

Mathematics, 2009 EC, entrance exam

Grade 12 · Mathematics · 2009 E.C.

Time allowed: 120 minutes

Questions: 61

Total marks: 61

Name _____ Grade _____ ID No. _____ Date _____

1. If $A = (a_{ij})_{3 \times 3}$ is a square matrix with $A^{-1} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 0 & 4 & 5 \end{bmatrix}$, then what is the cofactor of a_{23} ? (1)
- A. $-3/14$ B. $-2/7$
C. $2/7$ D. $-3/7$
-
2. A team of 10 researchers consist 4 biologists and 6 chemists. If 3 persons are chosen randomly from the team, what is the probability that at least one is a biologist? (1)
- A. $2/3$ B. $2/5$
C. $5/6$ D. $7/10$
-
3. The probability that an electronic device produced by a company does not function properly is equal to 0.1. If 2 devices are bought, then what is the probability that at least one device function properly? (1)
- A. 0.81 B. 0.09
C. 0.18 D. 0.99
-
4. In how many ways can a committee of 3 members be formed from 7 candidates? (1)
- A. 7 B. 21
C. 28 D. 35
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5. The following is a frequency distribution table of a grouped data with variable X. What is the mean ($\bar{x}$) and the variance (s^2) of the data respectively? (1)
- [Figure: A two-row bordered frequency table with five columns. Top row: first cell labelled 'X', then the class intervals 3-7, 8-12, 13-17, 18-22. Bottom row: first cell labelled 'Frequency', then the values 4, 6, 8, 2 under those intervals respectively.]
- A. $\bar{x} = 12, s^2 = 21$ B. $\bar{x} = 12, s^2 = 25$
C. $\bar{x} = 13, s^2 = 9$ D. $\bar{x} = 13, s^2 = 16$
-
6. The expenditure of 100 families is given below. The mode of the data is 23.5, what are the values of f_1 and f_2 (1)
- [Figure: A two-row bordered table with six columns. Top row: first cell labelled 'Expenditure', then the classes 0-9, 10-19, 20-29, 30-39, 40-49. Bottom row: first cell labelled 'No. of Families', then the values 14, 23, f_1 , 21, f_2 under those classes respectively.]
- A. $f_1 = 27, f_2 = 15$ B. $f_1 = 15, f_2 = 27$
C. $f_1 = 25, f_2 = 17$ D. $f_1 = 17, f_2 = 25$
-
7. The first group of 10 children has a mean weight of 15.6kg, the second group of another 10 children has a mean weight of 16kg, and the third group of children has a mean weight of 20kg. If the mean weight of all the children is 17kg, what is the total number of children in all of the three group? (1)
- A. 28 B. 29
C. 30 D. 32

8. Which of the following is a valid argument? (1)
- A. $\neg p \Rightarrow \neg q, q \vdash \neg p$ B. $p \Rightarrow \neg q, p, r \Rightarrow q \vdash \neg r$
 C. $\neg p \vee q, r \Rightarrow p, r \vdash \neg q$ D. $\neg p, p \vee q, r \Rightarrow q \vdash \neg r$
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9. Consider the following open propositions: $P(x)$ = x is a prime number, $C(x)$ = x is a composite number, and $E(x)$ = x is an even number, which one of the following has a truth value of True in the set of positive integers? (1)
- A. $(\forall x)[P(x) \Rightarrow \neg E(x)]$ B. $(\forall x)[P(x) \Rightarrow \neg E(x)]$
 C. $(\forall x)[P(x) \Rightarrow \neg E(x)]$ D. $(\forall x)[P(x) \Rightarrow \neg E(x)]$
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10. Which of the following functions is a one to one correspondence? (1)
- A. $f : [0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = |x|$ B. $f : \mathbb{R} \rightarrow [0, \infty)$ defined by $f(x) = x^2$
 C. $f : \mathbb{R} \rightarrow [0, \infty)$ defined by $f(x) = 3^x$ D. $f : (0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = \log_2 x$
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11. If $f(x) = \sqrt{x^3}$ and $(f \circ g)(x) = \sqrt[4]{x}$, then what is the value of $g(8)$? (1)
- A. $\sqrt[3]{2}$ B. 2
 C. $\sqrt{2}$ D. $2\sqrt{2}$
-
12. Which one of the following is the inverse of $f(x) = 8x^3 + 2$? (1)
- A. $f^{-1}(x) = \frac{1}{8x^3 + 2}$ B. $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-2}$
 C. $f^{-1}(x) = 8x^{-3} + 2$ D. $f^{-1}(x) = \frac{1}{8}\sqrt[3]{x-2}$
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13. What is the solution set of $\frac{1}{1 + \frac{1}{x}} - \frac{1}{1 - \frac{1}{x}} = \frac{x + \frac{1}{x}}{x - \frac{1}{x}}$? (1)
- A. $\{ \}$ B. $\{-1\}$
 C. $\{1\}$ D. $\{-1, 1\}$
-
14. Which one of the following is true about the graph of $\frac{x^3}{x^3(x-1)}$? (1)
- A. the vertical asymptotes of the graph are $x = 0$ and $x = 1$
 B. a horizontal asymptote of the graph is $y = 1$
 C. the graph intersects its horizontal asymptote at a point $(-1, 0)$
 D. the graph intersects the vertical line $x = 1$ at a point $(1, 2)$
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15. What is the equation of a line that passes through point (a, a) in xy -plane if it is parallel to a line that passes through points (a, b) and (b, a) where $a \neq b$? (1)
- A. $y = x$ B. $y = -x$
 C. $y = -x + 2a$ D. $y = 2x - a$
-
16. What are the values of the center (C) and radius (r) of a circle $x^2 + y^2 - 4x + 6y = 5$? (1)
- A. $C = (-2, 3), r = 3\sqrt{2}$ B. $C = (2, -3), r = 3\sqrt{2}$
 C. $C = (2, -3), r = 2\sqrt{3}$ D. $C = (-2, 3), r = 2\sqrt{3}$
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17. What is the radius of the largest possible circle that can be inscribed in the ellipse given by $5(x-1)^2 + 3y^2 = 15$? (1)
- A. $\sqrt{3}$ B. $\sqrt{5}$
 C. 3 D. 5

18. Suppose the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is reciprocal to that of the eccentricity of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through the focus of the ellipse, then what is the equation of the hyperbola? (1)
- A. $x^2 - 2y^2 = 2$ B. $x^2 - 3y^2 = 3$
C. $x^2/3 - y^2/2 = 1$ D. $x^2/2 - y^2/3 = 1$
19. Let p and q stands for the statements "Nejat is intelligent" and "Almaze is hardworking". Respectively, which of the following represent the statement "Almaze is hardworking if Nejat is intelligent"? (1)
- A. $\neg p \wedge q$ B. $\neg p \vee q$
C. $p \wedge q$ D. $\neg q \vee p$
20. Let f be differentiable function with $f(1) = -1$ and $f'(1) = 1$. If $g(x) = [f(2x + 1) + 2]^2$, then what is the value of $g'(0)$? (1)
- A. 4 B. 2
C. -2 D. -4
21. If $f(x) = \ln(x^2 + 2)$, then what is the value of $f''(1)$? (1)
- A. $3/2$ B. $5/9$
C. $2/3$ D. $2/9$
22. If $x^2 + xy = 10$, then what is the value of $\frac{dy}{dx}$ when $x = 2$? (1)
- A. $-7/2$ B. $2/7$
C. $3/2$ D. $7/2$
23. What is the equation of the tangent line to the graph of $f(x) = 3x^2 + 4x - 5$ at $(1, 2)$? (1)
- A. $10x - y - 8 = 0$ B. $-10x + y - 8 = 0$
C. $-10x - y - 8 = 0$ D. $10x + y - 8 = 0$
24. If $f(x) = \pi^2 + 1$, then what is the value of $f'(x)$? (1)
- A. $2\pi + 1$ B. 2π
C. 2 D. 0
25. Suppose f is continuous on $[2, 6]$ and the only solutions of the equation $f(x) = 7$ are $x = 2$ and $x = 5$. If $f(3) = 9$, then one of the following Can Not be the value of $f(4)$? (1)
- A. 5 B. 7.5
C. 8 D. 9
26. What is the value of k so that $f(x) = \begin{cases} \frac{\tan 2x}{x}, & \text{if } x > 0 \\ k - e^x, & \text{if } x \leq 0 \end{cases}$ if continuous at $x = 0$? (1)
- A. 2 B. 3
C. 1 D. 0

27. If f is continuous at $x = 0$ and $g(x) = \sqrt{x} \left(2f(x) + \frac{3}{\sqrt{x}} \right)$ for all $x > 0$, then what is the value of $\lim_{x \rightarrow 0^+} g(x)$? (1)
- A. 0
B. 2
C. 3
D. 5

28. What is the value of $\lim_{x \rightarrow 0} \frac{\sin x \cos 2x}{x^2 + 3x}$? (1)
- A. $1/3$
B. $2/3$
C. 1
D. 2

29. If $a \neq 0$, then what is the value of $\lim_{x \rightarrow 0} \frac{\sin x \cos 2x}{x^2 + 3x}$? (1)
- A. $1/a^2$
B. $1/2a^2$
C. $1/6a^2$
D. 0

30. What are the values of λ and μ so that the system $\begin{cases} x + y + z = 6 \\ x + 2y + 3z = 10 \\ x + 2y + \lambda z = \mu \end{cases}$ has infinitely many solutions? (1)
- A. $\lambda \neq 3$ and $\mu \in \mathbb{R}$
B. $\lambda = 3$ and $\mu \neq 10$
C. $\lambda = 3$ and $\mu = 10$
D. $\lambda \in \mathbb{R}$ and $\mu = 10$

31. When $\begin{vmatrix} a & b & c \\ a & -a & a \\ a & a & -a \end{vmatrix} = a^3$, and $a \neq 0$, what is the solution of $\begin{bmatrix} a & b & c \\ a & -a & a \\ a & a & -a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$? (1)
- A. $x = 0, y = 2a, z = 2a$
B. $x = 1/a, y = -2a, z = 2a$
C. $x = 1/a, y = 2/a, z = 2/a$
D. $x = 0, y = 2/a, z = 2/a$

32. If $z_1 = \frac{2-i}{1+i}$ and $z_2 = \frac{1+i}{1-i}$, then what is the value of $z_1 + 2z_2$? (1)
- A. $1+i$
B. $\frac{i+1}{2}$
C. $\frac{i+2}{2}$
D. $1-i$

33. Which one of the following is the conjugate of $|3+4i| - \frac{25i}{3+4i}$? (1)
- A. $5+3i$
B. $1+3i$
C. $3-5i$
D. $1-3i$

34. If $z = \sqrt{2} \cos \left(\frac{\pi}{12} \right) + i \sqrt{2} \sin \left(\frac{\pi}{12} \right)$, then what is the value of z^3 ? (1)
- A. $2+2i$
B. $\sqrt{2} + \sqrt{2}i$
C. $2\sqrt{2} + 2\sqrt{2}i$
D. $3\sqrt{2} + 3\sqrt{2}i$

35. Which of the following relations holds for the sequence: -10, -3, 4, 11, ...? (1)
- A. $a_n = a_{n-1} - 8$
B. $a_n = a_{n-1} + 7$
C. $a_n = a_{n-1} - 7$
D. $a_n = a_{n-1} + 8$

36. What is the sum of $\sum_{n=1}^{30} (-1)^n \left(\frac{1}{n} + \frac{1}{n+1} \right)$? (1)

- A. $-29/30$
C. $30/31$

- B. $29/30$
D. $-30/31$

37. Which of the following is the sum of the series $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$? (1)

- A. -5
C. 3

- B. -3
D. 5

38. Which one of the following is a convergent sequence? (1)

- A. $\left\{ \left(\frac{5}{3} \right)^n \right\}$
C. $\left\{ \frac{n^2}{n+1} \right\}$

- B. $\left\{ \frac{2n}{n+1} \right\}$
D. $\left\{ \frac{(-1)^n}{3} \right\}$

39. What is the value of the area of the region enclosed by the graph of $f(x) = e^x$ and $g(x) = x$ between the lines $x = -1$ and $x = 1$? (1)

A. $\frac{e^2 - 1}{e}$

B. $e^2 - \frac{1}{e}$

C. $e^2 - \frac{1}{e} + 2$

D. $e - \frac{1}{e} + 2$

40. If $f(0) = -1$, $f(1) = 2$ and $f'(x)$ is continuous on $[0, 1]$, then which of the following is equal to $\int_0^1 f'(x) \sqrt{2 + f(x)} dx$? (1)

- A. $16/3$
C. $8/3$

- B. $14/3$
D. $4/3$

41. Which of the following is equal to the volume of a solid generated when the region bounded by the graph of $y = 2\sqrt{x+1}$ and x-axis, when $0 \leq x \leq 2$, rotates about the x-axis? (1)

- A. 8π
C. 16π

- B. $8\pi^2$
D. $16\pi^2$

42. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t = 0$. If the particle is at position $x = 2$ at a time $t = 0$, what is the position of the particle at $t = 1$? (1)

- A. 6
C. 11

- B. 9
D. 12

43. Which of the following is equal to $\int x (e^x + \sin(x^2)) dx$? (1)

- A. $e^x(x+1) + \frac{1}{2} \sin(x^2) + c$
C. $e^x(x-1) - \frac{1}{2} \cos(x^2) + c$

- B. $e^x(x-1) - \frac{1}{2} \sin(x^2) + c$
D. $e^x(x-1) + \frac{1}{2} \cos(x^2) + c$

44. What is the value of $\int_0^{\ln \sqrt{3}} \frac{e^x}{e^{-x} + e^x} dx$? (1)

- A. $\frac{1}{2} \ln 2$
C. $\ln 2$

- B. $\ln 4$
D. 1

45. Suppose $f : (-\infty, \infty) \rightarrow \mathbb{R}$ is differentiable and the graph of its derivative $y = f'(x)$, is as shown in the figure below. Which one of the following is true about f ? (1)
- A. f is increasing on $(1, \infty)$ B. f is concave upward on $(0, \infty)$
 C. f has no relative maximum value D. f has a relative minimum value at $x = 2$
-
46. If $2 \leq f'(x) \leq 4$ for all values of x , then the value of $f(8) - f(2)$ is between which of the following numbers? (1)
- A. 14 and 24 B. 12 and 24
 C. 12 and 18 D. 8 and 10
-
47. A tin can of volume $54\pi \text{ cm}^3$ is to be made in the form of a right circular cylinder that has both flat top and flat bottom. What is the base radius of the tin if it is to be made of the least amount of metal? (1)
- A. 2cm B. 3 cm
 C. 4 cm D. 6 cm
-
48. Air is being pumped into a spherical balloon so that its volume increase at a rate of $50 \text{ cm}^3/\text{s}$. How fast is the radius of the balloon increasing when the diameter is 5 cm? (1)
- A. $\frac{1}{50\pi} \text{ cm/s}$ B. $\frac{1}{25\pi} \text{ cm/s}$
 C. $\frac{5}{\pi} \text{ cm/s}$ D. $\frac{2}{\pi} \text{ cm/s}$
-
49. Which one of the following is a valid assertion that can be proved by principle of mathematical induction? (1)
- A. the sum of any two positive rational numbers is positive.
 B. $r^2 \geq 1$, for every real number $r \geq 1$
 C. $n^2 \geq 4n$, for every integer $n \geq 4$
 D. $2^2 \leq 2^2$, for every integer $n \leq 100$
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50. Consider the assertion: "The sum of positive irrational numbers is positive irrational number". Which one of the following is correct about the assertion? (1)
- A. taking irrational numbers such as $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}$, and so on, if we add any two of them, the sum is irrational. Therefore, the assertion is true.
 B. the sum of $1 + \sqrt{2}$ and $1 - \sqrt{2}$ is 2, which is rational. This is a counter example that disproves the assertion.
 C. the sum of $\sqrt{7}$ and $\sqrt{2}$ is a counter example that shows the assertion is false.
 D. the assertion can be disproved by taking the sum of $1 + \sqrt{2}$ and $2 - \sqrt{2}$ as a counter example.
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51. Suppose "If $x \in A$, then $y \in B$ " is a True statement. Then, which one of the following is necessarily true? (1)
- A. $y \in B$ B. if $x \notin A$, then $y \notin B$
 C. if $y \in B$, then $x \in A$ D. if $y \notin B$, then $x \notin A$
-
52. Let $\vec{A}$ and $\vec{B}$ be vectors in space such that $\vec{A} \cdot \vec{B} = -2$ and $\vec{B} = 6\vec{i} - 7\vec{j} + \sqrt{15}\vec{k}$. If θ is the angle between $\vec{A}$ and $\vec{B}$, then what is the value of $|\vec{A}|$? (1)
- A. $\frac{1}{5} \cos \theta$ B. $\frac{1}{5 \cos \theta}$
 C. $-\frac{1}{5} \cos \theta$ D. $\frac{-1}{5 \cos \theta}$

53. If $P(2, \sqrt{5}, 1)$ and $Q(3, 0, 9)$ are points on a sphere whose center is on z-axis, then which one of the following point is outside of the sphere? (1)
- A. $(-4, 3, 5)$ B. $(2, -2, 1)$
C. $(3, 1, 1)$ D. $(0, 0, 0)$
-
54. If $A(x, 0, 2)$, $B(3, 0, 2)$ and $C(2, \sqrt{3}, 2)$ are vertices of an equilateral triangle in space, then what is the value of x? (1)
- A. 5 B. 3
C. 2 D. 1
-
55. A patrol boat on a sea sailed from its station 7km to the North; and changed its course and sailed $4\sqrt{2}$ km in the direction of 45° South-East. What is the shortest (straight) distance the boat should travel in order to return to its station? (1)
- A. 5km B. 7km
C. $5\sqrt{2}$ km D. $5+\sqrt{2}$ km
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56. Let $\overrightarrow{PQ}$ be a vector with initial point $P = (1, 5)$ and terminal point $Q = (4, 0)$. If $\vec{v} = x\vec{i} + 2\vec{j}$ is parallel to $\overrightarrow{PQ}$, then what is the value of x? (1)
- A. $-6/5$ B. $-2/5$
C. -3 D. 3
-
57. For two non-zero vectors $\vec{a}$ and $\vec{b}$ if $\|\vec{a} + \vec{b}\| = \|\vec{a}\|$ which of the following is true? (1)
- A. $2\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{b}$ B. $2\vec{a} + \vec{b}$ and $\vec{b}$ are perpendicular
C. $2\vec{a} + \vec{b}$ and $2\vec{a} + \vec{b}$ are not parallel D. $\vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{b} = 0$
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58. A line given by the vector equation $(x, y) = (-t, 6 + 2t)$, $t \in \mathbb{R}$, is tangent to a circle at a point $(1, 4)$. What is the radius of the circle if its center is on the y-axis? (1)
- A. $\sqrt{5}$ B. $\frac{1}{2}\sqrt{5}$
C. $2\sqrt{5}$ D. $\sqrt{10}$
-
59. If $\cot \theta = \sqrt{8}$ and θ is first quadrant angle, then what is the value of $\csc \theta$? (1)
- A. $1/3$ B. 3
C. $\sqrt{8}/3$ D. $1/\sqrt{8}$
-
60. If $\theta = \arctan(2)$, then what is the value of $\sin(2\theta)$? (1)
- A. $2/5$ B. $4/5$
C. $4/\sqrt{5}$ D. $2/\sqrt{5}$
-
61. Which one of the following is true? (1)
- A. the amplitude of $f(x) = \sin 3x$ is 3 B. the period of $f(x) = 2 \sin 4x$ is π
C. the period of $f(x) = 3 \cos(0.5x - \pi/3)$ is 4π D. the amplitude of $f(x) = -5 \cos(3x + 2) - 2$ is 7

— END OF EXAMINATION —