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Mathematics, 2005 EC, entrance exam

Grade 12 · Mathematics · 2005 E.C.

Time allowed: 120 minutes | **Questions:** 60 | **Total marks:** 60

Name _____ Grade _____ ID No. _____ Date _____

1. If $\{a_n\}$ is a sequence such that $a_1 = 2$, and $a_{n+1} = a_n + 4$ for all $n \geq 1$, then $\sum_{n=1}^{35} a_n$ is equal to : (1)

A. 2460 **B.** 2458

C. 2450 **D.** 2442

2. If $f(x) = x^2 + 2\ln x$, then $\lim_{h\to 0} \dfrac{f(2+h) - f(2)}{h}$? (1)

A. 5 **B.** 4

C. 2 **D.** 0

3. If $f(x) = e^{2x} + x - 3\cos x$, then what is $f''(x)$? (1)

A. $e^{2x} + 1 - 3\sin(x)$ **B.** $e^{2x} + 1 + 3\sin(x)$

C. $4e^{2x} - 3\cos(x)$ **D.** $4e^{2x} + 3\cos(x)$

4. Which one of the following intervals does $f(x) = x^4 + 4x$ increase? (1)

A. $(-\infty, -1]$ **B.** $(-\infty, 0]$

C. $[-1, \infty)$ **D.** $(-\infty, \infty)$

5. Which one of the following is the simplest form of $|3 + 4i| - \dfrac{25i}{3 + 4i}$ (1)

A. 5 ? 5i **B.** 5 + 5i

C. 1 + 3i **D.** 1 [illegible] 3i

6. If $Z = \cos(\pi/10) + i\sin(\pi/10)$, then what is the value of Z^5? (1)

A. $\pi/2 + \pi/2\, i$ **B.** $1/2 + 1/2\, i$

C. i **D.** $1 + i$

7. What is the value of $|x| + 2x$ if $x \leq 0$? (1)

A. $-3x$ **B.** $3x$

C. [illegible]x **D.** x

8. Suppose $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ if x is a 2 x 2 matrix such that $AX - A^T = 2A$, then what is the value of x? (1)

A. $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ **B.** $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$

C. $\begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$ **D.** $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

9. What is the 50^{th} term of the sequence 3, 10, 17, 24, 31,[illegible]? (1)

A. 310 **B.** 346

C. 510 **D.** 531

10. For real members x and y, which one of the following statements is true? (1)

A. $(\forall x)(\exists y)(x^2 + y + 1 = 0)$
B. $(\exists x)(\forall y)(x^2 + y + 1 = 0)$
C. $(\exists y)(\forall x)(x^2 + y + 1 = 0)$
D. $(\forall y)(\exists x)(x^2 + y + 1 = 0)$

11. Which one of the following functions has NO vertical asymptote? (1)

A. $f(x) = \ln(x + 1)$
B. $f(x) = \dfrac{x^2 + 1}{x^3 + 8}$
C. $f(x) = \dfrac{x^2 - 9}{x - 3}$
D. $f(x) = \dfrac{x - 1}{x^2 - x}$

12. Let p, q and r be propositions such that $p \Rightarrow (r \vee \neg q)$ is false. Then, which one of the following propositions is true? (1)

A. $p \Rightarrow r$
B. $\neg r \Rightarrow q$
C. $\neg q \Rightarrow q$
D. $q \Leftrightarrow r$

13. If $f(x) = \dfrac{1}{e^x + 1}$, then which one of the following is equal to $f^{-1}(x)$ for $0 < x < 1$? (1)

A. $\ln(1 - x) - \ln(x)$
B. $e^{-x} + 1$
C. $\ln(1/x + 1)$
D. $1/e^{-x} + 1$

14. If $x^2 - 6x + y^2 + k = 0$ is equation of a circle with radius 2, then what is the value of k? (1)

A. 13
B. 5
C. 4
D. -4

15. If a line with angle of inclination of $3\pi/4$ passes through (0, 1), which one of the following is the equation of the line? (1)

A. $y = -x + 1$
B. $y = x + 1$
C. $y = -x - 1$
D. $y = x - 1$

16. If $g(x) = xf(x) - \sqrt{f(x)}$ and $f(2) = f'(2) = 4$, then which of the following is equal to $g'(2)$? (1)

A. 11
B. 8
C. 2
D. 0

17. What is the sum of the series $\sum_{n=1}^{\infty} \dfrac{2^{2n+1}}{5^{n-1}}$ (1)

A. 40
B. 20
C. 10
D. 8

18. What is the value of $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^{\frac{-x}{2}}$? (1)

A. $\dfrac{1}{\sqrt{e}}$
B. $\sqrt{e}$
C. e^{-2}
D. ∞

19. Among students who took a quiz, 15 students scored 6, 20 students scored 7, 10 students scored 8 and 5 students scored 10. What is the average score of the students? (1)

A. 7.8
B. 7.5
C. 7.2
D. 7.0

20. A parabola with focus (3, -1) has directrix $y = 3$. Which one of the following is the equation of the parabola? (1)

A. $(x-3)^2 = -4(y+1)$
B. $(x-3)^2 = -8(y-1)$
C. $(x-3)^2 = 4(y+1)$
D. $(x-3)^2 = 8(y-1)$

21. How many four digit even numbers can be formed from 1, 2, 3, 4 and 5 if the numbers start with 3? (1)

A. 40
B. 50
C. 100
D. 120

22. A satellite moves along a hyperbolic curve whose horizontal transverse axis is 24 km and an asymptote $y = 5/12\ x + 2$. Then, what is the eccentricity of the hyperbola? (1)

A. 5/13
B. 12/13
C. 13/12
D. 15/3

23. What is an anti-derivative of $f(x) = \dfrac{2}{4x^2 + 4x + 1}$? (1)

A. $\dfrac{1}{2x+1}$
B. $\dfrac{-2}{2x+1}$
C. $-\dfrac{1}{2x+1}$
D. $\ln(4x^2\ \ 4x + 1)$

24. At which valeu(s) of x does $f(x) = 0.25x^4 - 2x^2$ have a local maximum? (1)

A. $x = 4$
B. $x = 0$
C. $x = -2$ and $x = 2$
D. $x = 0$ and $x = 2$

25. In the set of complex numbers, which one of the following is the solution set of $Z^3 - iZ^2 + 2Z = 0$? (1)

A. $\{0\}$
B. $\{0, -i\}$
C. $\{0, -i, 2i\}$
D. $\{0, i, -2i\}$

26. The population of certain country is currently 80 million with growth rate of 2% per year. Given: $(0.02)^9 = 5.12 \times 10^{-16}$, $(1.02)^9 = 1.19$, $(0.02)^{10} = 1.024 \times 10^{-17}$, $(1.02)^{10} = 1.22$, which one of the following is the best approximation of the population after 10 years? (1)
[Figure: A boxed 'Given:' panel sits between the first and second sentences of the stem, holding four values laid out in two rows of two columns. Left column: $(0.02)^9 = 5.12 \times 10^{-16}$ on the first row and $(0.02)^{10} = 1.024 \times 10^{-17}$ on the second row, each followed by a comma. Right column: $(1.02)^9 = 1.19$ on the first row and $(1.02)^{10} = 1.22$ on the second row. All four values are reproduced verbatim in the stem.]

A. 81.9 million
B. 86.8 million
C. 95.2 million
D. 97.6 million

27. The volume V of a melting ice cube after t seconds is $V = 2000 - 40t + 0.2t^2$ (in cm^3). How fast is the volume changing when t=40 seconds? (1)

A. $24\ \text{cm}^3/\text{sec}$
B. $15\,\text{cm}^3/\text{sec}$
C. $-15\ \text{cm}^3/\text{sec}$
D. $-24\,\text{cm}^3/\text{sec}$

28. Which one of the following is equal to $\lim_{n\to\infty} \dfrac{1 - n - 3n^2}{6n^2 + 1}$? (1)

A. 1/6
B. -1/2
C. -1/6
D. $-\infty$

29. What is the solution set of $\frac{2}{x} - \frac{x-2}{x^2-2x} = 1 - \frac{2x-2}{3x-2}$? (1)

A. $\{1, -2\}$ **B.** $\{1, 2\}$

C. $\{-1\}$ **D.** $\{1\}$

30. Let $f(x) = \begin{cases} 3^x + k, & \text{if } x \leq 0 \\ 3\dfrac{\sin(2x)}{x}, & \text{if } x > 0 \end{cases}$ if f is continuous at x=0, then what is the value of k? (1)

A. 6 **B.** 5

C. 2 **D.** 0

31. Which one of the following is equal to $\int (1+x)3^x \, dx$? (1)

A. $(1+x)3^x - 3x + c$ **B.** $(1+x)3^x + (\log_3 e)3^x + c$

C. $(1+x)3^x \log_3 e - (\log_3 e)^2 3^x + c$ **D.** $(1+x)3^x \log_3 e - 3^x(\log_3 e) + c$

32. A committee consisting of 3 students is to be selected from 10 candidates among which 4 are girls. What is the probability that at least one girl is selected? (1)

A. 5/6 **B.** 2/3

C. 1/3 **D.** 1/6

33. A group of six students take their seats at random in a round table for a discussion. What is the probability that two specific students do NOT sit together? (1)

A. 3/5 **B.** 2/3

C. 2/5 **D.** 1/3

34. The mark that students scored in an examination is grouped in class intervals as shown in the following table. What is the median of the mark? (1)

[Figure: A two-column frequency table sits between the two sentences of the stem. Header row: 'Class Interval (Mark)' and 'Number of Students'. Rows, in order: 55 - 64 with 8 students; 65 - 74 with 12 students; 75 - 84 with 20 students; 85 - 94 with 6 students; 95 - 100 with 4 students.]

A. 25.0 **B.** 75.5

C. 77.0 **D.** 79.5

35. A box seen below is to have a square base, an open top and volume of 32 cubic units. If x is the length of each side of its base and y is its height, how many units should x and y be in order to make the box with the smallest amount of material? (1)

[Figure: A scanned line drawing of a rectangular box in three-quarter perspective with a square base and no lid. A short horizontal leader line at the top right carries a downward-pointing arrow into the open mouth of the box and is labelled 'Open - top'. The vertical left edge of the box is labelled y and the bottom front edge is labelled x. The front and right faces are shaded/stippled; the top rim is drawn as an open rectangle.]

A. $x = 4,\ y = 2$ **B.** $x = 2,\ y = 8$

C. $x = \sqrt{8},\ y = 4$ **D.** $x = \sqrt{2},\ y = 16$

36. Which one of the following is equal to $\int \frac{1}{x^2 + x} \, dx$? (1)

A. $\ln|x^2 + x| + c$ **B.** $2\ln|x+1| + \ln|x| + c$

C. $\ln|x| - \ln|x+1| + c$ **D.** $\ln|x| + \ln|x+1| + c$

37. Consider the following argument: "If he does not love her, she will not marry him. He loves her. Therefore, she will marry him." If p? he loves her and q ? she will marry him, which one of the following is the correct representation of the argument and its validty? (1)

A. $\neg p \Rightarrow \neg q,\ p \vdash q$: valid argument

B. $\neg p \Rightarrow \neg q,\ p \vdash q$; invalid argument

C. $p \Rightarrow q,\ p \vdash q$; valid argument

D. $p \Rightarrow q,\ p \vdash q$; valid argument

38. What is the value of $\int_0^{\frac{\pi}{2}} 2x \cos x \, dx$? (1)

A. $\pi - 2$

B. $\pi/2 + 1$

C. $\pi + 2$

D. $\pi/2 - 1$

39. A box contains 5 white, 6 red and 4 black balls of all identical size. If 3 balls are randomly taken out of the box after the other, what is the probability that the first ball is white and both the second and third balls are red? (1)

A. 2/15

B. 3/15

C. 4/75

D. 5/91

40. Which one of the following is the equation of the line tangent to the graph of $f(x) = 1/(x+1) + \cos x$ at $(0, f(0))$? (1)

A. $x + y = 1$

B. $x - y = -2$

C. $x + y = 2$

D. $x + 4y = 2$

41. Which one of the following is equal to $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x^2 - 1}$? (1)

A. ∞

B. 0

C. -1/4

D. 1/4

42. Which one of the following is equal to $\lim_{x \to 1} \dfrac{1 - x}{1 - \frac{1}{x^2}}$? (1)

A. 1

B. 0

C. -1/2

D. Doesn't exist

43. Which one the following is equal to $\int \dfrac{\ln(xe^x)}{x}\, dx$ (1)

A. $\ln|x| + \dfrac{1}{2}e^x + c$

B. $\dfrac{1}{2}(\ln x)^2 + x + c$

C. $\ln|x| + e^{2x} + c$

D. $-\dfrac{1}{x^2} + \ln x)^2 + c$

44. Suppose f is differentiable on $(-\infty, \infty)$ and the graph of its derivative is as shown below. Which one of the following is true about f ? (1)

[Figure: A scanned sketch of a cubic-looking curve on x-y axes, labelled $y = f'(x)$ beside its upper-left branch. The x-axis carries marked dots and labels at -3, -2, -1, 1, 2 and 3; the y-axis is drawn as an upward arrow and the x-axis as a rightward arrow. The curve comes down steeply from the upper left, crosses the x-axis at $x = -2$, continues down to a local minimum at about $x = -1$ (a dashed vertical segment runs from the tick at -1 down to that minimum), then rises, crossing the x-axis at the origin $x = 0$, reaches a local maximum above $x = 1$ (a dashed vertical segment joins the tick at 1 to that peak), then falls, crossing the x-axis at $x = 2$ and continuing steeply down to the lower right past $x = 3$. No y-axis values are marked.]

A. f is decreasing on $(-\infty, -1) \cup [1, \infty)$

B. f has a local minimum at $x = -2$

C. f is concave down on $[0, \infty)$

D. f is concave up on $(-1, 1)$

45. Which one of the following is true about the derivative of $f(x) = x|x|$? (1)

A. f is not differentiable at $x = 0$

B. $f'(x) = 2|x|$, for every $x \in (-\infty, \infty)$

C. $f'(x) = 2x$, for every $x \in (-\infty, \infty)$

D. $f'(x) = |x| + x$, for every x ?$(-\infty, \infty)$

46. What is the area of the region between the graphs of $y = x^2$ and $y = -x + 2$, where $0 \leq x \leq 2$? (1)

A. 3

B. 2

C. 3/2

D. 2/3

47. For what value of b does the parabola $p(x) = ax^2 + x + b$ pass through the points (-1, 5) and (2, -1)? (1)

A. 9

B. 3

C. -3

D. -15

48. Which one of the following is equal to $\sec(\pi/2 - x)\sin^3 x + \cos 2x$? (1)

A. $2\cos x$

B. $2\sin x$

C. $\cos 2x$

D. $\sin 2x$

49. Suppose P(1, 2, 1) and Q(1, 0, 2) are points in space and $\vec{A} = \overrightarrow{PQ}$. If $\vec{B}$ is parallel to $\overrightarrow{PQ}$ and $\vec{A} \cdot \vec{B} = -10$,, then which one of the following is true? (1)

A. $\vec{A}$ and $\vec{B}$ has the same direction

B. $\|\vec{B}\| = 10\|\vec{A}\|$

C. $\|\vec{B}\| = \frac{1}{10}\|\vec{A}\|$

D. $\|\vec{B}\| = 2\|\vec{A}\|$

50. What is the solution of $\cos^2 x + 0.5\sin 2x = 1$ in the interval $[0, 2\pi)$? (1)

A. $\{0,\ \pi/4,\ \pi,\ 5\pi/4\}$

B. $\{0,\ \pi/4,\ 3\pi/4,\ \pi\}$

C. $\{0,\ \pi\}$

D. $\{0,\ \pi/4,\ \pi\}$

51. If $\vec{u} = (-3, x)$ and $\vec{v} = (x, y - 2)$ are vectors, what is the value of y so that $\vec{u} + \vec{v} = 3\vec{u} - \frac{1}{2}\vec{v}$? (1)

A. 2/3

B. -10/3

C. -4

D. -22/3

52. Which one of the following points is closer to the sphere $x^2 + y^2 + z^2 - 2x + 6z + 9 = 0$? (1)

A. (1, 0, 0)

B. (0, 0, 0)

C. (0, -1, 0)

D. (0, 0, -1)

53. What is the cot(arc sinx) if $0 < x < 1$? (1)

A. $\frac{x}{\sqrt{1 - x^2}}$

B. $\frac{1}{x}\sqrt{1 - x^2}$

C. $\sqrt{1 - x^2}$

D. $\frac{1}{\sqrt{1 - x^2}}$

54. Suppose $\vec{A} = 2\vec{j} - \vec{k}$ and $\vec{B} = 5\vec{i} + 15\vec{k}$, where $\vec{i}, \vec{j}$ and $\vec{k}$ are the standard unit vectors in the directions of positive x, y and z axis, respectively. Which one of the following is the unit vector in the direction of $\vec{A} + \frac{1}{5}\vec{B}$? (1)

A. $\frac{3}{5}\vec{i} + \frac{4}{5}\vec{k}$

B. $\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$

C. $\frac{4}{5}j - \frac{3}{5}k$

D. $\frac{2}{3}i - \frac{1}{3}j + \frac{2}{3}k$

55. A line given by a vector equation r(t)=(0, 3) + t(1, 1) is tangent to a circle at point (0,3). If the radius of the circle is ?2 which one of the following is the center of the circle? (1)

A. (1,4)
B. (1,-4)
C. (-1,2)
D. (1,2)

56. Suppose the following statements are the premises of an argument. ﷺHe was lazy or he did not like the classroom. If he was lazy, he could not pass the exam. He passed the exam.ﷺ Which one of the following can be a conclusion that makes the argument valid? (1)

A. He did like the classroom.
B. He did not like the classroom.
C. If he was not lazy, he did like the classroom.
D. He was not lazy and he did like the classroom.

57. What is the image of the ellipse whose equation is $2(x+2)^2+(y-1)^2=2$ under a translation that takes (2, 1) to (4, 0) followed by a rotation of 90^0? (1)

A. $x^2+2y^2=2$
B. $2x^2+y^2=2$
C. $2(x \text{ ﷺ } 4)^2+y^2=2$
D. $(x \text{ ﷺ } 4)^2+2y^2=2$

58. Let $\vec{a}=2\vec{i}+(x-1)\vec{j}+\vec{k}$ and $\vec{c}=\vec{i}-\vec{j}+y\vec{k}$ be vectors. If $\vec{a}\cdot\vec{c}=0$ and $\|\vec{a}\|=3$ which one of the following is a possible value of y? (1)

A. -4
B. -1
C. 3
D. 4

59. If $\vec{A}$ is perpendicular to $\vec{B}$, what is the cosine of the angle between $\vec{A}$ and $\vec{A}-\vec{B}$? (1)

A. $\dfrac{\|\vec{A}-\vec{B}\|}{\|\vec{A}\|}$
B. $\dfrac{\|\vec{A}\|}{\|\vec{A}-\vec{B}\|}$
C. $\dfrac{\|\vec{A}-\vec{B}\|}{\|\vec{B}\|}$
D. $\dfrac{\|\vec{B}\|}{\|\vec{A}-\vec{B}\|}$

60. Which one of the following is necessarily true? (1)

A. If $\|\vec{A}\|=\|\vec{B}\|$, then $\vec{A}=\vec{B}$
B. $\|k\vec{A}\|=K\|\vec{A}\|$, for any real number k
C. if $\vec{A}$ is parallel to $\vec{B}$, then $\vec{A}\cdot\vec{B}=0$
D. if $\vec{u}$ is a unit vector in the direction of $\vec{A}$, then $\vec{A}\cdot\vec{u}=\|\vec{A}\|$

— END OF EXAMINATION —