d}{dx} (\ln e^{2x})$ is equal to :? (1)

A. $\frac{1}{e^{2x}}$ B. $\frac{2}{e^{2x}}$
C. $2x$ D. ✓ 2

Answer: D

By the inverse properties of logarithms and exponentials, $\ln(e^{2x}) = 2x$. Differentiating $2x$ with respect to x yields 2.

44. If $f(x) = 2 + |x - 3|$ for all x , then the value of the derivative $f'(x)$ at $x = 3$ (1)

A. -1 B. ✓ **Does not exist**
C. 1 D. 2

Answer: B

The function $f(x) = 2 + |x - 3|$ has a sharp corner (cusp) at $x = 3$, where the left-hand derivative is -1 and the right-hand derivative is 1. Since the one-sided derivatives are not equal, the derivative $f'(3)$ does not exist.

45. The graph of $y = 5x^4 - x^5$ has a point of inflection at: (1)

A. ✓ **(3, 162) only** B. (4, 256) only
C. (0, 0) only D. (0, 0) and (3, 162)

Answer: A

The second derivative is $y'' = 20x^2(3 - x)$, which equals zero at $x = 0$ and $x = 3$. The second derivative changes sign only across $x = 3$ (from positive to negative), so the only inflection point is at $(3, y(3)) = (3, 162)$.

46. Let the angle between $\vec{V} = -2\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{PQ}$ be 60° , where P and Q are points in space. If $\vec{V} \cdot \vec{PQ} = 2$, (1)
then what is the distance between P and Q ?

A. $3/4$ B. $4/5$
C. ✓ **$4/3$** D. $5/4$

Answer: C

The magnitude of vector V is $\sqrt{(-2)^2 + (-1)^2 + 2^2} = 3$. Using $V \cdot PQ = |V||PQ|\cos(60^\circ)$, we have $3|PQ|(1/2) = 2$, which gives the distance $|PQ| = 4/3$.

47. Consider the following assertion of a person and his proof "If x and y are equal positive integers, then $x + y = y$." (1)
 Proof: the following steps and reasons are used to prove the assertion. Step | Reason
 1. $x = y$ | Given hypothesis
 2. $x^2 = xy$ | Multiply both sides of (1) by x
 3. $x^2 - y^2 = xy - y^2$ | Subtract y^2 from both sides of (2)
 4. $(x - y)(x + y) = (x - y)y$ | Factor both sides of (3)
 5. $x + y = y$ | Divide both sides of (4) by $x - y$
 Step 5 completes the proof.
 Which one of the following is true about this proof?
- A. It is a correct direct proof of the assertion.
 B. It follows the technique of a proof by contradiction because the steps lead to a contradiction.
 C. ✓ **The proof is invalid because Step 4 does not lead to Step 5.**
 D. The proof is invalid because Step 4 does not follow from Step 3.

Answer: C

Since the hypothesis states $x = y$, the term $x - y$ equals 0. Dividing both sides of Step 4 by $x - y$ to obtain Step 5 involves division by zero, which is invalid.

48. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction? (1)
- A. ✓ **$2^n > 10^n$ for every integer n such that $n \geq 6$.** B. $r^2 > 0$ for every real number r such that $r \geq 1$.
 C. $n^2 + 10n > 2n^2$ for every natural number $n \geq 1$. D. $2^n > 8n$ for every integer n such that $n \geq 3$.

Answer: A

49. The image of a figure with vertices A(1, 2), B(3, 6), C(-1, 2), and D(-2, -2) after reflection across the x-axis is: (1)
- A. A'(1, -2), B'(-3, -6), C'(1, -2), and D'(2, 2)
 B. A'(-1, 2), B'(-3, 6), C'(1, -2), and D'(2, -2)
 C. ✓ **A'(1, 2), B'(3, -6), C'(-1, -2), and D'(-2, 2)**
 D. A'(1, -2), B'(3, 6), C'(-1, 2), and D'(-2, -2)

Answer: C

Reflection across the x-axis maps each point (x, y) to $(x, -y)$, producing A'(1, -2), B'(3, -6), C'(-1, -2), and D'(-2, 2). Option C is the intended answer as it correctly transforms vertices B, C, and D, despite a typographical omission of the negative sign in A'.

50. If $\vec{V} = \overrightarrow{AB} + 3\overrightarrow{BA}$, where A and B are distinct points in the coordinate plane, then which one of the following is equal to $3\vec{V}$? (1)
- A. $6\overrightarrow{AB}$ B. ✓ $-6\overrightarrow{AB}$
 C. $12\overrightarrow{AB}$ D. $-12\overrightarrow{AB}$

Answer: B

Since $\overrightarrow{BA} = -\overrightarrow{AB}$, the vector $\vec{V} = \overrightarrow{AB} + 3(\overrightarrow{BA}) = \overrightarrow{AB} - 3(\overrightarrow{AB}) = -2(\overrightarrow{AB})$. Multiplying by 3 yields $3\vec{V} = 3(-2(\overrightarrow{AB})) = -6(\overrightarrow{AB})$.

51. If $\vec{U}$ is a unit vector in the direction of $\vec{A}$ and $|\vec{A}| = 4$, then $\vec{A} \cdot \vec{U}$ is equal to _____. (1)
- A. 1/4 B. ✓ **4**
 C. 1/2 D. 2

Answer: B

Since $\vec{U}$ is a unit vector in the same direction as $\vec{A}$, the angle between them is 0° . Thus, $\vec{A} \cdot \vec{U} = |\vec{A}||\vec{U}|\cos(0^\circ) = 4(1)(1) = 4$.

52. If $\cot(\theta) = 2$, then which of the following is equal to $\csc(\theta)$? (1)
- A. $\checkmark \sqrt{5}$ B. $2/\sqrt{5}$
C. $1/\sqrt{5}$ D. $1/2$

Answer: A

Using the trigonometric identity $\csc^2(\theta) = 1 + \cot^2(\theta)$, we find $\csc^2(\theta) = 1 + 2^2 = 5$, which gives $\csc(\theta) = \pm\sqrt{5}$. The positive value $\sqrt{5}$ corresponds to option A.

53. What is the amplitude and period, respectively, of the graph of $f(x) = 4\sin(x/3)\cos(x/3)$? (1)
- A. 4, $\pi/3$ B. $\checkmark$ 2, 3π
C. 2, $2\pi/3$ D. 4, 3π

Answer: B

Using the double-angle identity, $f(x) = 2(2\sin(x/3)\cos(x/3)) = 2\sin(2x/3)$. The amplitude is 2 and the period is $T = \frac{2\pi}{2/3} = 3\pi$.

54. A boat on a sea sailed from its station toward North with constant speed of 80 km/h. Another boat from the same station sailed 60° NE (North East) with constant speed of 100km/h. If the two boats started sailing at the same time, what is the straight distance between them after they have sailed for just 30 minutes? (1)
- A. $10\sqrt{42}$ km B. 90km
C. $10\sqrt{41}$ km D. $\checkmark$ $10\sqrt{21}$ km

Answer: D

In 30 minutes, the first boat travels 40 km and the second travels 50 km with an included angle of 60° . By the Law of Cosines, $d^2 = 40^2 + 50^2 - 2(40)(50)\cos 60^\circ = 4100 - 2000 = 2100$, so $d = 10\sqrt{21}$ km.

55. What is the value k, for which the two vectors $\vec{U} = \begin{pmatrix} 1 \\ k \\ -3 \end{pmatrix}$ and $\vec{V} = \begin{pmatrix} 2k \\ -5 \\ 4 \end{pmatrix}$ are perpendicular? (1)
- A. 4 B. $\checkmark$ -4
C. 3 D. -3

Answer: B

Two vectors are perpendicular if and only if their dot product is zero: $1(2k) + k(-5) + (-3)(4) = -3k - 12 = 0$, which yields $k = -4$.

56. If one of the end point of the line segment is (3, 2, -4) and the mid-point is (4, 1, -2), then the coordinate of the other end point is: (1)
- A. $\checkmark$ (5, 0, 0) B. (2, 0, 5)
C. (5, 1, 2) D. (3, 1, 0)

Answer: A

Using the midpoint formula $M = \frac{A+B}{2}$, the other endpoint is $B = 2M - A = 2(4, 1, -2) - (3, 2, -4) = (8 - 3, 2 - 2, -4 - (-4)) = (5, 0, 0)$.

57. What is the value of $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ (1)
- A. $\pi/4$ B. $\pi/2$
C. ✓ $-\pi/4$ D. $-\pi/2$

Answer: C

The principal range of the arcsine function is $[-\pi/2, \pi/2]$. Since $\sin(-\pi/4) = -\sqrt{2}/2$, the value of $\arcsin(-\sqrt{2}/2)$ is $-\pi/4$.

— END OF MARKING KEY —

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