Temari.et

Mathematics, 2006 EC, entrance exam

Grade 12 · Mathematics · 2006 E.C.

Time allowed: 120 minutes | **Questions:** 60 | **Total marks:** 60

Name _____ Grade _____ ID No. _____ Date _____

Answers, explanations and timed practice for this paper are free on Temari.et.
temari.et/exam-prep/papers/grade-12-mathematics-2006

1. A water tank is a circular cylinder with base radius 2m and height 3m. If the tank is empty and water is pumped into it at rate of $2\mathrm{m}^3$/min, how long does it take for the tank to be full? (1)

A. 1.5 min
B. 3/2 πmin
C. 6π
D. 12 min

2. If $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and $(2A+B)^T = A^T A$, then which one of the following is equal to B? (1)

A. $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$
B. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
C. $\begin{pmatrix} 8 & 0 & -4 \\ 4 & 8 & 0 \\ 0 & 0 & -4 \end{pmatrix}$
D. $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix}$

3. Which one of the following represents a geometric sequence? (1)

A. 3, 1, 1/3, 1/9, 1/27,?
B. 1/2, -1/3, 1/4, -1/5, 1/6,?
C. 1,3, 6, 10, 15,?
D. -3, 6, -9, 12, -15,?

4. The sequence $\left\{ \frac{(n-1)(2n+1)}{1-n^2} \right\}_{n=1}^{\infty}$ converges to: (1)

A. $-\infty$
B. -2
C. 0
D. 1

5. If F(x) is an ant derivative of $f(x) = 1 - 2/x^2$ and F(1) = 0, then F(2) is equal to: (1)

A. 0
B. 1/2
C. -1/2
D. 3

6. Which one of the following expression is a polynomial expression? (1)

A. $x^2 - 3x + \sin x$
B. $\frac{4x^3+12x^2-x}{\pi x^2}$
C. $\frac{2+\pi}{1+\pi^2}$
D. $2 - 3x^{\frac{2}{3}} + 7x^{\frac{5}{2}} + 3x^{-1}$

7. What is the distance from the origin to the line that passes through (1, 0) and (0, 1)? (1)

A. $1/2\sqrt{2}$
B. 1
C. 1/2
D. $\sqrt{2}$

8. If the list of a measurement is 10, α, 5, α,5, 10, 20, 15,20,5 with mean a then α in terms of a is equal to: (1)

A. 10a-90 **B.** 9a-90

C. 5a-90 **D.** 5a-45

9. The total cost (in Birr) of producing x iron sheets per day is $C(x) = 1{,}000 + 100x - 0.5x^2, 0 \leq x \leq 100.$ What is the marginal (rate of change of) cost at a production level of 80 iron sheets? (1)

A. 8.5 **B.** 20

C. 1,800 **D.** 5,800

10. If z = x + yi is a complex number, then $|z|^2 + \frac{1}{2}(z - \bar{z})^2 = 1$ is equivalent to which one of the following equations? (1)

A. $x^2 - y^2 = 1$ **B.** $x^2 - 3iy^2 = 1$

C. $x^2 - y^2 = 2$ **D.** $2x^2 - y^2 = 1$

11. The following is the frequency distribution of a grouped data. (1)

Class Intervals	Frequency (f)
3-7	2
8-12	2
13-17	10
18-22	6

What is the mean and standard deviation of the distribution, respectively?

A. $15, 2\sqrt{5}$ **B.** $15, \sqrt{7.5}$

C. $12.5, 5\sqrt{2}$ **D.** $12.5, \sqrt{15}$

12. If $(p \vee q) \Rightarrow (\neg r \wedge r)$ is true, then which one of the following is necessarily true? (1)

A. $((p \vee r) \Rightarrow q$ **B.** $\neg p \wedge r$

C. $\neg p \Rightarrow r$ **D.** $\neg p \vee r$

13. What is the actual value of the sum $\sum_{n=1}^{\infty} \left(\frac{2^n+5^n}{10^n}\right)$ (1)

A. 0.325 **B.** 1

C. 5/4 **D.** 37/9

14. If $f(x) = \frac{x+1}{x-1}$ and $f(a) = 5$ then $f(2a)$ is equal to: (1)

A. 2 **B.** 4

C. 6 **D.** 8

15. If $f(x) = \sqrt[3]{1 + e^{-x}}$, which of the following is equal to $f^{-1}(x)$? (1)

A. $\ln\left(\frac{1}{x^3-1}\right)$ **B.** $\frac{1}{\ln(x^3-1)}$

C. $\ln(1 - x^3)$ **D.** $(1 + e^{-x})^3$

16. What is the equation of the directrix for the parabola whose equation is $y^2 + 8x + 6y + 25 = 0$? (1)

A. y = 3 **B.** x = 2

C. x = 0 **D.** x = 4

17. If $f(x) = \frac{x^2}{1+xg(x)}$, g(2) = 1 and g(2) = 10, then which one of the following is equal to f'(2)(the derivative of f at 2)? (1)

A. -8 **B.** -8/9

C. 4/3 **D.** 8/9

18. If $w = \frac{16i}{1+i} + (1-3i)^2$ and $z = |w| + \bar{w}$, which one of the following is the simplest form of z? (1)

A. $\sqrt{2} + 2i$

B. 2 + 2i

C. 4 - 2i

D. 2 - 2i

19. If $M = \begin{pmatrix} 0 & 1 & 2 \\ 3 & -1 & 0 \\ 5 & 2 & 4 \end{pmatrix}$ and $A^T M = 21$, where A is a 3 x 3 matrix and I is the identity matrix of order 3, then what is det(A)? (1)

A. 0.2

B. 4/17

C. 0.8

D. 1/17

20. If distinct codes (words of eight letters are formed by rearranging the letters in the word'ABBEBAYE', how many of the codes begin with B or Y? (1)

A. 840

B. 630

C. 1680

D. 420

21. What should be the value of k so that the system of equation $\begin{cases} x - y + z = 1 \\ -x + 5y - 4z = 1 \\ 2x + 2y - z = k \end{cases}$ has a solution? (1)

A. 0

B. 1

C. -4

D. 4

22. Let $f(x) = \begin{cases} a\dfrac{\sin x}{x - |x|} & \text{if } x < 0 \\ e^{-x} + \cos x, & \text{if } x \geq 0 \end{cases}$ if f is continuous at x = 0, then what is the value of a? (1)

A. 4

B. 2

C. 1/2

D. -4

23. What is the area of the region between the graphs of $y = -x^2 + 2$ and $y = |x|$, where $-1 \leq x \leq 2$? (1)

A. 11/6

B. 25/6

C. 7/3

D. 11/3

24. What is the derivative of $f(x) = \int_0^{x^2+\pi} \frac{dt}{\sin t + 1}$? (1)

A. $\frac{\cos x}{\sin(x^2+\pi)+1}$

B. $\frac{2x}{\sin(x^2+\pi)+1}$

C. $\frac{2xcox}{\sin(x^2+\pi)+1}$

D. $\int_0^{2x} \frac{dt}{\sin t + 1}$

25. If two lines y=x and y= x - 4 are tangent to a circle at (2, 2) and (4, 0), respectively, then what is the equation of the circle? (1)

A. $(x-2)^2 + y^2 = 4$

B. $(x-4)^2 + (y-2)^2 = 4$

C. $(x-3)^2 + (y-1)^2 = 2$

D. $(x-1)^2 + (y+1)^2 = 10$

26. Which one of the following is the set of all critical numbers of $f(x) = \frac{1}{3}x^3 - |4x - 1|$? (1)

A. {1/4, 2}

B. {-2, 1/4, 2}

C. {-2, 2}

D. {1/4}

27. Which one of the following is equal to $\int_0^{\frac{\pi}{2}} \frac{x-\sin x}{\sec x}\, dx$? (1)

A. $\frac{\pi-3}{2}$

B. $\frac{\pi-1}{2}$

C. $\frac{3-\pi}{2}$

D. $\frac{\pi+3}{2}$

28. Which one of the following is true? (1)

A. a polynomial can have infinitely many vertical asymptotes

B. the graph of a rational function can never cross its horizontal asymptote.

C. the graph of $f(x) = \frac{3x-1}{x-1}$ has no horizontal asymptote.

D. the graph of $f(x) = \frac{x^3-x}{x^2-x}$ has no vertical asymptote.

29. The simplified form of the derivative of $f(x) = \frac{1+\sin x}{\cos x}$ is (1)

A. $\sec x + \tan x$

B. $\frac{1+\sin x}{\cos^2 x}$

C. $\frac{1}{1+\tan x}$

D. $\frac{\cos x}{\sin^2 x}$

30. If Qi, Di and Pi are respectively the ith ?quartile, decile and percentile of a data arranged in an increasing order, then which one of the following is necessarily true? (1)

A. $Q_2 = \frac{Q_1+Q_3}{2}$

B. $D_3 > P_{25}$

C. $P_{25} > Q_1$

D. Q_2 = mean of the data

31. If [the function is missing from the capture], what is the slope of the tangent line to the graph of f at x=2? (1)

A. -4

B. 2

C. 18

D. 17

32. If $f(x) = e^{2x} \sin x$, then $f''(x)$ is equal to (1)

A. $3e^{2x} \sin x - 4e^{2x} \cos x$

B. $4e^{2x} \sin x + 2e^{2x} \cos x$

C. $e^{2x}(3 \sin x + 4 \cos x)$

D. $e^{2x}(4 \sin x - 3 \sin x)$

33. If the equation $(x-2)^2 - (y-2)^2 = 1$ represents a hyperbola, which one of the following represents equation of an asymptote to the hyperbola? (1)

A. y = 4 - x

B. x + y = 1

C. x = 2 - y

D. x + 2y = 3

34. What is the sum of the series $\sum_{n=1}^{\infty} (-1)^n 3^{-2n}$? (1)

A. -1/8

B. -0.13

C. -0.1

D. 1/8

35. Which of the equations below is represented by the following parabola? (1)

[Figure: A hand-drawn sketch of x-y coordinate axes, with an arrowhead on the upward y-axis and on the rightward x-axis. An upward-opening parabola is drawn. Its vertex sits on the x-axis at the point marked by a short tick on the positive x-axis labelled '1' below the axis, so the curve just touches the x-axis there. The left branch rises steeply to the upper left and crosses the y-axis at the point labelled '2' (the label '2' is written to the left of the y-axis at that height). The right branch rises to the upper right. No other values or gridlines are shown.]

A. $y = x^2 + 2$

B. $y = (2x-1)^2$

C. $y = 2(x-1)^2$

D. $y = (2x+1)^2$

36. A company produced 25,000 bulbs and randomly tested 2% of the product. Among the tested bulbs, if 40 have defect of type D_1, 60 have defect of type D_2 and 25 have both types of defects, what is the probability that a bulb produced by the company had none of the defects? (1)

A. 0.95
B. 0.80
C. 0.85
D. 0.20

37. A semi-elliptical arc over a tunnel for a road through a mountain has a major axis of length 80 meters and a height of 30 meters at the center. What is the equation of the semi-elliptical arc over the tunnel, if the center is considered as the origin? (1)

A. $\frac{x^2}{6400} + \frac{y^2}{900} = 1$
B. $\frac{x^2}{1600} + \frac{y^2}{900} = 1$
C. $\frac{x^2}{900} + \frac{y^2}{6400} = 1$
D. $\frac{x^2}{8100} + \frac{y^2}{6400} = 1$

38. Suppose AX=b, where A is a 3 x 3 matrix, $b = (b_1, b_2, b_3)^T$ and $X = (x, y, z)^T$. Which one of the following is necessarily true about this system of linear equations? (1)

A. The system has a solution only when $\det(A) \neq 0$.
B. the Cramer?s rule is suitable to solve the system if two rows of A are identical
C. if $\det(A) \neq 0$ and the second column of A is a multiple of b, then x = 0
D. if b = 0, then $x = (0, 0, 0)^T$ is the only solution of the system.

39. If $y = \sin(3x^2)$, then the simplified form of $\frac{d^2y}{dx^2}$ is : (1)

A. $-6\sin(3x^2)$
B. $\cos(6x) - 6\sin(3x^2)$
C. $6\cos(3x^2) + 36x2\sin(3x^2)$
D. $x^2\cos(3x^2) + 6\sin(3x^2)$

40. If S is a set with 10 elements and $A \subseteq S$, what is the probability that A has 3 or more elements? (1)

A. 7/10
B. 8/11
C. 121/128
D. 7/128

41. If $a_n = \left(\frac{n+3}{n+1}\right)^n$, then the limit of the sequence $\{a_n\}_{n=1}^{\infty}$ is equal to : (1)

A. 1
B. 1/2 e
C. e^2
D. $+\infty$

42. Which one of the following is equal to $\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^{-3x}$? (1)

A. e^6
B. e^{-3}
C. $e^{-3/2}$
D. e^{-6}

43. If a box with square base open top is made from $1{,}200\text{cm}^2$ material, what is the largest volume of the box in cm^3 ? (1)

A. 4000
B. 8000
C. 15,000
D. 3000

44. Which one of the following is true about the graph of $f(x) = \frac{x^2-1}{x-x^2}$? (1)

A. x = 0 and x = 1 are its vertical asymptotes.
B. y = 1 is tis horizontal asymptote
C. y = x - 1 is its oblique asymptote
D. it is almost the same as the horizontal line y = -1 as $x \to \pm\infty$

45. Which one of the following is a valid logical argument? (1)

A. $p \Rightarrow q,\ q \vdash p$

B. $p \Leftrightarrow q,\ p \Rightarrow q \vdash q$

C. $\neg p \wedge q,\ \Box q \Rightarrow r \vdash r$

D. $\neg p,\ p \vee q,\ r \Rightarrow q \vdash r$

46. Suppose that equal squares are cut from each of the four corners of a square cardboard whose sides are 72 centimetres long. [see the figure below.] The resulting flaps are then folded up to form a box without a top. How long should be each of the four squares that has to be cut off to maximize the volume of the box? (1)

[Figure: A hand-drawn square (the cardboard sheet) with its outline in heavy dashes. Two dashed lines run horizontally and two run vertically, each parallel to and set in from an edge by the same short distance, so that they mark off a small square at each of the four corners and leave a larger rectangle in the middle. The side of a corner square is labelled 'x' in five places: once along the top edge (just right of the top-left corner square), once at the left (in the left-middle corner strip), once at the right (in the right-middle strip), and twice along the bottom (at the bottom-left and bottom-right corner squares). No numeric dimensions are shown on the drawing.]

A. 6cm

B. 12cm

C. 15cm

D. 24cm

47. What is the area of the region between the graph of $f(x) = -x^2 + 4x - 3$ and the x axis from x = 0 to x = 3? (1)

A. -2/3

B. 2/3

C. 4/3

D. 8/3

48. Which one of the following is equal to $\int \frac{x+\ln(x+1)}{(x+1)^2}\, dx$? (1)

A. $\ln(x+1) + \frac{x}{x+1} + c$

B. $(x+1)^2 \frac{1}{x+1} + c$

C. $(x+1)^2 - \frac{x}{x+1} + c$

D. $\frac{x\ln(x+1)}{x+1} + c$

49. If a point (2, 5) is reflected under a line to the point(-3, 1), what is the line of reflection? (1)

A. 2x + 3y= 7

B. x + 3y= 7

C. 8y + 10x= 19

D. 2x + 3y + 5= 0

50. Let the equation $x^2 + 2x + y^2 = 8$ represents a circle. Then which one of the following lines cut the circle at exactly two points? (1)

A. 4x + 3y + 19 = 0

B. 3x + 4y + 14 = 0

C. 2y= 5x + 43

D. 2x =y - 50

51. If A= (-2, 3), B= (3, 1) and C is any other point on the plane, then which one of the following is the coordinate form of $\overrightarrow{AC} - \overrightarrow{BC}$? (1)

A. (-5, 2)

B. (5, -2)

C. (1, 4)

D. (-1, -4)

52. What is the equation of a line that passes through the point (-1, -2) and parallel to the vector (1, -1)? (1)

A. 2x - y= 1

B. x + y - 1 = 0

C. x - 2y = 3

D. y - 2x + 1 = 0

53. Suppose P and Q are point in space such that the midpoint of PQ is on the negative z-axis and the distance between P and Q is 6. If P=(2, -1, 0), then what is the coordinate of Q? (1)

A. (-2, 1, 4)

B. (2, -1, 6)

C. (2, -1, -6)

D. (-2, 1, -4)

54. Which one of the following is equivalent to $\neg\left[(\forall x)(p(x) \Rightarrow Q(x))\right]$? (1)

A. $(\forall x)(\neg p(x) \Rightarrow \neg q(x))$

B. $(\exists x)(\neg p(x) \Rightarrow \neg q(x))$

C. $(\exists x)(\neg p(x) \wedge q(x))$

D. $(\exists x)(p(x) \wedge \neg q(x))$

55. An observer on level ground is at a distance $10\sqrt{3}$ m form a building. The angles of elevation to the bottom of the windows on the second and third floors are 30^0 and 60^0, respectively. What is the distance h between the bottoms of the windows? [you may use the values: $\sin 30^0 = \cos 60^0 = 1/2$ and $\sin 60^0 = \cos 30^0 = \sqrt{3}/2$] (1)

A. 15m

B. 20m

C. $15\sqrt{3}$ m

D. 32m

56. What is the image of the line given by (x, y) = (-1, 0) + t(3, 6), $t \in \mathbb{R}$, under the transition that takes (1, 0) to (0, 1) following by the reflection about the line y = 2x? (1)

A. y = 2x + 3

B. y= 2x - 3

C. y = 2x + 6

D. y = 2x - 5

57. If $\theta = 2\arctan(1/2)$, then which one of the following is equal to $\sec(\theta)$? (1)

A. 35/3

B. 4/5

C. 5/3

D. 5/4

58. If a translation T takes the circle $x^2 + y^2 - 2x + 6y + 3 = 0$ into the circle whose equation is $(x+2)^2 + (y-4)^2 = 7$, then what is the image of the origin under T? (1)

A. (-3, 7)

B. (1, 2)

C. (1, -3)

D. (-2, 4)

59. If L is the line passing through (0, 2) & parallel to $\vec{v} = \vec{i} + 3\vec{j}$. Which one of the following is true about L and the circle $(x-2)^2 + (y-1)^2 = 5$? (1)

A. L is tangent to the circle at (0, 2)

B. L is tangent to the circle at some point P, where $P \neq (0, 2)$.

C. L intersects the circle at two distinct points.

D. the distance between L and the centre of the circle is greater than $\neq 5$

60. Suppose $\vec{A} = 3\vec{i} - 4\vec{j}$ and $\vec{B}$ is a vector in the xy- plane such that the angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{3}$, if $\vec{u}$ is the unit vector in the direction of $\vec{B}$, then $\vec{A} \cdot (\vec{A} - 2\vec{u})$ is equal to: (1)

A. 20

B. 5

C. 15

D. 30

END OF THE EXAMINATION

Check your answers at temari.et/exam-prep/papers/grade-12-mathematics-2006.
More exam papers are at temari.et/exam-prep/papers.