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Mathematics, 2016 EC, common, entrance exam

Grade 12 · Mathematics · 2016 E.C.

MARKING KEY — CONFIDENTIAL

Time allowed: 120 minutes **Questions:** 38 **Total marks:** 38

1. In the set of natural numbers, which one of the following defines a composite number? (1)

A. A number that has only two distinct factors

B. A number that is divisible by numbers from 2 to 9

C. A number that has two or more different factors

D. ✓ **A number that has three or more different divisors**

Answer: D

The topic is the classification of natural numbers by how many divisors they have.

Count divisors:
A prime such as 7 has exactly two divisors, 1 and 7.
A composite such as 4 has the divisors 1, 2 and 4, which is three.
A composite such as 6 has 1, 2, 3 and 6, which is four.

So a composite number is one that has more divisors than a prime, that is three or more different divisors. That is option D.

The classic wrong turn is option C. Saying *two or more* factors still lets every prime in, because a prime has exactly two. Option A is the definition of a prime, and option B fails for 121, which is composite yet is divisible by none of the numbers 2 to 9.

Textbook: Mathematics Grade 9, Unit 1 The Number System, section 1.1 (page 18).

2. Which of the following is true about rationalizing the denominator? (1)

A. Only denominators can be multiplied by an irrational number

B. Only numerators can be rationalized when there is a radical

C. ✓ **It is a method of changing the denominator into a rational number**

D. It is a technique of multiplying a given number by an irrational expression

Answer: C

Rationalizing the denominator means rewriting a fraction so that no radical is left underneath.

You multiply the numerator and the denominator by the same well chosen expression, so the value of the fraction never changes:
$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$
With a two term denominator you use the conjugate:
$\frac{1}{3-\sqrt{2}} = \frac{3+\sqrt{2}}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{3+\sqrt{2}}{7}$
In both cases the denominator ends up rational, so option C states the purpose correctly.

The classic wrong turn is option D. The multiplier is not always irrational, and multiplying the whole number by it would change the value. You multiply by a fraction equal to 1.

Textbook: Mathematics Grade 9, Unit 1 The Number System, section 1.3 (page 39).

3. When dividing 142 by 13 and expressed using the division algorithm, which is correct? (1)

A. ✓ 142 = 10(13) + 12

B. 142 = 11(13) − 1

C. 142 = 11(13) + 1

D. 142 = 10(14) + 2

Answer: A

The division algorithm says that for a dividend a and a divisor $b > 0$ there are unique numbers q and r with $a = qb + r$ and 0 \le r < b. The remainder must be non negative and smaller than the divisor.

Divide 142 by 13:

$13 \times 10 = 130$

$142 - 130 = 12$

0 \le 12 < 13, so the remainder is allowed.

Therefore $142 = 10(13) + 12$, which is option A.

The classic wrong turn is option B. It is true as arithmetic, since $11 \times 13 - 1 = 142$, but -1 is not a legal remainder, so it is not the division algorithm form.

Textbook: Mathematics Grade 9, Unit 1 The Number System, section 1.1 (page 18).

4. The solution set of the system 2x − y = 5 and 3y + 15 = 6x is: (1)

A. The empty set

B. {(t, 2t − 15): t ∈ R}

C. ✓ {(t, 2t − 5): t ∈ R}

D. {(0, −5), (1, −3), (2, −2), ...}

Answer: C

This is a system of two linear equations in two variables. Put both equations in the form $y = mx + c$ and compare them.

First equation:

$2x - y = 5$

$y = 2x - 5$

Second equation:

$3y + 15 = 6x$

$3y = 6x - 15$

$y = 2x - 5$

The two equations describe the same line, so the system is dependent and has infinitely many solutions. Write them with a parameter: let $x = t$, then $y = 2t - 5$.

The solution set is $\{(t,\ 2t - 5) : t \in \mathbb{R}\}$, which is option C.

The classic wrong turn is option A. Students see two equations, subtract, get $0 = 0$ and read that as no solution. $0 = 0$ means every point of the line works, not that the set is empty.

Textbook: Mathematics Grade 9, Unit 2 Solving Equations, section 2.2 (page 91).

5. Which one of the following refers to the definition of similar plane figures? (1)

A. Figures whose corresponding angles have the same ratio

B. ✓ **Figures where one is an enlargement of the other**

C. Figures whose ratios of corresponding sides are different

D. Figures that have different shapes but same size

Answer: B

Two plane figures are similar when they have the same shape but not necessarily the same size. Formally, corresponding angles are congruent and corresponding sides are in the same ratio.

Saying one figure is an enlargement (or a reduction) of the other is the same statement in everyday words: every length is multiplied by one scale factor k, and the angles stay as they are. So option B is the definition.

Why the others fail:
Option A: angles are equal, not in a ratio.
Option C: the ratios of corresponding sides must be equal, not different.
Option D: same size with different shape is not similarity. Figures with the same shape and the same size are congruent, which is the special case $k = 1$.

Textbook: Mathematics Grade 9, Unit 6 Congruency and Similarity, section 6.2 (page 214).

6. In a right triangle where angle A is 30° and side opposite it is 11 cm, what is the hypotenuse? (1)

A. ✓ **22 cm**

B. 22√3 cm

C. 11√3 cm

D. 33 cm

Answer: A

In a right angled triangle the sine of an acute angle is the opposite side over the hypotenuse.

Here $A = 30^\circ$ and the side opposite A is 11 cm. Let the hypotenuse be c.

$\sin 30^\circ = \frac{11}{c}$
$\frac{1}{2} = \frac{11}{c}$
$c = 2 \times 11 = 22$

The hypotenuse is 22 cm, which is option A. A quick check: in a 30, 60, 90 triangle the side facing 30° is always half the hypotenuse.

The classic wrong turn is using $\cos 30^\circ$ or $\tan 30^\circ$, which gives the answers with $\sqrt{3}$ in them. Those belong to the other two sides, not to the pair given here.

Textbook: Mathematics Grade 9, Unit 4 Introduction to Trigonometry, section 4.2 (page 167).

7. A circle has diameter 16 cm and a chord 6 cm from the center. What is the chord length? (1)

A. $2\sqrt{7}$ cm

B. 8 cm

C. ✓ **$4\sqrt{7}$ cm**

D. 16 cm

Answer: C

The perpendicular from the centre of a circle to a chord bisects that chord. So the radius, half the chord and the distance from the centre form a right angled triangle.

Radius: $r = \frac{16}{2} = 8$ cm.

Distance from the centre to the chord: $d = 6$ cm.

Let half the chord be h.

$$h^2 + d^2 = r^2$$
$$h^2 + 36 = 64$$
$$h^2 = 28$$
$$h = \sqrt{28} = 2\sqrt{7}$$

The full chord is twice that:

$2h = 4\sqrt{7}$ cm, which is option C.

The classic wrong turn is stopping at $2\sqrt{7}$, option A. That is only half of the chord.

Textbook: Mathematics Grade 10, Unit 5 Circles, section 5.1 (page 238).

8. Which of the following is the approximate surface area of the base of a regular pentagonal prism of radius 6 cm? (1)

A. 54 sq. cm

B. ✓ **85.5 sq. cm**

C. 27 sq. cm

D. 27.5 sq. cm

Answer: B

The base of a regular pentagonal prism is a regular pentagon. Its radius (the distance from the centre to a vertex) is $R = 6$ cm.

Join the centre to all five vertices. This cuts the pentagon into 5 congruent triangles, each with two sides R and included angle $\frac{360^\circ}{5} = 72^\circ$.

Area of one triangle: $\frac{1}{2}R^2 \sin 72^\circ$

Area of the pentagon: $A = \frac{5}{2}R^2 \sin 72^\circ$

$$A = \frac{5}{2}(36)(0.9511)$$
$$A = 90 \times 0.9511 \approx 85.6$$

So the base area is about 85.5 square cm, which is option B.

The classic wrong turn is treating 6 cm as the side length instead of the radius, or forgetting the factor $\frac{1}{2}$, which lands on the much smaller distractors.

Textbook: Mathematics Grade 9, Unit 5 Regular Polygons, section 5.4 (page 197).

9. If each observation is multiplied by 0.5, what is the new mean? (1)

A. 1/2

B. ✓ **1/2 x̄**

C. 2 x̄

D. x̄ - 0.5

Answer: B

This is the effect of scaling on the mean. If every observation is multiplied by a constant, the mean is multiplied by the same constant.

Start from the definition:
$\bar{x} = \frac{x_1+x_2+\cdots+x_n}{n}$

Multiply each value by 0.5 and take the new mean:
$\bar{y} = \frac{0.5x_1+0.5x_2+\cdots+0.5x_n}{n}$
$\bar{y} = 0.5 \cdot \frac{x_1+x_2+\cdots+x_n}{n}$
$\bar{y} = 0.5\,\bar{x} = \frac{1}{2}\bar{x}$

So the new mean is half the old mean, which is option B.

The classic wrong turn is option D. Subtracting 0.5 is what happens when 0.5 is *taken away* from every value, not when every value is multiplied by it.

Textbook: Mathematics Grade 11, Unit 7 Statistics, section 7.4 (page 371).

10. A regular triangular pyramid labeled 1, 2, and 3 is thrown twice. What is the probability that the sum of the numbers facing down is less than 5? (1)

A. 1/3

B. ✓ **2/3**

C. 4/9

D. 5/9

Answer: B

Each throw shows one of the labels 1, 2 or 3, each equally likely. Two throws give $3 \times 3 = 9$ equally likely ordered pairs, so this is a simple counting problem.

List the pairs and their sums:
(1,1) = 2, (1,2) = 3, (1,3) = 4
(2,1) = 3, (2,2) = 4, (2,3) = 5
(3,1) = 4, (3,2) = 5, (3,3) = 6

Sums below 5: (1,1), (1,2), (1,3), (2,1), (2,2) and (3,1). That is 6 favourable pairs.

$P = \frac{6}{9} = \frac{2}{3}$, which is option B.

The classic wrong turn is counting the pairs as unordered. Then you find only 4 favourable cases out of 9 and get $\frac{4}{9}$. Order matters here, because (1,2) and (2,1) are two different throws.

Textbook: Mathematics Grade 11, Unit 8 Probability, section 8.7 (page 453).

11. Given vectors u with initial point (-1, 0) and terminal point (2, 3), and v with initial point (5, -2) and terminal point (2, 3), what is v - u? (1)

A. (-6, 3)

B. (2, -6)

C. (0, -2)

D. ✓ **(-6, 2)**

Answer: D

A vector given by two points is terminal point minus initial point, taken coordinate by coordinate.

$\vec{u} = (2 - (-1),\ 3 - 0) = (3,\ 3)$
$\vec{v} = (2 - 5,\ 3 - (-2)) = (-3,\ 5)$

Now subtract:
$\vec{v} - \vec{u} = (-3 - 3,\ 5 - 3)$
$\vec{v} - \vec{u} = (-6,\ 2)$

That is option D.

The classic wrong turn is subtracting in the wrong order or mixing up initial and terminal points. Both slips give $(6, -2)$ or $(-6, 3)$, and option A is exactly that trap.

Textbook: Mathematics Grade 9, Unit 7 Vectors in Two Dimensions, section 7.3 (page 257).

12. Which one of the following is the definition of a polynomial function? (1)

A. ✓ **A function f defined by f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0, where n is non-negative integer, a_n ≠ 0 and a_i ∈ R**

B. A function f defined by f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0, where n is positive integer, a_n ≠ 0 and a_i ∈ R

C. A function f defined by f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0, where n is an integer, a_n ≠ 0 and a_i ∈ R

D. A function f defined by f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0, where n is a rational number, a_n ≠ 0 and a_i ∈ R

Answer: A

A polynomial function is $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ where the coefficients are real and the exponents are whole numbers. The point being tested is what n is allowed to be.

The degree n must be a non negative integer, so $n = 0, 1, 2, 3, \ldots$ That is option A.

Why $n = 0$ has to be allowed: the constant function $f(x) = 7$ is a polynomial of degree 0. Option B says positive integer, which throws every non zero constant function out of the family, so it is the classic wrong turn.

Options C and D fail as well. An integer exponent allows x^{-2}, and a rational exponent allows $x^{1/2} = \sqrt{x}$. Neither of those is a polynomial term.

Textbook: Mathematics Grade 10, Unit 2 Polynomial functions, section 2.1 (page 66).

13. When f(x) = 4x + kx - 1 is divided by x - 1/2, the remainder is 3. What is the value of k? (1)

A. 14

B. ✓ **7**

C. 7

D. 9

Answer: B

This uses the remainder theorem: when $f(x)$ is divided by $x - c$, the remainder is $f(c)$. Here $c = \frac{1}{2}$, so set $f\left(\frac{1}{2}\right) = 3$.

Careful with this item: the first term has lost its exponent in printing. Reading it as $f(x) = 4x^3 + kx - 1$ is the only version that lands on an offered value.

$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + k\left(\frac{1}{2}\right) - 1$
$= 4 \cdot \frac{1}{8} + \frac{k}{2} - 1$
$= \frac{1}{2} + \frac{k}{2} - 1 = \frac{k}{2} - \frac{1}{2}$

Set that equal to 3:
$\frac{k}{2} - \frac{1}{2} = 3$
$\frac{k}{2} = \frac{7}{2}$
$k = 7$

So the value is 7, printed at both option B and option C.

The classic wrong turn is using $f(-\frac{1}{2})$. The divisor is $x - \frac{1}{2}$, so you substitute the number that makes the divisor zero, that is $+\frac{1}{2}$.

Textbook: Mathematics Grade 10, Unit 2 Polynomial functions, section 2.3 (page 81).

14. What is the solution set of $\log_4(x + 1) - 2 \log_4 x = 0.5$? (1)

A. {1, -1/2}

B. {1/2, -1}

C. {1/2}

D. ✓ **None**

Answer: D

Use the logarithm laws to write one logarithm, then convert to exponential form. Check the domain at the end, because that is where this question is decided.

Domain first: $\log_4 x$ needs x > 0, and $\log_4(x + 1)$ needs x > -1. So any answer must satisfy x > 0.

$\log_4(x + 1) - 2\log_4 x = 0.5$
$\log_4(x + 1) - \log_4 x^2 = 0.5$
$\log_4 \frac{x+1}{x^2} = \frac{1}{2}$
$\frac{x+1}{x^2} = 4^{1/2} = 2$
$x + 1 = 2x^2$
$2x^2 - x - 1 = 0$
$(2x + 1)(x - 1) = 0$
$x = -\frac{1}{2}$ or $x = 1$

Now apply the domain. $x = -\frac{1}{2}$ is negative, so $\log_4 x$ does not exist and that root is extraneous. Only $x = 1$ survives, and $\log_4 2 - 2\log_4 1 = 0.5 - 0 = 0.5$ checks out.

The true solution set is $\{1\}$. It is not printed, so the honest choice is option D.

The classic wrong turn is option A: solving the quadratic and writing both roots without testing them in the original logarithms.

Textbook: Mathematics Grade 10, Unit 3 Exponential and Logarithmic Functions, section 3.4 (page 162).

15. Which of the following is the characteristic of log(20 + 1/5)? (1)

A. -1

B. ✓ **1**

C. 2.02

D. 0.2

Answer: B

Any common logarithm splits into a characteristic and a mantissa. The characteristic is the integer part, the mantissa is the decimal part, which is always kept non negative.

First simplify the number inside:
$20 + \frac{1}{5} = 20 + 0.2 = 20.2$

Now trap it between powers of 10:
10^1 = 10 < 20.2 < 100 = 10^2

so 1 < \log 20.2 < 2.

The integer part is therefore 1, and indeed $\log 20.2 \approx 1.3054$. The characteristic is 1, which is option B.

A quick rule: for a number written in standard form $a \times 10^n$ with 1 \le a < 10, the characteristic is simply n. Here $20.2 = 2.02 \times 10^1$.

The classic wrong turn is option C, reading 2.02 off the standard form and calling that the characteristic. That number is the mantissa part of the story, not the characteristic.

Textbook: Mathematics Grade 10, Unit 3 Exponential and Logarithmic Functions, section 3.1 (page 117).

16. Which of the following statements about trigonometric values is true? (1)

A. sin 270° = 0

B. ✓ **cos(-180°) = -1**

C. tan(-90°) = 0

D. tan(540°) = -1

Answer: B

Test each statement on the unit circle, where a point at angle θ has coordinates $(\cos\theta,\ \sin\theta)$.

Option A: 270° is the point $(0, -1)$, so $\sin 270^\circ = -1$, not 0. False.

Option B: cosine is an even function, so $\cos(-180^\circ) = \cos 180^\circ$. The point at 180° is $(-1, 0)$, so the value is -1. True.

Option C: $\tan\theta = \frac{\sin\theta}{\cos\theta}$, and $\cos(-90^\circ) = 0$. Division by zero, so $\tan(-90^\circ)$ is undefined, not 0. False.

Option D: $540^\circ - 360^\circ = 180^\circ$, and $\tan 180^\circ = \frac{0}{-1} = 0$, not -1. False.

Only option B survives.

The classic wrong turn is mixing sine and cosine at the quadrantal angles. Remember the pattern of $(\cos, \sin)$ at $0^\circ, 90^\circ, 180^\circ, 270^\circ$: $(1, 0), (0, 1), (-1, 0), (0, -1)$.

Textbook: Mathematics Grade 10, Unit 4 Trigonometric functions, section 4.2 (page 194).

17. For 0° ≤ θ ≤ 90°, which relationship describes the co-function of θ? (1)

A. cosθ = sec(90° - θ)

B. cscθ = sin(180° - θ)

C. ✓ **cotθ = tan(90° - θ)**

D. secθ = sin(180° - θ)

Answer: C

Co-function identities pair each ratio with its co-ratio across the complementary angle $90^\circ - \theta$. In a right triangle the two acute angles add to 90°, so the side that is opposite one of them is adjacent to the other, which is where these identities come from.

The pairs are:

$\sin\theta = \cos(90^\circ - \theta)$

$\cos\theta = \sin(90^\circ - \theta)$

$\tan\theta = \cot(90^\circ - \theta)$

$\cot\theta = \tan(90^\circ - \theta)$

$\sec\theta = \csc(90^\circ - \theta)$

$\csc\theta = \sec(90^\circ - \theta)$

Option C matches the fourth line exactly. Check it at $\theta = 30^\circ$: $\cot 30^\circ = \sqrt{3}$ and $\tan 60^\circ = \sqrt{3}$.

The classic wrong turn is options B and D, which use $180^\circ - \theta$. That is a supplementary angle identity, and it never swaps a ratio for its co-ratio.

Textbook: Mathematics Grade 10, Unit 4 Trigonometric functions, section 4.3 (page 221).

18. For 0° ≤ θ ≤ 90°, which of the following identities is correct? (1)

A. ✓ **csc²(90° - θ) = 1 + tan²θ**

B. sec²θ + tan²θ = 1

C. cot²(90° - θ) = 1 + csc²θ

D. cot²θ + csc²θ = 1

Answer: A

Two tools settle this: the Pythagorean identities and the co-function rule.

Pythagorean identities:

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

Option A: $\csc(90^\circ - \theta) = \sec\theta$, so the left side is $\sec^2\theta$. The second identity rearranges to $\sec^2\theta = 1 + \tan^2\theta$, so both sides agree. True.

Option B: the identity is $\sec^2\theta - \tan^2\theta = 1$, with a minus sign. Test $\theta = 45^\circ$: $2 + 1 = 3$, not 1. False.

Option C: $\cot(90^\circ - \theta) = \tan\theta$, so the left side is $\tan^2\theta$. At $\theta = 45^\circ$ that is 1, while $1 + \csc^2 45^\circ = 1 + 2 = 3$. False.

Option D: the identity is $\csc^2\theta - \cot^2\theta = 1$, again a minus sign. At $\theta = 45^\circ$, $1 + 2 = 3$, not 1. False.

So option A is the correct identity. The classic wrong turn is turning the minus sign in these identities into a plus. Only the sine and cosine identity is a sum of squares.

Textbook: Mathematics Grade 10, Unit 4 Trigonometric functions, section 4.3 (page 221).

19. Let ABCD be a rhombus and E be the midpoint of diagonal AC. Which of the following is NOT true? (1)

A. ΔABC is isosceles

B. AC is perpendicular to BD

C. ΔBEC is a right triangle

D. ✓ **∠DAB is acute**

Answer: D

Facts about a rhombus: all four sides are equal, the diagonals bisect each other, and the diagonals meet at right angles. Since the diagonals bisect each other, the midpoint E of AC is also the point where the two diagonals cross.

Option A: in triangle ABC the sides AB and BC are two sides of the rhombus, so $AB = BC$ and the triangle is isosceles. Always true.

Option B: the diagonals of a rhombus are perpendicular, so AC is perpendicular to BD. Always true.

Option C: E is the crossing point of the diagonals, so the angle BEC is a right angle and triangle BEC is right angled at E. Always true.

Option D: nothing forces the angle DAB to be small. A square is a rhombus, and there the angle is exactly 90°; a flatter rhombus can make it obtuse. So this statement need not hold, and it is the one that is NOT true.

The classic wrong turn is picking A because the triangle is not equilateral. Isosceles only asks for two equal sides, and AB = BC delivers that.

20. A regular hexagonal park has radius 12 m. What length of fencing material is needed? (1)

A. 72√3

B. ✓ **72**

C. 36√3

D. 36

Answer: B

Fencing a park means finding its perimeter, so the work is to get the side length of the regular hexagon.

Join the centre to all six vertices. Each of the six triangles has two sides equal to the radius and a central angle of $\frac{360^{\circ}}{6} = 60^{\circ}$. A triangle with two equal sides and a 60° angle between them is equilateral, so the side of a regular hexagon equals its radius.

$s = R = 12$ m
$P = 6s = 6 \times 12 = 72$ m

So 72 m of fencing is needed, which is option B.

The classic wrong turn is using the apothem relation and dragging in a $\sqrt{3}$, as in option A. The $\sqrt{3}$ belongs to the apothem and the area of a hexagon, never to its perimeter.

Textbook: Mathematics Grade 9, Unit 5 Regular Polygons, section 5.4 (page 197).

21. Given a right circular cylinder has radius 5 cm and lateral surface area 150π cm². What is its volume? (1)

A. ✓ **375π**
B. 350π
C. 315.5π
D. 200π

Answer: A

For a right circular cylinder the lateral (curved) surface area is $A_L = 2\pi rh$ and the volume is $V = \pi r^2 h$. Use the first formula to find the height, then the second.

Find h:

$2\pi rh = 150\pi$
$2\pi(5)h = 150\pi$
$10\pi h = 150\pi$
$h = 15$ cm

Now the volume:

$V = \pi r^2 h = \pi(5)^2(15)$
$V = \pi(25)(15) = 375\pi$ cubic cm

That is option A.

The classic wrong turn is using the total surface area $2\pi rh + 2\pi r^2$ in place of the lateral area. The question gives the lateral area only, so the two circular ends stay out of that step.

Textbook: Mathematics Grade 10, Unit 6 Solid Figures, section 6.1 (page 272).

22. A spherical container has diameter 3 m. What is the volume of water it holds? (1)

A. 27/4 π
B. 27/2 π
C. ✓ **9/2 π**
D. 9/4 π

Answer: C

A full spherical container holds a volume equal to the volume of the sphere, $V = \frac{4}{3}\pi r^3$.

The diameter is 3 m, so the radius is half of that:

$r = \frac{3}{2}$ m

Cube the radius:

$r^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

Now the volume:

$V = \frac{4}{3}\pi \cdot \frac{27}{8}$
$V = \frac{108}{24}\pi = \frac{9}{2}\pi$ cubic m

That is option C.

The classic wrong turn is putting the diameter straight into the formula. Using $r = 3$ gives 36π, eight times too big, because the radius is cubed.

Textbook: Mathematics Grade 10, Unit 6 Solid Figures, section 6.2 (page 277).

23. The domain and range of f(x) = 1 + [2x - 4] are: (1)

A. [2, ∞) and [1, ∞)

B. ℝ and [2, ∞)

C. [2, ∞) and ℝ

D. ✓ **ℝ and [1, ∞)**

Answer: D

The brackets here stand for absolute value, so the function is $f(x) = 1 + |2x - 4|$. Read the domain first, then the range.

Domain: an absolute value accepts every real input. There is no square root and no denominator to worry about, so the domain is $\mathbb{R}$.

Range: build it up from the inside.
$|2x - 4| \geq 0$ for every x, and it reaches 0 when $2x - 4 = 0$, that is at $x = 2$.
Adding 1: $f(x) = 1 + |2x - 4| \geq 1$, with the value 1 actually reached at $x = 2$.
As x moves away from 2 in either direction the value grows without bound.

So the range is $[1, \infty)$ and the answer is domain $\mathbb{R}$, range $[1, \infty)$, which is option D.

The classic wrong turn is option A. Students set $2x - 4 \geq 0$ and restrict the domain to $x \geq 2$. That condition belongs to a square root, not to an absolute value.

Textbook: Mathematics Grade 11, Unit 1 Relations and Functions, section 1.3 (page 22).

24. Which of the following is the inverse of f(x) = √(x - 3)? (1)

A. ✓ **$f^{-1}(x) = x^2 + 3, x \geq 0$**

B. $f^{-1}(x) = x^2 + 3, x \geq 3$

C. $f^{-1}(x) = \sqrt{x + 3}$

D. $f^{-1}(x) = 1/\sqrt{x - 3}$

Answer: A

To invert a function, swap the roles of x and y and solve again for y. Then remember that the domain of the inverse is the range of the original function.

Original: $y = \sqrt{x - 3}$, which needs $x \geq 3$ and produces $y \geq 0$.

Swap and solve:
$x = \sqrt{y - 3}$
$x^2 = y - 3$
$y = x^2 + 3$

Now the domain. The outputs of f were $y \geq 0$, so the inputs of f^{-1} are $x \geq 0$.

$f^{-1}(x) = x^2 + 3,\ x \geq 0$, which is option A.

Check one value: $f(7) = \sqrt{4} = 2$ and $f^{-1}(2) = 4 + 3 = 7$.

The classic wrong turn is option B. It carries the domain $x \geq 3$ over from the original function. The two functions trade their domain and range, so that restriction does not travel with the formula.

Textbook: Mathematics Grade 11, Unit 1 Relations and Functions, section 1.5 (page 68).

25. Which of the following is the simplified form of [x - (x + 3)/(x - 1)]² + (x - 3)/(x² - 2x + 1)? (1)

A. $(x^2 - 2x + 3)/(x - 1)$

B. $(x - 1)/(x - 3)$

C. ✓ **$x^2 - 1$**

D. $x^2 - 2x + 1$

Answer: C

This is a complex fraction. The printed form has lost its division bar, and the item is

$$\left(x - \frac{x+3}{x-1}\right) \div \frac{x-3}{x^2-2x+1}$$

Step 1. Combine the first bracket over one denominator:

$x - \frac{x+3}{x-1} = \frac{x(x-1)-(x+3)}{x-1}$
$= \frac{x^2-x-x-3}{x-1} = \frac{x^2-2x-3}{x-1}$
$= \frac{(x-3)(x+1)}{x-1}$

Step 2. Factor the other denominator:

$x^2 - 2x + 1 = (x-1)^2$

Step 3. Divide by multiplying with the reciprocal:

$\frac{(x-3)(x+1)}{x-1} \cdot \frac{(x-1)^2}{x-3}$
$= (x+1)(x-1)$
$= x^2 - 1$

So the simplified form is $x^2 - 1$, which is option C. It holds for $x \neq 1$ and $x \neq 3$, the values that were never allowed.

The classic wrong turn is the sign in step 1. Writing $x(x-1) - (x+3) = x^2 - 2x + 3$ forgets to distribute the minus over the 3, and that mistake lands exactly on option A.

Textbook: Mathematics Grade 11, Unit 2 Rational Expressions and Rational Functions, section 2.1 (page 84).

26. A data on scores of 32 students is listed as follows: (1)

Score	Number
52	48
76	35
60	45
50	75

If the data is to be grouped in 6 classes, what has to be the class interval (w)?

A. 6
B. 7
C. ✓ **8**
D. 9

Answer: C

When raw data is grouped, the class width is

$$w = \left\lceil \frac{\text{range}}{\text{number of classes}} \right\rceil$$

where range = highest value minus lowest value, and the result is always rounded UP.

Rounding up is the whole point. If you round down, the classes together are shorter than the data and the largest value falls outside the last class.

Here the number of classes is 6, so you divide the range by 6 and round up. A range anywhere from 43 to 48 gives
$\frac{43}{6} = 7.2$ up to $\frac{48}{6} = 8$
and each of those rounds up to $w = 8$, which is option C.

Warning about this item: the table of scores did not print correctly, so the range cannot be read off it with confidence. Learn the method, and check the table against a clean copy of the paper.

The classic wrong turn is rounding 7.6 down to 7. Class width is always rounded up.

Textbook: Mathematics Grade 11, Unit 7 Statistics, section 7.2 (page 360).

27. Five English books (3 identical) and 3 different Mathematics books are arranged in a row. If one specific Math book is first, how many arrangements are possible? (1)

A. 240
B. ✓ **840**
C. 5040
D. 6720

Answer: B

There are 8 books in all: 5 English, of which 3 are identical, and 3 different Mathematics books. One named Mathematics book must sit in the first place.

Step 1. Place the named book. It is fixed in position 1, so there is only 1 way to do this.

Step 2. Arrange the rest. Seven books remain: 3 identical English, 2 different English and 2 different Mathematics books.

Step 3. Use the permutation rule for repeated items. With n items of which one group of p is identical, the number of distinct arrangements is $\frac{n!}{p!}$:
$\frac{7!}{3!} = \frac{5040}{6} = 840$
So there are 840 arrangements, which is option B.

The classic wrong turn is option C. It gives $7! = 5040$ and forgets to divide by $3!$. Swapping identical books produces the same row, so those orders must not be counted more than once.

Textbook: Mathematics Grade 11, Unit 8 Probability, section 8.3 (page 423).

28. Balls numbered 1 to 10 are placed in a box. What is the probability that a randomly selected ball is neither odd nor a multiple of 3? (1)

A. 3/10

B. ✓ **2/5**

C. 3/5

D. 2/5

Answer: B

The sample space is the ten balls 1 to 10, all equally likely. The word *neither* means the ball must fail both conditions, so list the balls that pass either one and take what is left.

Odd numbers: 1, 3, 5, 7, 9.
Multiples of 3: 3, 6, 9.
Together (counting 3 and 9 once): 1, 3, 5, 6, 7, 9, which is 6 balls.

The balls that are neither odd nor a multiple of 3 are the remaining ones: 2, 4, 8, 10. That is 4 balls.

$P = \frac{4}{10} = \frac{2}{5}$

The value $\frac{2}{5}$ appears at both option B and option D on this paper. Option B is kept as the key.

The classic wrong turn is counting 3 and 9 twice while joining the two lists. Then you find only 3 balls left and answer $\frac{3}{10}$.

Textbook: Mathematics Grade 11, Unit 8 Probability, section 8.7 (page 453).

29. Let A be a 3×3 matrix and k a real number. Which statement is true? (1)

A. det(kA) = 3det(A)

B. det(-A^T) = det(A)

C. Interchanging rows does not change determinant

D. ✓ **Multiplying one column by k multiplies determinant by k**

Answer: D

Test each claim against the properties of determinants for a 3×3 matrix.

Option A: multiplying the whole matrix by k multiplies every one of the three rows by k, so $\det(kA) = k^3 \det(A)$, not $3 \det(A)$. False.

Option B: transposing leaves the determinant alone, $\det(A^T) = \det(A)$, but the minus sign hits all three rows, so $\det(-A^T) = (-1)^3 \det(A) = -\det(A)$. False.

Option C: interchanging two rows reverses the sign, $\det \to -\det$. It changes the determinant unless the determinant is 0. False.

Option D: multiplying a single row or a single column by k multiplies the determinant by exactly k. True, and it is the property that makes option A come out as k^3, since a 3×3 matrix has three rows to scale.

So option D is the true statement.

The classic wrong turn is option A: pulling k out only once instead of once per row.

Textbook: Mathematics Grade 11, Unit 4 Determinants and Their Properties, section 4.4 (page 220).

30. Given the linear system, (1)

$$\begin{cases} a_{1x} + b_{1y} - c_{1z} = 3 \\ a_{2x} + b_{2y} + c_{2z} = -1 \\ a_{3x} + b_{3y} + c_{3z} = 2 \end{cases}$$

such that the determinant of the coefficient matrix is $\frac{1}{3}$. Which of the following is the value of y?

A. $\begin{vmatrix} a_1 & b_1 & c_1 \\ -3 & 1 & -2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

B. ✓ $\begin{vmatrix} a_1 & 3 & c_1 \\ a_2 & -1 & c_2 \\ a_3 & 2 & c_3 \end{vmatrix}$

C. $\begin{vmatrix} a_1 & b_1 & c_1 \\ -3 & 1 & -2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

D. $\begin{vmatrix} a_1 & -3 & c_1 \\ -3a_3 & 1 & c_2 \\ a_3 & -2 & c_3 \end{vmatrix}$

Answer: B

Cramer's rule solves a system by replacing one column of the coefficient matrix with the column of constants.

Write the system as $AX = B$. Here the constants are $B = (3,\ -1,\ 2)$, and $\det(A) = \frac{1}{3}$.

For the second unknown:
$y = \frac{\det(A_y)}{\det(A)}$
where A_y is A with its SECOND column, the y column of b values, replaced by the constants.

So A_y has first column a_1, a_2, a_3, second column $3, -1, 2$ and third column c_1, c_2, c_3. That is the determinant shown in option B.

Two things to note on this printed paper: the constants belong in the middle column only, and because $\det(A) = \frac{1}{3}$, the value of y is really 3 times that determinant. No option carries the factor 3, so option B is taken as the intended answer.

The classic wrong turn is options A and C, which put the constants into a ROW instead of a column. Cramer's rule always replaces a column, and the column it replaces is the one belonging to the unknown you want.

Textbook: Mathematics Grade 11, Unit 4 Determinants and Their Properties, section 4.6 (page 235).

31. Given the sequence −2, 0, 4, 10, ... what can be the fifth term? (1)

A. 14

B. 16

C. ✓ **18**

D. 20

Answer: C

The sequence is not arithmetic and not geometric, so look at the differences between neighbouring terms.

Terms: $-2,\ 0,\ 4,\ 10,\ \ldots$
First differences: $0-(-2)=2, 4-0=4, 10-4=6$

The differences are 2, 4, 6, going up by 2 each time. The next difference is 8, so
$a_5 = 10 + 8 = 18$

That is option C.

You can confirm it with a formula. Since the second differences are constant, the terms follow a quadratic rule, and fitting the first three terms gives
$a_n = n^2 - n - 2$
Check: $a_4 = 16 - 4 - 2 = 10$ and $a_5 = 25 - 5 - 2 = 18$.

The classic wrong turn is assuming the difference stays at 6 and answering 16.

Textbook: Mathematics Grade 12, Unit 1 Sequences and Series, section 1.1 (page 2).

32. Which one of the following gives the definition of integration for a function? (1)

A. ✓ **Reversing differentiation**

B. Derivative at a point

C. Rate of change

D. Derivative on an interval

Answer: A

Integration and differentiation are opposite operations. To integrate a function is to ask which function would have it as a derivative, so integration reverses differentiation. That is option A.

An example makes it concrete:

$\frac{d}{dx}\left(x^3\right) = 3x^2$

$\int 3x^2\,dx = x^3 + C$

The constant C appears because every function of the form $x^3 + C$ has the same derivative, so reversing the process gives a whole family of answers. For that reason the result is called an antiderivative.

The other options describe differentiation instead. The derivative at a point and the rate of change are two descriptions of $f'(x)$, which is the forward direction, not the reverse.

Textbook: Mathematics Grade 12, Unit 2 Introduction to Calculus, section 2.3 (page 123).

33. What is the mean deviation about the mean of the data: 15, 12, 10, 13, 9, 12, 11, 15, 14, 9? (1)

A. 1.4

B. 1.5

C. ✓ **1.8**

D. 2.0

Answer: C

Mean deviation about the mean is the average of the distances from the mean, $MD = \frac{\sum |x_i - \bar{x}|}{n}$. Find the mean, then the distances.

Step 1. The mean. There are 10 values:

$15 + 12 + 10 + 13 + 9 + 12 + 11 + 15 + 14 + 9 = 120$

$\bar{x} = \frac{120}{10} = 12$

Step 2. The distances $|x - 12|$, keeping every one positive:

3, 0, 2, 1, 3, 0, 1, 3, 2, 3

Step 3. Add them and divide:

$3 + 0 + 2 + 1 + 3 + 0 + 1 + 3 + 2 + 3 = 18$

$MD = \frac{18}{10} = 1.8$

That is option C.

The classic wrong turn is adding the deviations with their signs. They always cancel to zero, which is why the absolute value is taken before adding.

Textbook: Mathematics Grade 12, Unit 3 Statistics, section 3.1 (page 161).

34. Given the following grouped data (1)

x	8 – 12	13 – 17	18 – 22	23 – 27	28 – 32
f	3	4	5	6	2

Which of the following is the standard deviation of the data?

A. 6

B. 6.5

C. \sqrt{37}

D. ✓ **\sqrt{37.5}**

Answer: D

For grouped data you work with class midpoints. The standard deviation is

$$\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

Step 1. Midpoints, taken as the average of the two class limits:
10, 15, 20, 25, 30, with frequencies 3, 4, 5, 6, 2 and $\sum f = 20$.

Step 2. The mean:
$\sum fx = 3(10) + 4(15) + 5(20) + 6(25) + 2(30)$
$= 30 + 60 + 100 + 150 + 60 = 400$
$\bar{x} = \frac{400}{20} = 20$

Step 3. The squared deviations, weighted by frequency:
$3(10-20)^2 = 3(100) = 300$
$4(15-20)^2 = 4(25) = 100$
$5(20-20)^2 = 0$
$6(25-20)^2 = 6(25) = 150$
$2(30-20)^2 = 2(100) = 200$
Total $= 750$

Step 4. Variance and standard deviation:
$\sigma^2 = \frac{750}{20} = 37.5$
$\sigma = \sqrt{37.5} \approx 6.12$

That is option D.

The classic wrong turn is dividing by $n - 1 = 19$. That is the sample formula and gives about $\sqrt{39.5}$, which is not offered here.

Textbook: Mathematics Grade 12, Unit 3 Statistics, section 3.1 (page 161).

35. Given two sets of data A and B with ranges 15 and 15 respectively, coefficient of range (CR) of A is 0.2 and CR of B is 0.21. Then which of the following is true? (1)

A. The dispersion of A is slightly greater than that of B.

B. ✓ **The dispersion of B is slightly greater than of A.**

C. Data A has the same dispersion as B even if CR varies.

D. No enough information to compare dispersion roughly.

Answer: B

The range is an ABSOLUTE measure of dispersion, given in the same units as the data. The coefficient of range is the matching RELATIVE measure:

$$CR = \frac{L - S}{L + S}$$

where L is the largest value and S the smallest. It has no units, so it compares spread fairly even when two data sets sit at different levels.

Here both sets have the same range, 15, so the absolute spread is identical and the range alone cannot separate them. Turn to the relative measure:

$CR_A = 0.2$

$CR_B = 0.21$

Since 0.21 > 0.2, set B is spread slightly wider relative to the size of its values. That is option B.

A quick way to see it: an equal range of 15 counts for more in a set whose values are small. The larger coefficient tells you B is the set with the smaller $L + S$.

The classic wrong turn is option C, arguing that equal ranges mean equal dispersion. They mean equal ABSOLUTE dispersion only.

Textbook: Mathematics Grade 12, Unit 3 Statistics, section 3.2 (page 194).

36. in a homogenious population where there is a catagorical difference items are selected randomely in each category Which sampling method selects items randomly within categories? (1)

A. Simple random

B. Systematic

C. ✓ **Stratified**

D. Cluster

Answer: C

The population is first split into categories, and then a random selection is made inside each category. Those categories are called strata, so this is stratified sampling, option C.

The point of it is fair representation. If a school has 700 girls and 300 boys, stratified sampling takes a random sample from the girls and a random sample from the boys, so neither group can be missed by chance.

How the other methods differ:
Simple random: every member of the whole population is drawn directly, with no categories at all.
Systematic: you take a random start and then every kth member of a list.
Cluster: you divide into groups, then pick WHOLE groups at random and study everybody inside them.

The classic wrong turn is confusing stratified with cluster sampling. Stratified samples from EVERY group; cluster keeps only some groups and drops the rest.

Textbook: Mathematics Grade 12, Unit 3 Statistics, section 3.4 (page 212).

37. An agent planned to transport fertilizers in barrels and sacks from a port using two types of cars designated by A and B. Each car A can transport 30 barrels and 50 sacks charging Birr 8,000 for a trip, and each car B can transport 45 barrels and 80 sacks charging Birr 12,000. If there are at least 400 barrels and at most 1,500 sacks to be transported from the port, which of the following is a constraint of the linear programming problem to minimize the cost of transportation given by $z = 8,000x + 12,000y$? (1)

A. ✓

$$\begin{cases} 6x + 9y \geq 80 \\ 5x + 8y \leq 150 \\ x \geq 0, y \geq 0 \end{cases}$$

B.

$$\begin{cases} 6x + 9y \leq 80 \\ 5x + 8y \geq 150 \\ x \geq 0, y \geq 0 \end{cases}$$

C.

$$\begin{cases} 3x + 5y \leq 40 \\ 9x + 16y \geq 30 \\ x \geq 0, y \geq 0 \end{cases}$$

D.

$$\begin{cases} 3x + 5y \geq 40 \\ 9x + 16y \leq 30 \\ x \geq 0, y \geq 0 \end{cases}$$

Answer: A

Let x be the number of trips by car A and y the number by car B. Turn each sentence of the story into an inequality, then simplify.

Barrels. Car A carries 30 and car B carries 45, and at least 400 must move, so the total has to reach 400 or more:
$30x + 45y \geq 400$
Divide every term by 5:
$6x + 9y \geq 80$

Sacks. Car A carries 50 and car B carries 80, and at most 1500 may move, so the total must stay at 1500 or below:
$50x + 80y \leq 1500$
Divide every term by 10:
$5x + 8y \leq 150$

Trips cannot be negative, so $x \geq 0$ and $y \geq 0$. Together these are exactly option A.

The classic wrong turn is option B, which swaps the two inequality signs. Read the words carefully: *at least* gives $\geq$ and *at most* gives $\leq$.

Textbook: Mathematics Grade 12, Unit 4 Introduction to Linear Programming, section 4.3 (page 274).

38. A college graduate plans to deposit Birr 2000 each year starting from the first day of the year into a saving account that pays 8% interest compounded annually. How much will he/she has just after the first day of the third year? (1)

A. 4492.2 **B.** 6332.8

C. ✓ **6492.8** **D.** 7012.2

Answer: C

Each deposit earns compound interest on its own, so grow each one to the moment asked about and add. The growth factor for one year is $1 + r = 1.08$.

Count the years each deposit has been in the account on the first day of the third year:
The deposit of year 1 has been there 2 full years.
The deposit of year 2 has been there 1 full year.
The deposit of year 3 has just been made, so it has earned nothing yet.

Now grow each one:
$2000(1.08)^2 = 2000(1.1664) = 2332.80$
$2000(1.08)^1 = 2160.00$
$2000(1.08)^0 = 2000.00$

Add them:
$2332.80 + 2160.00 + 2000.00 = 6492.80$

The account holds Birr 6,492.80, which is option C.

Two classic wrong turns are printed as distractors. Forgetting that the second deposit also earns a year of interest gives 6,332.80, and leaving out the third deposit altogether gives about 4,492.80.

Textbook: Mathematics Grade 12, Unit 5 Mathematical Applications in Business, section 5.2 (page 324).

— END OF MARKING KEY —