

Mathematics, 2007 EC, entrance exam

Grade 12 · Mathematics · 2007 E.C.

Time allowed: 120 minutes

Questions: 62

Total marks: 62

Name _____ Grade _____ ID No. _____ Date _____

1. Which one of the following is the equation of an ellipse with centre at (1, 4), vertices at (10, 4) and (1, 2) is : (1)

A. $4(x - 1)^2 + 81(y - 4)^2 = 324$

B. $(x - 1)^2 + 9(y - 4)^2 = 4$

C. $9(x - 1)^2 + 4(y - 4)^2 = 1$

D. $2(x - 1)^2 + 9(y - 4)^2 = 4$

2. Which of the following is convergent sequence? (1)

A. $1, 1/2, 1, 1/3, 1, 1/4, \dots$

B. $\{(-1)^n\}_{n=1}^{\infty}$

C. $\{10^{109} - \frac{1}{100^n}\}_{n=1}^{\infty}$

D. $\{n + \sin \frac{1}{n}\}_{n=1}^{\infty}$

3. Let $f(x) = 2x(x^2 + 1)^4$, then which of the following is an anti- derivative of f(x)? (1)

A. $\frac{2x}{5}(x^2 + 1)^5 + c$

B. $\frac{2}{5}(x^2 + 1)^5 + c$

C. $\frac{x}{5}(x^2 + 1)^5 + 1$

D. $\frac{1}{5}(x^2 + 1)^5 - 1$

4. Which one of the following is an arithmetic sequence? (1)

A. 3, 5, 7, 9, 11, ...

B. 3, 6, 12, 24, 28, ...

C. -3, 6, -9, 12, -15, ...

D. 1, 3, 6, 10, 15, 21, ...

5. Suppose that p represents the statement "He missed the tournament", q represents the statement "He got the gold medal." And r represents the statement "he took a trip abroad." then which of the following symbolic expressions represents the statement: "If He took a trip abroad and he does not miss the tournament, then he get the gold medal."? (1)

A. $(r \Rightarrow q) \neg p$

B. $r \wedge (p \Rightarrow q)$

C. $(r \wedge \neg p \Rightarrow q)$

D. $\neg(r \vee p) \vee q$

6. Let $f(x) = \begin{cases} a \frac{\sin x}{x}, & x < 0 \\ e^{2x} - 2, & x \geq 0 \end{cases}$ if f is continuous at x=0, then what is the value of a? (1)

A. 1/2

B. 2

C. -1/2

D. -2

7. The Ozone level (in ppb- parts per billion) on a sunny day in metropolitan area is given by formula $p(t) = 80 + 12t \diamond t^2$ where t is time in hours and t = 0 corresponds to 9A.M. what is the rate of increase of Ozone level after 3-hrs(i.e. at 12 A.M)? (1)

A. 6 ppb

B. 12 ppb

C. 107 ppb

D. 113 ppb

8. Which of the following functions is one to one correspondence? (1)

A. $f: R \rightarrow R, f(x) = \tan x$, where R' is domain of f

B. $g: R \rightarrow R, g(x) = 2^x$

C. $h: [-\infty, \infty) \rightarrow [0, \infty): h(x) = x^2$

D. $r: [0, \infty) \rightarrow [0, \infty): r(x) = x + 5$

9. Different codes, each of which consisting of five characters, are to be generated in such a way that the first two characters are any of the English capital letters (A to Z) and the remaining three are any of the digits (0, 1, 2,...,9). How many distinct codes can be generated ? (1)
- A. 468,000
B. 260
C. 676,000
D. $26! \times 10!$
-
10. A certain meeting hall has 20 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. How many seats are there on the last (20th) row of the hall? (1)
- A. 46
B. 58
C. 76
D. 5240
-
11. Two perpendicular lines l_1 and l_2 are intersecting at $(-1, 2)$. If the angle of inclination of l_1 is 45° , then what is the equation of l_2 ? (1)
- A. $y = -x + 3$
B. $y = x + 3$
C. $y = -x + 1$
D. $y = x + 1$
-
12. What is the area of the region between the graph of $y = \sin x$ and x-axis where $0 \leq x \leq 2\pi$? (1)
- A. 4
B. 4π
C. 2
D. 2π
-
13. The following is the set of data representing the average mark of students: 91, 89, 93, 91, 87, 94, 92, 85, 91, 90, 96, 93, 89. Then which one of the following statements is true about the data? (1)
- A. the median is 90.5
B. the upper quartile is 92
C. the range of marks is 11
D. the mean is 91.5
-
14. If $f(x) = \ln(\sqrt{x^2 + 1})$, which of the following is equal to $f'(x)$? (1)
- A. $\frac{x}{\sqrt{x^2 + 1}}$
B. $\frac{x}{x^2 + 1}$
C. $\frac{2x}{\sqrt{x^2 + 1}}$
D. $\frac{2x}{x^2 + 1}$
-
15. If $z = \frac{3+i}{i-2}$ is a given complex number, then what is the conjugate $\bar{z}$, of z ? (1)
- A. $\bar{z} = \frac{-3+i}{2+i}$
B. $\bar{z} = \frac{i-3}{2-i}$
C. $\bar{z} = -6 - 2i$
D. $\bar{z} = -1 - i$
-
16. What is the principal argument of $(5 + 5i)^{11}$? (1)
- A. $\pi/2$
B. $2\pi/3$
C. $\pi/4$
D. $3\pi/4$
-
17. Which one of the following is equal to $\lim_{x \rightarrow 9} \frac{x-9}{3-\sqrt{x}}$? (1)
- A. 6
B. -3
C. -6
D. ∞
-
18. For arbitrary propositions p and q , which one of the following is a valid equivalence? (1)
- A. $\neg(p \Rightarrow q) \equiv (q \Rightarrow p)$
B. $[\neg(p \Rightarrow) \wedge p] \equiv (p \wedge \neg q)$
C. $[p \vee \neg q] \equiv p \Rightarrow q$
D. $[(p \vee q) \Rightarrow q] \equiv [p \Rightarrow \neg q]$

19. If f and g are continuous on $\mathbb{R}$ and $a, b \in \mathbb{R}$, which one of the following is necessarily true? (1)

- A. if $\int_a^b f(x)dx = \int_a^b g(x)dx$, then $f(x) = g(x)$ for all $x \in [a, b]$
- B. if $f'(x) = g'(x)$ for all $x \in [a, b]$, then $\int_a^b f(x)dx = \int_a^b g(x)dx$
- C. if $f(x) \geq 3$ for all $x \in [-2, 2]$, then $\int_{-2}^2 f(x)dx \geq 12$
- D. $\int_a^b f(x)dx = \int_b^a f(x)dx$

20. Consider the following system of equations:
$$\begin{cases} ax + by = 2 \\ x + 3y + 2z = 0 \\ 2x + y + z = 0 \end{cases}$$
 if the determinant of the coefficient matrix is 2, then what is the solution set of the system? (1)

- A. $\{(1, 3, -5)\}$
- B. $\{(1/a, 1/b, 0)\}$
- C. $\{(-2, -6, 10)\}$
- D. $\emptyset$

21. For what values of a and b is the function $f(x) = \begin{cases} 1 - 3x^2, & \text{for } x \leq 1 \\ ax + b, & \text{for } x > 1 \end{cases}$ is differentiable at $x=1$? (1)

- A. $a = 6, b = 0$
- B. $a = -3, b = 1$
- C. $a = 0, b = -2$
- D. $a = -6, b = 4$

22. Which one of the following is equal to $\int \frac{\ln x + x^2 e^x}{x} dx$? (1)

- A. $\frac{1}{2} \ln^2 x + e^x(x^2 - 1) + c$
- B. $\frac{1}{2} \ln^2 x + e^x(x - 1) + c$
- C. $\frac{1}{x^2} \ln x + e^x(x - 1) + c$
- D. $\frac{-1}{x^2} \ln x + e^x(x^2 - 1) + c$

23. What is the focus of the parabola $y^2 + 4y + 8x = 4$? (1)

- A. $(1, -2)$
- B. $(-1, -2)$
- C. $(3, -2)$
- D. $(-3, -2)$

24. A city has two daily newspapers, X and Y. the following information was obtained from the survey of 100 residents of the city: 35 people subscribe to X, 60 people subscribe to Y and 20 subscribe to both newspapers. Then how many of the people in the survey do not subscribe to either of the newspapers? (1)

- A. 5
- B. 25
- C. 40
- D. 55

25. Which one of the following is necessarily true? (1)

- A. if $f'(x) = 0$ for all x in the interval I , then $f(x) = 0$ for all x in I .
- B. if $f(x) = x^2 \sin x + 5$, then there is $c \in (0, \pi)$ such that $f'(c) = 0$
- C. $f(x) = e^x$ is increasing on $(-\infty, \infty)$
- D. $f(c) = 0$, then f attains its maximum or minimum value at $x = c$

26. The derivative of the function $f(x) = \int_{-x}^x \frac{dt}{1+t}$ is : (1)

- A. $\frac{2}{1-x^2}$
- B. $\ln|1+x|$
- C. $\frac{1}{1+x}$
- D. $\ln\left|\frac{1+x}{1-x}\right|$

27. If $p(x) = 3x^2$ and $q(x) = x^2 + x$, then what is the solution set of $\frac{p(x)}{3q(x)} - \frac{1}{x} = \frac{1}{q(x)}$? (1)
- A. $\{-1, 2\}$ B. $\{2\}$
C. $\{-3, 2\}$ D. $\{-3\}$
-
28. What are the values of u and v that satisfy the equation: $\frac{u + 3i}{4 - 2i} = \frac{2 + vi}{20}$? (1)
- A. $u=2, v=3$ B. $u=-6, v=10$
C. $u=2, v=16$ D. $u=-4, v=6$
-
29. What is the sum of all multiples of 3 between 20 and 200? (1)
- A. 7,227 B. 6,570
C. 6,150 D. 5,166
-
30. The invers of the function defined by $g(x) = \frac{2x}{x+3}$ is equal to: (1)
- A. $g^{-1}(x) = -\frac{2x}{x-3}$ B. $g^{-1}(x) = -\frac{3x}{x-2}$
C. $g^{-1}(x) = -\frac{x+3}{2x}$ D. $g^{-1}(x) = -\frac{x+2}{3x}$
-
31. If $A = \begin{pmatrix} 0 & x & 0 \\ 1 & -1 & 1 \\ 0 & y & -1 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & 0 \\ 2 & 0 & -1 \end{pmatrix}$, then what are the value of x and y ? (1)
- A. $x = 3, y = -2$ B. $x = 2/3, y = 1/3$
C. $x = -3, y = 2$ D. $x = 1/3, y = 2/3$
-
32. If $f(2) = -e$, $f'(2) = 4$, $g(1) = -5$, $g'(1) = 1$ and $F(x) = f(2x+2)(g(x-x^2))$, then what is the value of $F'(0)$? (1)
- A. 19 B. 0
C. -20 D. -40
-
33. A ladder 6m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate (speed) of 0.5 m/s, how fast the angle between the top of ladder and the wall changing when the angle is $\pi/4$ rad? (1)
- A. $\sqrt{2}/2$ rad/sec B. $\sqrt{2}/12$ rad/sec
C. $\sqrt{2}/6$ rad/sec D. $\sqrt{2}/3$ rad/sec
-
34. The value(s) of x where the graph of the function $y = \frac{x^2 - 1}{x^3}$ crosses its horizontal asymptote is (are): (1)
- A. $x = -2$ B. $x = -1$ and $x = 1$
C. $x = 0$ D. $x = -\sqrt{2}$ and $x = +\sqrt{2}$
-
35. If $A = \begin{pmatrix} 3 & -2 & 8 \\ 0 & 6 & 7 \\ 0 & 4 & 5 \end{pmatrix}$, then $\det(A^T A)$ is equal to : (1)
- A. 12 B. 36
C. 30 D. 15

36. Suppose that the first 3 letters (A, B, and C) and number digits are to be used to form car plates in a small town. (1)
How many different plates can be formed in total that contains 1, 2 or 3 letters and then followed by 3 digits?
- A. 3,000
B. 27,000
C. 39,000
D. 100,000
-
37. A ball is thrown vertically from ground up to a height of 16m. Each time it drops h meters, it rebounds $0.80h$ m. (1)
noting that the ball travels every height of h twice, what is the total vertical distance travelled by the ball before it comes to rest?
- A. 40m
B. 80m
C. 160m
D. 320m
-
38. Suppose that a function f has a property that $f(x+y) = f(x)f(y)$ for all x and y and that $f(0) = 2$, $f'(0) = 1$. Then which (1)
one of the following represents the formula for the derivative $f'(x)$?
- A. $f(x) = 2f'(x) + 1$
B. $f'(x) = f(x) + 2f'(x)$
C. $f'(x) = f(x) + 2$
D. no such function
-
39. Which one of the following is true about a conic section represented by the equation $\frac{x^2}{k} + \frac{y^2}{k-9} = 1$? (1)
- A. it is a circle whose center is at origin for some $k \in \mathbb{R}$.
B. it is an ellipse whose major axis is vertical when $k > 9$.
C. it is a hyperbola whose foci are at $(-3, 0)$ & $(3, 0)$ when $0 < k < 9$.
D. it is a hyperbola whose foci are at $(-3k, 0)$ & $(3k, 0)$ when $0 < k < 9$.
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40. Which of the following is NOT a tautology? (1)
- A. $[p \vee (q \Rightarrow r)] \Leftrightarrow [\neg p \Rightarrow (q \Rightarrow r)]$
B. $p \vee (q \Rightarrow \neg q)$
C. $p \Rightarrow [(p \Rightarrow q) \vee q]$
D. $[p \Leftrightarrow (q \wedge \neg r)] \Leftrightarrow [\neg p \Leftrightarrow (\neg q \vee r)]$
-
41. What is the solution of the system $\begin{cases} x + y - z = 1 \\ x + 2y - 3z = 1 \\ 2x + 3y - 4z = 2 \end{cases}$? (1)
- A. $\{(0, 2, 1)\}$
B. $\{(1 - k, 2k, k) \mid k \in \mathbb{R}\}$
C. $\{(2k + 1, -k, k) \mid k \in \mathbb{R}\}$
D. $\emptyset$
-
42. Let $\begin{cases} 3 - 2^{2x}, & \text{if } x < 0.5 \\ \frac{2^x - 5}{x + 1}, & \text{if } x \geq 0.5 \end{cases}$, if c is a zero of f , that is, $f(c) = 0$, then which one of the following intervals must (1)
contain c ?
- A. $(-\infty, 0]$
B. $[0, 1]$
C. $[1, 2]$
D. $[2, 3]$

43. Which of the following functions could most likely be drawn as in the Figure 1 below? (1)

[Figure: A hand-drawn sketch of a rational function on x- and y-axes. The horizontal axis carries an arrowhead on the right and is labelled x; the vertical axis carries an arrowhead at the top and is labelled y. A dashed vertical line to the LEFT of the y-axis is labelled "x=-2" and acts as a vertical asymptote. A dashed horizontal line ABOVE the x-axis is labelled "y=1" and acts as a horizontal asymptote. A single tick value "2" is marked on the x-axis to the right of the origin. The curve has two branches. The left branch (for x to the left of the asymptote) enters from the far left just BELOW the line y=1, runs almost flat, then bends downward, crosses the x-axis a little to the left of x=-2, and plunges steeply toward minus infinity as it approaches the dashed line x=-2 from the left. The right branch descends from high above (plus infinity) just to the right of the dashed line x=-2, drops steeply, crosses the y-axis above y=1, then flattens and decreases slowly toward the dashed line y=1 from above as x increases; this branch is drawn with a small visible break (a gap, i.e. a hole) at about x=2, over the marked tick "2". No other points, values or shading are shown.]

- A. $\frac{x+3}{x+2}$ B. $\frac{x^2-2x}{x^2-4}$
C. $\frac{-x^2-x+6}{x^2-4}$ D. $\frac{x^2+x-6}{x^2-4}$

44. A measurement is grouped in to five class intervals with the following frequency distribution. Class Intervals: 5-15 | 15-25 | 25-35 | 35-45 | 45-55 Frequency: 22 | 40 | 68 | 50 | 20 What are the first quartile Q_1 & the 75th percentile P_{75} of the measurement? (1)

[Figure: A bordered two-row table with six columns. The first column holds the row headings. Row 1 heading "Class Intervals", then the five cells: 5-15, 15-25, 25-35, 35-45, 45-55. Row 2 heading "Frequency", then the five cells: 22, 40, 68, 50, 20.]

- A. $Q_1 = 20, P_{75} = 40$ B. $Q_1 = 22, P_{75} = 40$
C. $Q_1 = 20, P_{75} = 39$ D. $Q_1 = 22, P_{75} = 39$

45. The number of shoes s that a factory can produce per day is a function of the number of hours t it operates: $s(t) = 40t$ for $0 \leq t \leq 12$ the daily cost c, in Birr, to manufacture s shoes is given by the function $c(s) = 0.1s^2 + 90s + 800$ If the factory operates for 10 hours, what is the cost it incurs in producing as much shoes it can with in this time? (1)

- A. Birr 400 B. Birr 1,600
C. Birr 52,800 D. Birr 124,600

46. Three persons P1, P2 and P3 are firing at a target independently and have a probability 0.7, 0.5 and 0.4, respectively, of hitting the target. What is the probability that at least one of them hits the target? (1)

- A. 0.95 B. 0.85
C. 0.91 D. 0.99

47. Which one of the following is equal to $\lim_{x \rightarrow \infty} \left(\frac{3x}{3x+2} \right)^{-3x}$? (1)

- A. e^2 B. e^{-3}
C. e^{-2} D. e^3

48. A company manufactures x computer sets per month. The monthly marginal profit (in Birr) is given by: $p(x) = 165 - 0.1x$, for $0 \leq x \leq 400$ The company is currently manufacturing 10 sets of computer per month, but is planning to increase production. What is the total change in the monthly profit if the monthly production increased to 60 sets? (1)

- A. Birr 50 B. Birr 1,865
C. Birr 8,075 D. Birr 18,635

49. If angle θ is an acute angle of a right triangle, what is the length of the side adjacent to θ , given the hypotenuse has 6 units length and $\sec \theta = 10/3$? (1)
- A. 1.8 units
B. 2 units
C. 18 units
D. 20 units
-
50. If $A = (1, -2)$, $B = (-3, 2)$ and $\vec{v}$ is a position vector such that $2\vec{v} + \overrightarrow{AB} = \vec{o}$, then $\vec{v}$ is equal to (1)
- A. (2, 0)
B. (-1, 0)
C. (-2, 2)
D. (2, -2)
-
51. If each of the compound propositions $P \vee Q$, $P \Rightarrow R$ and $\neg R$ is true, then which one of the following is True? (1)
- A. P
B. Q
C. $Q \Rightarrow P$
D. $P \wedge \neg R$
-
52. If $\vec{A} = 4\vec{i} - 3\vec{j}$ and $\vec{u}$ is a unit vector such that $|\vec{A} + \vec{u}|^2 = 27$, then the cosine of the angle between $\vec{A}$ and $\vec{u}$ is equal to (1)
- A. 0.1
B. 0.2
C. 0.3
D. 0.4
-
53. What is the possible value of x that solves the equation: $\sin^{-1} x + \cos^{-1} \left(\frac{3}{5} \right) = \pi$ (1)
- A. $\pi/3$
B. $3/5$
C. $5\pi/2$
D. $4/5$
-
54. If $P = (3, \alpha - 1, \alpha + 2)$ and $Q = (2\alpha + 1, 3, 3\alpha)$ are points in space, what should be the value(s) of α so that the distance between the two points is 6? (1)
- A. $\alpha = 1$, or $\alpha = 5$
B. $\alpha = \frac{4 + 2\sqrt{7}}{3}$, or $\alpha = \frac{4 - 2\sqrt{7}}{3}$
C. $\alpha = -1$, or $\alpha = 3$
D. $\alpha = -3$, or $\alpha = 2$
-
55. If $(-1, 2, 2)$ and $(1, 0, -2)$ are end points of a diameter of a sphere, then which one of the following is true about the sphere? (1)
- A. $(0, 1, 0)$ is a point on the sphere
B. the equation of the sphere is $x^2 + (y - 1)^2 + z^2 = 6$
C. the equation of the sphere is $x^2 + (y - 2)^2 + z^2 = 6$
D. the radius of the sphere is 6
-
56. Suppose that the equation $x^2 + y^2 + z^2 + 2x + 8z = 6(y + 1)$, represents a sphere. Where is the point $(1, -1, 4)$ located relative to the sphere? (1)
- A. inside the sphere
B. on the sphere
C. at the center of the sphere
D. outside the sphere
-
57. Two ships, one with angle of depression 60° due east and the other with 30° due west are observed from a plane 1,000 metres above sea. If the two ships are on the same line, what is the distance between the two ships? [you may use the values: $\sin 30^\circ = \cos 60^\circ = 1/2$ and $\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$] (1)
- A. $\frac{1}{\sqrt{3}} 4,000 \text{ m}$
B. 2,000m
C. $\sqrt{3} 500 \text{ m}$
D. $\frac{1}{\sqrt{3}} 600 \text{ m}$

58. What is the image of the ellipse $(x - 1)^2 + 4y^2 = 1$ under the translation that takes $(1, 1)$ to $(0, 2)$ followed, by the reflection through the x-axis?
- A. $x^2 + 4(y - 1)^2 = 1$ B. $4x^2 + (y - 1)^2 = 1$
C. $x^2 + 4(y + 1)^2 = 1$ D. $4x^2 + (y + 1)^2 = 1$
-
59. What is the amplitude and period, respectively, of the graph of $f(x) = -6 \sin x \cos x$?
- A. $3, \pi$ B. $6, \pi$
C. $3, \pi/2$ D. $6, 2\pi$
-
60. What is the work done (in joule) when a force of 50 Newton is used to pull a cart 20 meters along path if the force is at an angle of 60° ? [Remember that: Work=(Force) x (distance travelled)]
- A. 360 B. 500
C. 760 D. $1500\sqrt{2}$
-
61. Suppose L is the line through the center of the sphere $x^2 + y^2 + (z - 2)^2 = 9$ and the line intersects the sphere at $(1, 2, 4)$. What is the cosine of the angle between L and positive z-axis?
- A. $2/3$ B. $1/3$
C. $3/5$ D. $4/5$
-
62. Consider the formula for a natural number $n \in \mathbb{N}$: $2 + 4 + 8 + \dots + 2^n = 2^{n+1} + 1$ To prove this formula a person has used the following argument. "Assume the formula is true for $n=k$, for some $k \in \mathbb{N}$. Then the person has shown that the formula is also true for $n=k + 1$. And then, the person has concluded that, by the Principle of Mathematical induction, the formula is true for all natural numbers $n \in \mathbb{N}$." which one of the following statements is true about the above argument?
- A. the formula holds true though it does not work for $n=1$
B. since the left-hand-side is an even number and the right hand side an odd number, the Principle of Mathematical Induction is False.
C. this is one example where the Principle of Mathematical Induction fails to work.
D. the above formula does not work for all natural numbers $n \in \mathbb{N}$

— END OF EXAMINATION —