Temari.et

Mathematics, 2013 EC, common, entrance exam

Grade 12 · Mathematics · 2013 E.C.

MARKING KEY — CONFIDENTIAL

Time allowed: 120 minutes | **Questions:** 20 | **Total marks:** 20

1. What is the determinant of the matrix: (1)

$$\begin{pmatrix} 1 & -7 \\ 6 & 5 \end{pmatrix}$$

A. ✓ **47**
B. 37
C. -37
D. -47

Answer: A

For a 2 by 2 matrix the determinant is $ad - bc$, where a and b are the top row and c and d are the bottom row.

Here $a = 1, b = -7, c = 6, d = 5$.

Step 1: $ad = 1 \times 5 = 5$
Step 2: $bc = (-7) \times 6 = -42$
Step 3: $ad - bc = 5 - (-42) = 5 + 42 = 47$

The determinant is 47, so the answer is A.

Watch the sign: if you drop the minus on -7 you get $5 - 42 = -37$, which is the trap in option C.

Textbook: Mathematics Grade 11, Unit 4 Determinants and Their Properties, section 4.1 Determinants of Matrices of Order 2 (page 209).

2. The additive inverse of the complex number 3 - 4i is (1)

A. 3 - 4i
B. ✓ **-3 + 4i**
C. 3 + 4i
D. -3 - 4i

Answer: B

The additive inverse of a number z is the number you add to z to get 0. It is $-z$.

Step 1: write the inverse: $-(3 - 4i)$
Step 2: change the sign of both parts: $-3 + 4i$

Check: $(3 - 4i) + (-3 + 4i) = 0$. So the answer is B.

Do not mix this up with the conjugate. The conjugate of $3 - 4i$ is $3 + 4i$, which is the trap in option C.

3. What is the modulus of the complex number -6 + 8i? (1)

A. ✓ **10**
B. 8
C. 5
D. 2

Answer: A

The modulus of a complex number $a + bi$ is its distance from the origin: $|a + bi| = \sqrt{a^2 + b^2}$.

Here $a = -6$ and $b = 8$.

Step 1: $a^2 = (-6)^2 = 36$
Step 2: $b^2 = 8^2 = 64$
Step 3: $|z| = \sqrt{36 + 64} = \sqrt{100} = 10$

So the answer is A.

Remember that squaring removes the minus sign, so the -6 contributes $+36$, never -36.

4. Given two vectors u = (1 , 3) and v= (-3, 5) in the plane. Then 6u + 2v is equal to (1)

A. (2, 28)
B. ✓ **(0, 28)**
C. (0,18)
D. (3, 3)

Answer: B
To find $6u + 2v$, multiply each vector by its number, then add the matching coordinates.
Step 1: $6u = 6(1, 3) = (6, 18)$
Step 2: $2v = 2(-3, 5) = (-6, 10)$
Step 3: add: $(6 + (-6),\ 18 + 10) = (0, 28)$
So the answer is B.
The trap in option A is adding $6 + (-6)$ wrongly; the first coordinates cancel to 0 exactly.
Textbook: Mathematics Grade 11, Unit 5 Vectors, section 5.1 Revision on Vectors and Scalars (page 252).

5. Which one of the following is the standard form of the equation ofthe circle centered at (2, -3) and radius 5? (1)

A. (x - 2)^(2) + (y + 3)^(2) = 5
B. (x + 2)^(2) + (y + 3)^(2) = 25
C. ✓ **(x - 2)^(2) + (y + 3)^(2) = 25**
D. (x - 2)^(2) + (y - 3)^(2) = 25

Answer: C
The standard form of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.
Here $h = 2, k = -3, r = 5$.
Step 1: $x - h = x - 2$
Step 2: $y - k = y - (-3) = y + 3$
Step 3: $r^2 = 5^2 = 25$
So the equation is $(x - 2)^2 + (y + 3)^2 = 25$, which is C.
Two traps here: option A forgets to square the radius, and option D keeps $y - 3$ instead of flipping the sign of -3.

6. Which of the following defines the equation of a sphere whose centeris at (0, -1, 2) and radius 6 units? (1)

A. x^2 + (y + 1)^2 + (z - 2)^2 = 12
B. x^2 + (y + 1)^2 + (z + 2)^2 = 36
C. ✓ **x^2 + (y + 1)^2 + (z - 2)^2 = 36**
D. x^2 + (y - 1)^2 + (z - 2)^2 = 36

Answer: C
A sphere with center (h, k, l) and radius r has the equation $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$. It is the circle formula with one more coordinate.
Here the center is $(0, -1, 2)$ and $r = 6$.
Step 1: $x - 0 = x$, so the first term is x^2
Step 2: $y - (-1) = y + 1$, giving $(y + 1)^2$
Step 3: $z - 2$ gives $(z - 2)^2$
Step 4: $r^2 = 6^2 = 36$
So the equation is $x^2 + (y + 1)^2 + (z - 2)^2 = 36$, which is C.
Option A uses $2 \times 6 = 12$ instead of $6^2 = 36$, and option B flips the sign on z. Both are common slips.

7. Which of the following represents the vector (3, -5, 7) in the spaceusing the standard unit vectors i, j and k ? (1)

A. ✓ **3i - 5j + 7k**

B. -3i - 5j - 7k

C. 3i - 51 - 7k

D. 3i + 5j + 7k

Answer: A

In space, $i = (1, 0, 0), j = (0, 1, 0)$ and $k = (0, 0, 1)$. Any vector (a, b, c) can be written as $ai + bj + ck$: the coordinates become the coefficients.

Step 1: first coordinate 3 gives $3i$

Step 2: second coordinate -5 gives $-5j$

Step 3: third coordinate 7 gives $7k$

So $(3, -5, 7) = 3i - 5j + 7k$, which is A.

Keep every sign exactly as it appears in the coordinates; only the middle one is negative here.

Textbook: Mathematics Grade 11, Unit 5 Vectors, section 5.2 Representation of Vectors (page 254).

8. What are the domain and range of the function f(x) = $2x^{3/4}$ respectively? (1)

A. [0, ∞) and R

B. ✓ **[0, ∞) and [0, ∞)**

C. R and [0, ∞)

D. [0, ∞) and (0, ∞)

Answer: B

The function is $f(x) = 2x^{3/4}$. A power with a fraction exponent is a root: $x^{3/4} = \sqrt[4]{x^3}$, a fourth root.

Domain: a fourth root is an even root, so what is inside must not be negative. $x^3 \geq 0$ means $x \geq 0$. Domain: $[0, \infty)$.

Range: an even root is never negative, so $f(x) \geq 0$. At $x = 0$ we get $f(0) = 0$, and as x grows, $f(x)$ grows without limit. Range: $[0, \infty)$.

Both are $[0, \infty)$, so the answer is B.

Option D is the near miss: it leaves out 0 from the range, but $f(0) = 0$ really happens, so 0 belongs in the range.

Textbook: Mathematics Grade 11, Unit 1 Relations and Functions, section 1.3 Types of Functions (page 22).

9. Which one of the following defines a one-to-one function? (1)

A. f(x) = {(x, y): y = x^2 + 1}

B. f(x) = {(x, y): y is a father of x}

C. f(x) = {(x, y): y is a sister of x}

D. ✓ **f(x) = {(x, y): y = 3x - 1}**

Answer: D

A function is one to one when different inputs always give different outputs.

Check each option:

- A: $y = x^2 + 1$ gives the same output for x and $-x$. For example $f(1) = 2$ and $f(-1) = 2$. Not one to one.
- B: every person has exactly one father, so this is a function, but two children can share the same father. Not one to one.
- C: a person can have more than one sister, so one input can point to several outputs. This is not even a function.
- D: $y = 3x - 1$ is a line with slope 3. If $3a - 1 = 3b - 1$ then $a = b$, so different inputs never share an output. One to one.

The answer is D.

Textbook: Mathematics Grade 11, Unit 1 Relations and Functions, section 1.3 Types of Functions (page 22).

10. The equation of a circle is given by $(x - 3)^2 + (y + 2)^2 = 9$. What arethe center and the radius of the circle representatively? (1)

A. (3,-2) and 9

B. (3,2) and 9

C. ✓ **(3,-2) and 3**

D. (3,2) and 3

Answer: C

Compare the given equation with the standard form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

Step 1: $(x - 3)^2$ matches $(x - h)^2$, so $h = 3$.
Step 2: $(y + 2)^2 = (y - (-2))^2$, so $k = -2$.
Step 3: $r^2 = 9$, so $r = \sqrt{9} = 3$.

Center $(3, -2)$ and radius 3, which is C.

The classic trap is reading the radius as 9. The equation gives r^2, so you must take the square root.

11. Which of the following sentences has a truth value? (1)

A. ✓ **Man is mortal**

B. Tomorrow is a beautiful day

C. Man is selfish

D. We shall win!

Answer: A

In logic, a sentence has a truth value when it is a statement: a declarative sentence that is definitely true or definitely false, with no opinion in it.

Check each option:

- A: "Man is mortal" states a fact. It is true, so it has a truth value.
- B: "Tomorrow is a beautiful day" is about the future and "beautiful" is a matter of taste. No fixed truth value.
- C: "Man is selfish" is an opinion. Different people would judge it differently.
- D: "We shall win!" is an exclamation, not a claim that can be checked.

The answer is A.

12. Which one of the following is the standard form of the equation ofthe parabola with vertex (4, 6) and focus (-8, 6)? (1)

A. (y - 6)^2 = 48(x - 4)

B. ✓ **(y - 6)^2 = -48(x - 4)**

C. (x - 4)^2 = -12(y - 6)

D. (x - 4)2 = 12(y - 6)

Answer: B

The vertex is $(4, 6)$ and the focus is $(-8, 6)$. They share the same y value, so the axis of the parabola is horizontal and the equation has the form $(y - k)^2 = \pm 4p(x - h)$.

Step 1: the vertex gives $h = 4$ and $k = 6$.
Step 2: p is the distance from vertex to focus: $p = |4 - (-8)| = 12$.
Step 3: the focus is to the LEFT of the vertex, so the parabola opens left and the sign is negative: $(y - 6)^2 = -4p(x - 4)$.
Step 4: $-4p = -4 \times 12 = -48$.

So the equation is $(y - 6)^2 = -48(x - 4)$, which is B.

Option A is the sign trap: a parabola always opens toward its focus, and here the focus is on the left.

13. Which of the following is true about the hyperbola given by $(x^{2/4}) - y^2 = 1$? (1)

A. Its center is at (2, 1).

B. An asymptote to the hyperbola is y = 2x

C. Its foci are at (-5, 0) and (5, 0).

D. ✓ **Its vertices are at (-2, 0) and (2, 0).**

Answer: D

The equation $\frac{x^2}{4} - y^2 = 1$ has the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, a hyperbola centered at the origin that opens left and right.

Step 1: $a^2 = 4$, so $a = 2$. $b^2 = 1$, so $b = 1$.

Step 2: center: $(0, 0)$, so A is false.

Step 3: vertices: $(\pm a, 0) = (-2, 0)$ and $(2, 0)$.

Step 4: asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{1}{2}x$, so B (which says $y = 2x$) is false.

Step 5: $c^2 = a^2 + b^2 = 5$, so the foci are $(\pm\sqrt{5}, 0)$, not $(\pm 5, 0)$. C is false.

Only the statement about the vertices is true, so the answer is D.

Option C is the trap: $c^2 = 5$ means $c = \sqrt{5}$, not 5.

14. Two statements P and q are given by: (1)

p: 22/7 is an irrational number.

q: The sum of two odd integers is odd.

Then which of the following has truth value T?

A. ¬p ∧ q

B. p ∧ q

C. p ∧ ¬q

D. ✓ **¬q ∨ p**

Answer: D

First find the truth value of each statement.

p: "22/7 is an irrational number." This is FALSE. $22/7$ is a ratio of two integers, so it is rational. It is only an approximation of π; it is not π itself.

q: "The sum of two odd integers is odd." This is FALSE. For example $3 + 5 = 8$, which is even.

So $p \equiv F, q \equiv F, \neg p \equiv T, \neg q \equiv T$.

Now test each option:

A: $\neg p \wedge q = T \wedge F = F$

B: $p \wedge q = F \wedge F = F$

C: $p \wedge \neg q = F \wedge T = F$

D: $\neg q \vee p = T \vee F = T$

Only D is true.

The trap is calling $22/7$ irrational because it reminds you of π. Any fraction of integers is rational.

15. Which one of the following is the contrapositive of the statement: If 21 is a prime number, then 4 is an odd number? (1)

A. ✓ **If 4 is not an odd number, then 21 is not a prime number.**

B. If 21 is not a prime number, then 4 is an even number.

C. If 4 is an odd number, then 21 is not a prime number.

D. If 4 is an odd number, then 21 is a prime number.

Answer: A

The contrapositive of "if p then q" is "if $\neg q$ then $\neg p$": negate both parts AND swap their order.

Here:
p: 21 is a prime number.
q: 4 is an odd number.

Step 1: $\neg q$: 4 is not an odd number.
Step 2: $\neg p$: 21 is not a prime number.
Step 3: contrapositive: "If 4 is not an odd number, then 21 is not a prime number."

That is option A.

Do not confuse it with the converse (swap only, option D) or the inverse (negate only). Only the contrapositive always keeps the same truth value as the original statement.

16. In a certain health center there are 3 doctors, 8 nurses and 2 physicians. In how many ways one can form a group of 5 members consisting of 1 doctor, 3 nurses and 1 physician? (1)

A. ✓ **336**

B. 330

C. 168

D. 112

Answer: A

Order does not matter when forming a group, so use combinations, and multiply the choices for each profession.

Step 1: choose 1 doctor from 3: $\binom{3}{1} = 3$
Step 2: choose 3 nurses from 8: $\binom{8}{3} = \frac{8\times7\times6}{3\times2\times1} = \frac{336}{6} = 56$
Step 3: choose 1 physician from 2: $\binom{2}{1} = 2$
Step 4: multiply: $3 \times 56 \times 2 = 336$

So there are 336 ways, which is A.

The trap is adding the counts instead of multiplying them. Each doctor choice can pair with every nurse group and every physician, so the counts multiply.

Textbook: Mathematics Grade 11, Unit 8 Probability, section 8.3 Permutations and Combinations (page 423).

17. Which one of the following forms represents an exhaustive set of events if a die is thrown once? (1)

A. {1, 2}, {2}, {4, 5}, {5, 6}

B. {1, 2}, {4}, {4, 5, 6}

C. {1, 2, 3}, {3, 4}, {6}

D. ✓ **{1, 2, 3}, {4, 5}, {6}**

Answer: D

Events are exhaustive when, taken together, they cover the whole sample space. For one throw of a die the sample space is $\{1, 2, 3, 4, 5, 6\}$, so the union of the events must contain all six outcomes.

Check each option:
A: $\{1, 2\} \cup \{2\} \cup \{4, 5\} \cup \{5, 6\} = \{1, 2, 4, 5, 6\}$. Missing 3.
B: $\{1, 2\} \cup \{4\} \cup \{4, 5, 6\} = \{1, 2, 4, 5, 6\}$. Missing 3.
C: $\{1, 2, 3\} \cup \{3, 4\} \cup \{6\} = \{1, 2, 3, 4, 6\}$. Missing 5.
D: $\{1, 2, 3\} \cup \{4, 5\} \cup \{6\} = \{1, 2, 3, 4, 5, 6\}$. Complete.

Only D covers every outcome, so the answer is D.

Exhaustive is about covering everything; the events do not need to be equal in size, they just must leave no outcome out.

Textbook: Mathematics Grade 11, Unit 8 Probability, section 8.6 Events (page 444).

18. A bag contains 2 red, 3 yellow and 5 green balls. One ball is drawn at random. What is the probability that the drawn ball is NOT yellow? (1)

A. 4/5

B. ✓ **7/10**

C. 3/10

D. 1/5

Answer: B

Use the complement rule: $P(\text{not } E) = 1 - P(E)$.

Step 1: total balls: $2 + 3 + 5 = 10$
Step 2: $P(\text{yellow}) = \frac{3}{10}$
Step 3: $P(\text{not yellow}) = 1 - \frac{3}{10} = \frac{7}{10}$

Check directly: the non yellow balls are $2 + 5 = 7$ out of 10, so $\frac{7}{10}$. The answer is B.

Option C, $\frac{3}{10}$, answers the opposite question. Read whether the question asks for yellow or NOT yellow.

Textbook: Mathematics Grade 11, Unit 8 Probability, section 8.7 Probability of an Event (page 453).

19. Which one of the following is NOT true about the determinant of a given square matrix A ? (1)

A. The determinant of A is equal to the determinant of its transpose.

B. ✓ **Interchanging two rows of A gives the same determinant.**

C. If A contains a row of zeros, then its determinant is zero.

D. If A has identical rows, then its determinant is zero.

Answer: B

The question asks for the statement that is NOT true, so check each property of determinants.

- A: $\det(A^T) = \det(A)$. This is TRUE for every square matrix.
- B: interchanging two rows keeps the determinant the same. This is FALSE. Swapping two rows multiplies the determinant by -1, so the sign flips.
- C: a row of zeros makes the determinant zero. TRUE, since every term in the expansion uses one entry from that row.
- D: two identical rows make the determinant zero. TRUE. Swapping the identical rows changes nothing but must also flip the sign, so $\det(A) = -\det(A)$, which forces $\det(A) = 0$.

The false statement is B, so the answer is B.

Quick check with $a = 1, b = 2, c = 3, d = 4$: the determinant is $1 \times 4 - 2 \times 3 = -2$. After swapping the rows it becomes $3 \times 2 - 4 \times 1 = 2$. Same size, opposite sign.

Textbook: Mathematics Grade 11, Unit 4 Determinants and Their Properties, section 4.4 Properties of Determinants (page 220).

20. If f is a continuous function on [a, b] and F is its anti-derivative, then which of the following is true? (1)

 Question Diagram

A. ✓ **A**

B. B

C. C

D. D

Answer: A

This is the Fundamental Theorem of Calculus. If F is an antiderivative of f (that is, $F' = f$), then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

That is exactly option A: integrate the function, then evaluate its antiderivative at the two endpoints and subtract.

Why the others fail:
B: $\int_a^b f'(x)\,dx = f(b) - f(a)$, not $F(b) - F(a)$. The antiderivative of f' is f itself.
C: since $F' = f$, $\int_a^b F'(x)\,dx = F(b) - F(a)$, not $f(b) - f(a)$.
D: integrating F itself has no such shortcut; the theorem links a function with its OWN antiderivative.

The answer is A.

The trap is mixing up which letter is the derivative of which: here F is the antiderivative of f, so f goes inside the integral and F goes on the right side.

Textbook: Mathematics Grade 12, Unit 2 Introductions to Calculus, section 2.3 Introduction to Integration (page 123).

— END OF MARKING KEY —