

Since $|z|^2 = x^2 + y^2$ and $z - \bar{z} = 2iy$, we have $(z - \bar{z})^2 = -4y^2$. Substituting these into the equation gives $(x^2 + y^2) + \frac{1}{2}(-4y^2) = x^2 - y^2 = 1$.

11. The following is the frequency distribution of a grouped data. What is the mean and standard deviation of the distribution, respectively? (1)

[Figure: A two-column table. Column headings: 'Class Intervals' and 'Frequency (f)'. Rows: class interval 3-7 with frequency 2; class interval 8-12 with frequency 2; class interval 13-17 with frequency 10; class interval 18-22 with frequency 6.]

- A. ✓ $15, 2\sqrt{5}$ B. $15, \sqrt{7.5}$
C. $12.5, 5\sqrt{2}$ D. $12.5, \sqrt{15}$

Answer: A

The midpoints are 5, 10, 15, and 20. The mean is $\frac{2(5)+2(10)+10(15)+6(20)}{20} = \frac{300}{20} = 15$, and the standard deviation is $\sqrt{\frac{2(5-15)^2+2(10-15)^2+10(15-15)^2+6(20-15)^2}{20}} = \sqrt{\frac{400}{20}} = \sqrt{20} = 2\sqrt{5}$.

12. If $(p \vee q) \Rightarrow (\neg r \wedge r)$ is true, then which one of the following is necessarily true? (1)

- A. $((p \vee r) \Rightarrow q)$ B. $\neg p \wedge r$
C. $\neg p \Rightarrow r$ D. ✓ $\neg p \vee r$

Answer: D

Since $\neg r \wedge r$ is always false, the conditional statement being true implies $p \vee q$ must be false, which means both p and q are false. Because p is false, $\neg p$ is true, making $\neg p \vee r$ necessarily true regardless of the truth value of r .

13. What is the actual value of the sum $\sum_{n=1}^{\infty} \left(\frac{2^n+5^n}{10^n}\right)$ (1)

- A. 0.325 B. 1
C. ✓ $5/4$ D. $37/9$

Answer: C

Splitting the terms gives two geometric series: $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$. Evaluating these yields $\frac{1/5}{1-1/5} + \frac{1/2}{1-1/2} = \frac{1}{4} + 1 = \frac{5}{4}$.

14. If $f(x) = \frac{x+1}{x-1}$ and $f(a) = 5$ then $f(2a)$ is equal to: (1)

- A. ✓ 2 B. 4
C. 6 D. 8

Answer: A

From $f(a) = \frac{a+1}{a-1} = 5$, we find $a+1 = 5a-5$, so $a = \frac{3}{2}$. Thus, $2a = 3$ and $f(2a) = f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$.

15. If $f(x) = \sqrt[3]{1+e^{-x}}$, which of the following is equal to $f^{-1}(x)$? (1)

- A. ✓ $\ln\left(\frac{1}{x^3-1}\right)$ B. $\frac{1}{\ln(x^3-1)}$
C. $\ln(1-x^3)$ D. $(1+e^{-x})^3$

Answer: A

Setting $y = \sqrt[3]{1+e^{-x}}$ gives $y^3 = 1+e^{-x}$, so $e^{-x} = y^3 - 1$. Taking the logarithm gives $-x = \ln(y^3 - 1)$, which implies $x = -\ln(y^3 - 1) = \ln\left(\frac{1}{y^3-1}\right)$.

16. What is the equation of the directrix for the parabola whose equation is $y^2 + 8x + 6y + 25 = 0$? (1)

- A. $y = 3$ B. $x = 2$
C. ✓ $x = 0$ D. $x = 4$

Answer: C

Completing the square gives $(y+3)^2 = -8(x+2)$, which is of the form $(y-k)^2 = -4p(x-h)$ with vertex $(-2, -3)$ and $p = 2$. Since the parabola opens to the left, the directrix is the vertical line $x = h + p = -2 + 2 = 0$.

Multiplying the first equation by 3 and adding the second equation yields $3(x - y + z) + (-x + 5y - 4z) = 2x + 2y - z$. For the system to be consistent, the constant term must satisfy $k = 3(1) + 1(1) = 4$.

Since $1/\sec x = \cos x$, the integral becomes $\int_0^{\pi/2} (x \cos x - \sin x \cos x) dx$. Integrating by parts and using standard integrals yields $[x \sin x + \cos x - (1/2)\sin^2 x]_0^{\pi/2} = (\pi/2 - 1) - 1/2 = (\pi - 3)/2$.

28. Which one of the following is true?

(1)

- A. a polynomial can have infinitely many vertical asymptotes
- B. the graph of a rational function can never cross its horizontal asymptote.
- C. the graph of $f(x) = \frac{3x-1}{x-1}$ has no horizontal asymptote.
- D. ✓ the graph of $f(x) = \frac{x^3-x}{x^2-x}$ has no vertical asymptote.

Answer: D

Factoring the rational function $f(x)$ gives $[x(x-1)(x+1)] / [x(x-1)] = x+1$ for $x \neq 0, 1$. The discontinuities at $x = 0$ and $x = 1$ are removable (holes), so the graph has no vertical asymptotes.

29. The simplified form of the derivative of $f(x) = \frac{1+\sin x}{\cos x}$ is

(1)

- A. $\sec x + \tan x$
- B. ✓ $\frac{1+\sin x}{\cos^2 x}$
- C. $\frac{1}{1+\tan x}$
- D. $\frac{\cos x}{\sin^2 x}$

Answer: B

Using the quotient rule, $f'(x) = [\cos x(\cos x) - (1 + \sin x)(-\sin x)] / \cos^2 x = (\cos^2 x + \sin x + \sin^2 x) / \cos^2 x = (1 + \sin x) / \cos^2 x$.

30. If Q_i , D_i and P_i are respectively the i^{th} quartile, decile and percentile of a data arranged in an increasing order, then which one of the following is necessarily true? (1)

- A. $Q_2 = \frac{Q_1+Q_3}{2}$
- B. ✓ $D_3 > P_{25}$
- C. $P_{25} > Q_1$
- D. $Q_2 = \text{mean of the data}$

Answer: B

31. If [the function is missing from the capture], what is the slope of the tangent line to the graph of f at $x=2$? (1)

- A. -4
- B. 2
- C. 18
- D. ✓ 17

Answer: D

32. If $f(x) = e^{2x} \sin x$, then $f''(x)$ is equal to

(1)

- A. $3e^{2x} \sin x - 4e^{2x} \cos x$
- B. $4e^{2x} \sin x + 2e^{2x} \cos x$
- C. ✓ $e^{2x}(3 \sin x + 4 \cos x)$
- D. $e^{2x}(4 \sin x - 3 \cos x)$

Answer: C

33. If the equation $(x-2)^2 - (y-2)^2 = 1$ represents a hyperbola, which one of the following represents equation of an asymptote to the hyperbola? (1)

- A. ✓ $y = 4 - x$
- B. $x + y = 1$
- C. $x = 2 - y$
- D. $x + 2y = 3$

Answer: A

34. What is the sum of the series $\sum_{n=1}^{\infty} (-1)^n 3^{-2n}$?

(1)

- A. -1/8
- B. -0.13
- C. ✓ -0.1
- D. 1/8

Answer: C

35. Which of the equations below is represented by the following parabola? (1)

[Figure: A hand-drawn sketch of x-y coordinate axes, with an arrowhead on the upward y-axis and on the rightward x-axis. An upward-opening parabola is drawn. Its vertex sits on the x-axis at the point marked by a short tick on the positive x-axis labelled '1' below the axis, so the curve just touches the x-axis there. The left branch rises steeply to the upper left and crosses the y-axis at the point labelled '2' (the label '2' is written to the left of the y-axis at that height). The right branch rises to the upper right. No other values or gridlines are shown.]

- A. $y = x^2 + 2$ B. $y = (2x - 1)^2$
C. ✓ $y = 2(x - 1)^2$ D. $y = (2x + 1)^2$

Answer: C

36. A company produced 25,000 bulbs and randomly tested 2% of the product. Among the tested bulbs, if 40 have defect of type D_1 , 60 have defect of type D_2 and 25 have both types of defects, what is the probability that a bulb produced by the company had none of the defects? (1)

- A. 0.95 B. 0.80
C. ✓ 0.85 D. 0.20

Answer: C

37. A semi-elliptical arc over a tunnel for a road through a mountain has a major axis of length 80 meters and a height of 30 meters at the center. What is the equation of the semi-elliptical arc over the tunnel, if the center is considered as the origin? (1)

- A. $\frac{x^2}{6400} + \frac{y^2}{900} = 1$ B. ✓ $\frac{x^2}{1600} + \frac{y^2}{900} = 1$
C. $\frac{x^2}{900} + \frac{y^2}{6400} = 1$ D. $\frac{x^2}{8100} + \frac{y^2}{6400} = 1$

Answer: B

38. Suppose $AX=b$, where A is a 3×3 matrix, $b = (b_1, b_2, b_3)^T$ and $X = (x, y, z)^T$. Which one of the following is necessarily true about this system of linear equations? (1)

- A. The system has a solution only when $\det(A) \neq 0$.
B. the Cramer's rule is suitable to solve the system if two rows of A are identical
C. ✓ if $\det(A) \neq 0$ and the second column of A is a multiple of b , then $x = 0$
D. if $b = 0$, then $x = (0, 0, 0)^T$ is the only solution of the system.

Answer: C

39. If $y = \sin(3x^2)$, then the simplified form of $\frac{d^2y}{dx^2}$ is : (1)

- A. $-6 \sin(3x^2)$ B. $\cos(6x) - 6 \sin(3x^2)$
C. ✓ $6 \cos(3x^2) + 36x^2 \sin(3x^2)$ D. $x^2 \cos(3x^2) + 6 \sin(3x^2)$

Answer: C

40. If S is a set with 10 elements and $A \subseteq S$, what is the probability that A has 3 or more elements? (1)

- A. 7/10 B. 8/11
C. ✓ 121/128 D. 7/128

Answer: C

The total number of subsets of a 10-element set is $2^{10} = 1024$. The number of subsets with fewer than 3 elements is $\binom{10}{0} + \binom{10}{1} + \binom{10}{2} = 1 + 10 + 45 = 56$, so the probability of having 3 or more elements is $(1024 - 56)/1024 = 968/1024 = 121/128$.

41. If $a_n = \left(\frac{n+3}{n+1}\right)^n$, then the limit of the sequence $\{a_n\}_{n=1}^{\infty}$ is equal to : (1)
- A. 1
B. $1/2 e$
C. $\sqrt{e^2}$
D. $+\infty$

Answer: C

Rewriting the base gives $\left(1 + \frac{2}{n+1}\right)^n$. Taking the limit as $n \rightarrow \infty$ yields $e^{\lim_{n \rightarrow \infty} \frac{2n}{n+1}} = e^2$.

42. Which one of the following is equal to $\lim_{x \rightarrow \infty} \left(\frac{x}{x+2}\right)^{-3x}$? (1)
- A. $\sqrt{e^6}$
B. e^{-3}
C. $e^{-3/2}$
D. e^{-6}

Answer: A

Inverting the base gives $\lim_{x \rightarrow \infty} \left(\frac{x+2}{x}\right)^{3x} = \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{3x} = (e^2)^3 = e^6$.

43. If a box with square base open top is made from $1,200\text{cm}^2$ material, what is the largest volume of the box in cm^3 ? (1)
- A. $\sqrt{4000}$
B. 8000
C. 15,000
D. 3000

Answer: A

For a box with a square base of side x and height h , the surface area is $x^2 + 4xh = 1200$, giving $h = \frac{1200-x^2}{4x}$. Maximizing $V(x) = 300x - \frac{1}{4}x^3$ yields $V'(x) = 300 - \frac{3}{4}x^2 = 0 \implies x = 20$ cm, giving a maximum volume of $V(20) = 4000 \text{ cm}^3$.

44. Which one of the following is true about the graph of $f(x) = \frac{x^2-1}{x-x^2}$? (1)
- A. $x = 0$ and $x = 1$ are its vertical asymptotes.
B. $y = 1$ is its horizontal asymptote
C. $y = x - 1$ is its oblique asymptote
D. $\checkmark$ it is almost the same as the horizontal line $y = -1$ as $x \rightarrow \pm\infty$

Answer: D

Simplifying $f(x) = \frac{(x-1)(x+1)}{-x(x-1)} = -\frac{x+1}{x}$ for $x \neq 1$, we see there is a hole at $x = 1$ rather than a vertical asymptote. As $x \rightarrow \pm\infty$, $f(x) \rightarrow -1$, meaning the graph approaches the horizontal line $y = -1$.

45. Which one of the following is a valid logical argument? (1)
- A. $p \Rightarrow q, q \vdash p$
B. $p \Leftrightarrow q, p \Rightarrow q \vdash q$
C. $\checkmark \neg p \wedge q, \Box q \Rightarrow r \vdash r$
D. $\neg p, p \vee q, r \Rightarrow q \vdash r$

Answer: C

From the conjunction $\neg p \wedge q$, simplification yields q . Using Modus Ponens with $q \Rightarrow r$ (where the square symbol is a typographical artifact), one validly derives r .

46. Suppose that equal squares are cut from each of the four corners of a square cardboard whose sides are 72 centimetres long. [see the figure below.] The resulting flaps are then folded up to form a box without a top. How long should be each of the four squares that has to be cut off to maximize the volume of the box? (1)

[Figure: A hand-drawn square (the cardboard sheet) with its outline in heavy dashes. Two dashed lines run horizontally and two run vertically, each parallel to and set in from an edge by the same short distance, so that they mark off a small square at each of the four corners and leave a larger rectangle in the middle. The side of a corner square is labelled 'x' in five places: once along the top edge (just right of the top-left corner square), once at the left (in the left-middle corner strip), once at the right (in the right-middle strip), and twice along the bottom (at the bottom-left and bottom-right corner squares). No numeric dimensions are shown on the drawing.]

- A. 6cm
B. ✓ 12cm
C. 15cm
D. 24cm

Answer: B

The volume of the box is $V(x) = x(72 - 2x)^2$. Differentiating and setting $V'(x) = (72 - 2x)(72 - 6x) = 0$ gives critical points at $x = 36$ (which gives zero volume) and $x = 12$ cm, which maximizes the volume.

47. What is the area of the region between the graph of $f(x) = -x^2 + 4x - 3$ and the x axis from $x = 0$ to $x = 3$? (1)

- A. -2/3
B. 2/3
C. 4/3
D. ✓ 8/3

Answer: D

The function $f(x) = -(x - 1)(x - 3)$ crosses the x-axis at $x = 1$ and $x = 3$. The total area between the curve and the x-axis is $\int_0^1 (-f(x)) dx + \int_1^3 f(x) dx = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$.

48. Which one of the following is equal to $\int \frac{x + \ln(x+1)}{(x+1)^2} dx$? (1)

- A. $\ln(x + 1) + \frac{x}{x+1} + c$
B. $(x + 1)^2 \frac{1}{x+1} + c$
C. $(x + 1)^2 - \frac{x}{x+1} + c$
D. ✓ $\frac{x \ln(x+1)}{x+1} + c$

Answer: D

Differentiating $\frac{x \ln(x+1)}{x+1}$ using the product and quotient rules gives $\frac{(x+1)[\ln(x+1) + \frac{x}{x+1}] - x \ln(x+1)}{(x+1)^2} = \frac{x + \ln(x+1)}{(x+1)^2}$, which matches the integrand.

49. If a point (2, 5) is reflected under a line to the point(-3, 1), what is the line of reflection? (1)

- A. $2x + 3y = 7$
B. $x + 3y = 7$
C. ✓ $8y + 10x = 19$
D. $2x + 3y + 5 = 0$

Answer: C

The line of reflection is the perpendicular bisector of the segment joining (2, 5) and (-3, 1). The midpoint is (-1/2, 3) and the slope of the segment is $(1 - 5)/(-3 - 2) = 4/5$, giving a perpendicular slope of $-5/4$; the line equation is $y - 3 = -5/4(x + 1/2)$, which simplifies to $10x + 8y = 19$.

50. Let the equation $x^2 + 2x + y^2 = 8$ represents a circle. Then which one of the following lines cut the circle at exactly two points? (1)

- A. $4x + 3y + 19 = 0$
B. ✓ $3x + 4y + 14 = 0$
C. $2y = 5x + 43$
D. $2x = y - 50$

Answer: B

The circle $(x + 1)^2 + y^2 = 9$ has center (-1, 0) and radius $r = 3$. The perpendicular distance from (-1, 0) to $3x + 4y + 14 = 0$ is $|3(-1) + 4(0) + 14| / 5 = 11/5 = 2.2$, which is strictly less than 3, meaning the line cuts the circle at two distinct points.

51. If $A = (-2, 3)$, $B = (3, 1)$ and C is any other point on the plane, then which one of the following is the coordinate form of $\overrightarrow{AC} - \overrightarrow{BC}$? (1)

A. $(-5, 2)$ B. ✓ $(5, -2)$
C. $(1, 4)$ D. $(-1, -4)$

Answer: B

By vector subtraction, $AC - BC = AC + CB = AB$. In coordinate form, $AB = B - A = (3 - (-2), 1 - 3) = (5, -2)$.

52. What is the equation of a line that passes through the point $(-1, -2)$ and parallel to the vector $(1, -1)$? (1)

A. $2x - y = 1$ B. ✓ $x + y - 1 = 0$
C. $x - 2y = 3$ D. $y - 2x + 1 = 0$

Answer: B

53. Suppose P and Q are point in space such that the midpoint of PQ is on the negative z -axis and the distance between P and Q is 6. If $P = (2, -1, 0)$, then what is the coordinate of Q ? (1)

A. $(-2, 1, 4)$ B. $(2, -1, 6)$
C. $(2, -1, -6)$ D. ✓ $(-2, 1, -4)$

Answer: D

Let $Q = (x, y, z)$. The midpoint $((2 + x)/2, (-1 + y)/2, z/2)$ lies on the negative z -axis, so $x = -2$, $y = 1$, and $z < 0$. Using the distance formula, $(x - 2)^2 + (y + 1)^2 + z^2 = 16 + 4 + z^2 = 36$ gives $z^2 = 16$, so $z = -4$, yielding $Q = (-2, 1, -4)$.

54. Which one of the following is equivalent to $\neg[(\forall x)(p(x) \Rightarrow Q(x))]$? (1)

A. $(\forall x)(\neg p(x) \Rightarrow \neg q(x))$ B. $(\exists x)(\neg p(x) \Rightarrow \neg q(x))$
C. $(\exists x)(\neg p(x) \wedge q(x))$ D. ✓ $(\exists x)(p(x) \wedge \neg q(x))$

Answer: D

Negating a universal quantifier changes it to an existential quantifier: $\neg[(\forall x)(p(x) \Rightarrow q(x))] \equiv (\exists x)\neg(p(x) \Rightarrow q(x))$. Since the negation of an implication $p \Rightarrow q$ is $p \wedge \neg q$, this is equivalent to $(\exists x)(p(x) \wedge \neg q(x))$.

55. An observer on level ground is at a distance $10\sqrt{3}$ m from a building. The angles of elevation to the bottom of the windows on the second and third floors are 30° and 60° , respectively. What is the distance h between the bottoms of the windows? [you may use the values: $\sin 30^\circ = \cos 60^\circ = 1/2$ and $\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$] (1)

A. 15m B. ✓ 20m
C. $15\sqrt{3}$ m D. 32m

Answer: B

The heights to the second and third floor window bottoms are $h_1 = 10\sqrt{3} \tan(30^\circ) = 10$ m and $h_2 = 10\sqrt{3} \tan(60^\circ) = 30$ m, respectively. The distance between them is $h = h_2 - h_1 = 30 - 10 = 20$ m.

56. What is the image of the line given by $(x, y) = (-1, 0) + t(3, 6)$, $t \in \mathbb{R}$, under the transition that takes $(1, 0)$ to $(0, 1)$ following by the reflection about the line $y = 2x$? (1)

A. $y = 2x + 3$ B. $y = 2x - 3$
C. $y = 2x + 6$ D. ✓ $y = 2x - 5$

Answer: D

The original line has equation $y = 2x + 2$. The translation by vector $(-1, 1)$ shifts it to $y = 2x + 5$, which is parallel to the reflection line $y = 2x$; reflecting it across $y = 2x$ gives the line $y = 2x - 5$.

57. If $\theta = 2 \arctan(1/2)$, then which one of the following is equal to $\sec(\theta)$? (1)
- A. $35/3$ B. $4/5$
C. $\checkmark$ $5/3$ D. $5/4$

Answer: C

Using the double-angle formula, $\cos(\theta) = (1 - \tan^2(\arctan(1/2))) / (1 + \tan^2(\arctan(1/2))) = (1 - 1/4)/(1 + 1/4) = 3/5$, so $\sec(\theta) = 5/3$.

58. If a translation T takes the circle $x^2 + y^2 - 2x + 6y + 3 = 0$ into the circle whose equation is $(x + 2)^2 + (y - 4)^2 = 7$, then what is the image of the origin under T? (1)
- A. $\checkmark$ $(-3, 7)$ B. $(1, 2)$
C. $(1, -3)$ D. $(-2, 4)$

Answer: A

The center of the first circle is $(1, -3)$ and the center of the second circle is $(-2, 4)$. The translation vector is $(-2 - 1, 4 - (-3)) = (-3, 7)$, which is also the image of the origin $(0, 0)$.

59. If L is the line passing through $(0, 2)$ & parallel to $\vec{v} = \vec{i} + 3\vec{j}$. Which one of the following is true about L and the circle $(x - 2)^2 + (y - 1)^2 = 5$? (1)
- A. L is tangent to the circle at $(0, 2)$
B. L is tangent to the circle at some point P, where $P \neq (0, 2)$.
C. $\checkmark$ **L intersects the circle at two distinct points.**
D. the distance between L and the centre of the circle is greater than $\neq 5$

Answer: C

The point $(0, 2)$ lies on the circle because $(0 - 2)^2 + (2 - 1)^2 = 5$. The line L has slope 3, whereas the tangent line at $(0, 2)$ must have slope 2; hence, L is not tangent and intersects the circle at two distinct points.

60. Suppose $\vec{A} = 3\vec{i} - 4\vec{j}$ and $\vec{B}$ is a vector in the xy- plane such that the angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{3}$, if $\vec{u}$ is the unit vector in the direction of $\vec{B}$, then $\vec{A} \cdot (\vec{A} - 2\vec{u})$ is equal to: (1)
- A. $\checkmark$ **20** B. 5
C. 15 D. 30

Answer: A

The magnitude of A is $|\vec{A}| = 5$, and $\vec{A} \cdot \vec{u} = |\vec{A}||\vec{u}|\cos(\pi/3) = 5(1)(1/2) = 5/2$. Thus, $\vec{A} \cdot (\vec{A} - 2\vec{u}) = |\vec{A}|^2 - 2(\vec{A} \cdot \vec{u}) = 25 - 2(5/2) = 20$.

— END OF MARKING KEY —

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