

Total marks: 65

- Answer:** D

Answer: B

Answer: A

Answer: D

A function f from A to B is onto (surjective) if every element in B has at least one pre-image in A . Therefore, if there is even one element in B with no pre-image, f cannot be onto.

5. For a given ellipse, what do you call the ratio of the distance between the two foci to the length of the major axis? (1)
- A. ✓ **Eccentricity**
- B. Latus rectum
- C. Directrix
- D. Length of minor axis

Answer: A

By definition, the eccentricity (e) of an ellipse is the ratio of the distance between the two foci ($2c$) to the length of the major axis ($2a$), which simplifies to c/a .

6. Which one is the standard form of the equation of the parabola with vertex $(-2, 3)$ and focus $(2, 3)$? (1)
- A. $(x + 2)^2 = 16(y - 3)$ B. $(x + 2)^2 = -16(y - 3)$
- C. ✓ $(y - 3)^2 = 16(x + 2)$ D. $(y - 3)^2 = -16(x + 2)$

Answer: C

The vertex is $(h, k) = (-2, 3)$ and the focus is $(h + p, k) = (2, 3)$, which yields $p = 4$. Since the axis is horizontal, the standard equation is $(y - k)^2 = 4p(x - h)$, which gives $(y - 3)^2 = 16(x + 2)$.

- 7.** Which one of the following is equivalent to $(p \Rightarrow q) \wedge (q \Rightarrow p)$? (1)
- A. $\checkmark$ $p \Leftrightarrow q$
- B. $p \wedge q$
- C. $(p \Rightarrow q) \vee (q \Rightarrow p)$
- D. $(p \vee q) \wedge (q \vee p)$

Answer: A

The logical expression $(p \Rightarrow q)$ and $(q \Rightarrow p)$ represents the biconditional statement, which is equivalent to $p \Leftrightarrow q$.

8. When $3.1\overline{25}$ is written in the form of $\frac{a}{b}$, it is equal to
- A. $\checkmark \frac{3094}{990}$ B. $\frac{3094}{1000}$
C. $\frac{3100}{990}$ D. $\frac{3090}{1009}$

Answer: A

Let $x = 3.12525\dots$ then $10x = 31.2525\dots$ and $1000x = 3125.2525\dots$ Subtracting these gives $990x = 3094$, so $x = 3094/990$.

9. If z_1 and z_2 are complex numbers, then which one of the following statements is NOT true about them? (1)
- A. $z_1 - z_2$ is a complex number. B. $z_1 + z_2$ is a complex number.
- C. ✓ $z_1 - z_2$ is always a real number. D. $z_1 + z_2$ can be a real number.

Answer: C

The difference of two complex numbers is always a complex number, but it is not always a real number (e.g., $(1 + i) - 1 = i$, which is purely imaginary).

10. A system of linear equations is given as:
$$\begin{cases} x + 3y = 9 \\ -y + 4z = -2 \\ x - 5z = 0 \end{cases}$$
 Which one of the following is the augmented matrix (1)

for the above system?

- A. $\begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & -5 \end{pmatrix}$ B. $\begin{pmatrix} 1 & 3 & 9 \\ -1 & 4 & -2 \\ 1 & -5 & 0 \end{pmatrix}$
- C. $\begin{pmatrix} 1 & 3 & 9 & 0 \\ 0 & -1 & 4 & -2 \\ 1 & 0 & -5 & 0 \end{pmatrix}$ D. ✓ $\begin{pmatrix} 1 & 3 & 0 & 9 \\ 0 & -1 & 4 & -2 \\ 1 & 0 & -5 & 0 \end{pmatrix}$

Answer: D

The system of equations includes variables x, y, and z. Expressing each equation as $ax + by + cz = d$ gives the coefficients for the augmented matrix, matching option D.

11. Let $E = \begin{pmatrix} 2 & -2 & -4 \\ -4 & 8 & 6 \\ -6 & 10 & -8 \end{pmatrix}$ and $F = \begin{pmatrix} -1 & 1 & 2 \\ 2 & -4 & -3 \\ 3 & -5 & 4 \end{pmatrix}$. Which one of the following is true about E and F? (1)

- A. $F = \frac{1}{2}E$ B. $F = 2E$
- C. ✓ $F = -\frac{1}{2}E$ D. $F = -2E$

Answer: C

Multiplying matrix E by $-1/2$ yields exactly matrix F, since each entry F_{ij} satisfies the relation $F_{ij} = -1/2 * E_{ij}$.

12. Which one of the followings is NOT principle of taxation? (1)

- A. Taxation should seek to be neutral and equitable between forms of business activities.
- B. ✓ **Exemption of taxes are made to ensure revenue to cover government cost of administration.**
- C. Tax rules should be clear and simple to understand so that taxpayers know where they stand.
- D. Compliance costs to business and administration costs for governments should be minimized as far as possible.

Answer: B

Tax exemptions reduce government revenue rather than ensuring it, making statement B false and not a standard principle of taxation.

13. A function f is continuous on an interval if and only if (1)

- A. for some point c in the interval $f(x) \neq f(c)$.
- B. ✓ **it is continuous at every point of the interval.**
- C. for some point c in the interval $\lim_{x \rightarrow c} f(x) = 0$.
- D. for exactly one point c in the interval $f(x) \neq f(c)$.

Answer: B

By definition, a function is continuous on an interval if and only if it is continuous at every single point within that interval.

14. If $\{a_n\}$ and $\{b_n\}$ are two given sequences so that $\lim_{n \rightarrow \infty} a_n = 3$ and $\lim_{n \rightarrow \infty} b_n = -2$, then which one of the following is NOT true about them? (1)

- A. $\lim_{n \rightarrow \infty} a_n b_n = -6$ B. $\lim_{n \rightarrow \infty} a_n + b_n = 1$
- C. ✓ $\lim_{n \rightarrow \infty} \frac{a_n}{2} = -\frac{3}{2}$ D. $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = -\frac{3}{2}$

Answer: C

Since the limit of a_n as n approaches infinity is 3, the limit of $a_n / 2$ must be $3/2$, which makes the statement in option C incorrect.

15. Which one of the following sequence has no upper bound? (1)

- A. $\{1 - n\}$ B. $\left\{\frac{(-1)^n}{n}\right\}$
C. $\left\{\frac{4n-1}{2n}\right\}$ D. ✓ $\{2^n - 1\}$

Answer: D

The sequence $\{2^n - 1\}$ grows without bound as n approaches infinity, meaning it has no upper bound, unlike the other sequences which are bounded above.

16. If two lines $\ell_1 : 2x + 3y - 5 = 0$ and $\ell_2 : kx + 2y + p = 0$, are perpendicular at point $(-2, 3)$, then which one of the following pairs gives the correct values of k and p ? (1)

- A. $k = -3$ and $p = 9$ B. $k = 3$ and $p = -6$
C. $k = 3$ and $p = 15$ D. ✓ $k = -3$ and $p = -12$

Answer: D

Since the lines are perpendicular, the product of their slopes is -1 , giving $(-2/3)(-k/2) = -1$, which means $k = -3$. Substituting $k = -3$ and the point $(-2, 3)$ into the second line's equation yields $p = -12$.

17. Which one of the following is equal to $\lim_{x \rightarrow 5} \frac{x-5}{\sqrt{x+4}-3}$? (1)

- A. 0 B. ∞
C. ✓ 6 D. 1

Answer: C

Rationalizing the denominator by multiplying the numerator and denominator by $(\sqrt{x+4} + 3)$ simplifies the limit to the limit as x approaches 5 of $(\sqrt{x+4} + 3)$, which equals $3 + 3 = 6$.

18. Which one of the following matrices is the inverse of $\begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$? (1)

- A. $\begin{pmatrix} \frac{5}{13} & \frac{-3}{13} \\ \frac{-1}{13} & \frac{2}{13} \end{pmatrix}$ B. $\begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ 1 & \frac{1}{5} \end{pmatrix}$
C. ✓ $\begin{pmatrix} \frac{5}{7} & \frac{-3}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{pmatrix}$ D. $\begin{pmatrix} \frac{1}{5} & \frac{-1}{3} \\ -1 & \frac{1}{2} \end{pmatrix}$

Answer: C

The determinant of the matrix is $(2*5) - (3*1) = 7$. Using the formula for the inverse of a 2×2 matrix, we swap diagonal entries, negate off-diagonals, and divide by the determinant, yielding option C.

19. The fourth term of a geometric sequence with positive terms is 125. If the tenth term is $\frac{125}{64}$, then which one of the following is the first term this sequence? (1)

- A. 500 B. 250
C. ✓ 1000 D. 750

Answer: C

Dividing the tenth term by the fourth term gives $r^6 = 1/64$, which means $r = 1/2$. Since $a_1 * r^3 = 125$, we have $a_1 * (1/8) = 125$, giving $a_1 = 1000$.

20. If $\{a_n\}$ is a sequence with $a_1 = 3$ and $a_{n+1} = a_n + 5$ for all $n \geq 1$, then which one of the following is the 100th term of the sequence? (1)

A. 500
B. ✓ 498
C. 508
D. 503

Answer: B

This is an arithmetic sequence with first term $a_1 = 3$ and common difference $d = 5$. The 100th term is $a_{100} = 3 + 99 \cdot 5 = 498$.

21. Which one of the following statements gives the geometric interpretation of the Rolle's theorem stated for a continuous function f on $[a, b]$ that is differentiable in (a, b) ? (1)

- A. There exists at most one tangent line to the graph of the function which is parallel to the y-axis.
B. ✓ **There is a point c in the interval (a, b) where a tangent line to the graph of the function through this point is parallel to the x-axis.**
C. At every point within the interval, it is possible to draw a tangent line to the graph of f which is parallel to the x-axis.
D. There is a point on the graph of f where the tangent line through this point is not parallel to a secant line joining any two points of the graph.

Answer: B

Rolle's theorem states that if a function is continuous on $[a, b]$ and differentiable on (a, b) with $f(a) = f(b)$, then there is some c in (a, b) where the derivative is 0, which geometrically means the tangent line is parallel to the x-axis.

22. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence with $a_1 = 2$ and $a_{n+1} = na_n$ for $n \in \mathbb{N}$. Then which one of the following expressions correctly describes a_n ? (1)

- A. $(2n)!$
B. $2n!$
C. $2(n+1)!$
D. ✓ $2(n-1)!$

Answer: D

By checking the recursive definition, we find $a_1 = 2$, $a_2 = 2$, $a_3 = 4$, $a_4 = 12$, and $a_5 = 48$. This matches the formula $2(n-1)!$, since $2(0!) = 2$, $2(1!) = 2$, $2(2!) = 4$, and so on.

23. Which one of the following is the set of all critical numbers of $f(x) = x^3 - 2x^2 + x + 1$ on the interval $[-1, 3]$? (1)

- A. ✓ $\{\frac{1}{3}, 1\}$
B. $\{\frac{1}{3}, -1\}$
C. $\{\frac{1}{3}, -2\}$
D. $\{1\}$

Answer: A

The derivative of the function is $f'(x) = 3x^2 - 4x + 1$. Setting $f'(x) = 0$ and factoring gives $(3x - 1)(x - 1) = 0$, so the critical numbers are $1/3$ and 1 , both of which lie in the interval $[-1, 3]$.

24. What is the value of k so that $f(x) = \frac{x}{x^2+k}$ has a relative extremum at $x = 2$? (1)

- A. ✓ 4
B. 8
C. 0
D. 2

Answer: A

The derivative of $f(x)$ is $(k - x^2)/(x^2 + k)^2$. For a relative extremum to exist at $x = 2$, the derivative must equal 0, meaning $k - 2^2 = 0$, which gives $k = 4$.

- 25.** An object has a circular base of radius r cm. Its volume (V) is directly proportional to its height (h), the constant π and the square of its base radius (r). When $r = 5$ cm and $h = 12$ cm, its volume $V = 100\pi \text{ cm}^3$. If $r = 9$ cm and $h = 10$ cm, then which one of the following equals to its volume?
- A. $210\pi \text{ cm}^3$
- B. ✓ $270\pi \text{ cm}^3$
- C. $120\pi \text{ cm}^3$
- D. $150\pi \text{ cm}^3$

Answer: B

The relationship is described by $V = k \cdot \pi \cdot h \cdot r^2$. Substituting the initial values gives $100 \cdot \pi = k \cdot \pi \cdot 12 \cdot 25$, which yields $k = 1/3$. For $r = 9$ and $h = 10$, the volume is $V = (1/3) \cdot \pi \cdot 10 \cdot 81 = 270 \cdot \pi$.

- 26.** If 20,000 Birr is divided among three persons in the ratio 5:2:3, then which one of the following is the maximum share?
- A. 3,000 Birr B. 6,000 Birr
- C. ✓ **10,000 Birr** D. 4,000 Birr

Answer: C

The ratio 5:2:3 has a total of 10 parts. The maximum share corresponds to 5 parts, which is calculated as $(5/10) * 20,000$ Birr = 10,000 Birr.

- 27.** A sum of money is invested at a compound interest calculated annually which amounts to 21,632 Birr at the end of the second year and to 22,497.28 Birr at the end of the third year. Which one of the following pairs of values gives the rate of interest and the principal, respectively? (1)
- A. ✓ **0.04 and 20,000 Birr** B. 0.03 and 18,550 Birr
C. 0.06 and 21,550 Birr D. 0.05 and 21,000 Birr

Answer: A

Using the compound interest formula, the ratio of the amounts at the end of the third and second years gives $1 + r = 22,497.28 / 21,632 = 1.04$, so $r = 0.04$. The principal is then $P = 21,632 / (1.04)^2 = 20,000$ Birr.

- 28.** The n^{th} term of a sequence $\{a_n\}$ is given by $a_n = \frac{1-2n}{1+2n}$, then which one of the following statements is true about this sequence? (1)
- A.** All its terms are positive numbers and hence diverges.
- B.** It converges to its least upper bound 0.
- C. ✓ It is a monotonic sequence and converges to -1 .**
- D.** It is unbounded sequence and hence diverges.

Answer: C

The sequence can be rewritten as $a_n = 2/(1+2n) - 1$, which is strictly decreasing (monotonic) and converges to -1 as n approaches infinity.

29. For all $n \in \mathbb{N}$, which one of the following sequences is a null sequence? (1)
- A. $\left\{ \frac{n}{n+1} \right\}$ B. $\checkmark \left\{ \left(\frac{1}{8} \right)^n \right\}$
- C. $\left\{ \frac{n^2}{n^2+1} \right\}$ D. $\left\{ \left(\frac{7}{5} \right)^n \right\}$

Answer: B

A null sequence is a sequence that converges to 0. Since the base of $(1/8)^n$ is between -1 and 1, the limit of this sequence as n approaches infinity is 0.

30. Which one of the following is the value of $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x-2}$? (1)
- A. ✓ $\frac{1}{4}$ B. 4
C. $\frac{1}{2}$ D. 2

Answer: A

By rationalizing the numerator, the expression simplifies to $1 / (\sqrt{x+2} + 2)$ for x different from 2. Taking the limit as x approaches 2 yields $1 / (2 + 2) = 1/4$.

31. Let $A = \begin{pmatrix} -4 & 2 & 0 \\ 1 & 3 & 6 \\ 5 & 7 & 9 \end{pmatrix}$. Which of the following is NOT true about this matrix? (1)
- A. Cofactor of $a_{13} = -8$ B. Minor of $a_{23} = -38$
C. Cofactor of $a_{23} = 38$ D. ✓ **Minor of $a_{13} = \text{Cofactor of } a_{31}$**

Answer: D

The minor of a_{13} is $\det[[1, 3], [5, 7]] = -8$, whereas the cofactor of a_{31} is $(-1)^{(3+1)} * \det[[2, 0], [3, 6]] = 12$. Since -8 is not equal to 12, statement D is NOT true.

32. If $A = \begin{pmatrix} a & 1 & -1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$, then $\det(A) = \text{-----}$. (1)
- A. ✓ $a^3 - a$ B. $a^2 - a$
C. $-a(a^2 - 1)$ D. $a^3 - 1$

Answer: A

Evaluating the determinant along the first row gives $\det(A) = a(a^2 - 1) - 1(a - 1) - 1(1 - a) = a^3 - a - a + 1 - 1 + a = a^3 - a$.

33. If $f(x) = x^3 \log_{10} x$, then which one of the following is equal to $f'(x)$? (1)
- A. ✓ $\frac{x^2}{\ln 10} + 3x^2 \log_{10} x$ B. $\frac{x^3}{\ln 10} + 3x^2 \log_{10} x$
C. $\frac{x^3}{\log_{10} x} - 3x^2 \ln x$ D. $\frac{x^2}{\log_{10} x} - 3x^2 \ln x$

Answer: A

Using the product rule and the fact that $\log_{10}(x) = \ln(x)/\ln(10)$, the derivative is $f'(x) = 3x^2 * \log_{10}(x) + x^3 / (x * \ln(10)) = x^2 / \ln(10) + 3x^2 * \log_{10}(x)$.

34. Which one of the following events explains the meaning of derivative? (1)
- A. It gives the position of an object that is moving along a coordinate line at a certain given time t .
B. **It is the marginal cost of a product which is the rate of change of cost with respect to the level of production.**
C. It is the slope of a secant line to the graph of a function f where this secant line passes through two distinct points (x_0, y_0) and (x_1, y_1) of f .
D. It gives the amount of money at the end of n years, if someone deposit P birr in a bank that pays $r\%$ annual interest compounded monthly.

Answer: B

The derivative represents an instantaneous rate of change. In economics, the marginal cost is defined as the derivative of the cost function with respect to the quantity produced.

35. A committee of three members with the position of Chairman, Vice-chairman and Secretary is to be formed from 10 candidates. If every candidate has equal chance to be selected and to take any of the three positions, in how many ways can the committee with the specified positions be formed? (1)

A. 360
B. 120
C. ✓ 720
D. 840

Answer: C

Since the positions are distinct, the order of selection matters. Thus, we calculate the number of permutations of 10 items taken 3 at a time: $P(10, 3) = 10 * 9 * 8 = 720$.

36. What is the 3rd decile (D_3) of the data 2, 5, 3, 4, 11, 9, 8, 7, 12, 19? (1)

A. 5
B. 4
C. 5.5
D. ✓ 4.5

Answer: D

Sorting the data gives: 2, 3, 4, 5, 7, 8, 9, 11, 12, 19. For $N = 10$, the 3rd decile (30th percentile) is located at index $np = 10 * 0.3 = 3$, so we take the average of the 3rd and 4th terms: $(4 + 5)/2 = 4.5$.

37. If $f(x) = 4 - x^2 \sin x$, then which of the following expressions describes $f'(x)$? (1)

A. ✓ $-x(x \cos x + 2 \sin x)$
B. $1 + \cos x + 2 \sin x$
C. $x(x \cos x - 2 \sin x)$
D. $1 - \cos x - 2 \sin x$

Answer: A

Applying the product rule to the second term of $f(x) = 4 - x^2 * \sin(x)$ yields $f'(x) = -(2x * \sin(x) + x^2 * \cos(x)) = -x(x * \cos(x) + 2 * \sin(x))$.

38. Which of the following is an equation of the line tangent to the graph of $y = x^2 + e^x$ at $x = 0$? (1)

A. $y = x - 1$
B. $y = x$
C. $y = 2x$
D. ✓ $y = x + 1$

Answer: D

At $x = 0$, $y = 1$. The derivative is $y' = 2x + e^x$, which equals 1 at $x = 0$. The equation of the tangent line is $y - 1 = 1(x - 0)$, which simplifies to $y = x + 1$.

39. Consider the function $f(x) = x^3 + x^2 - 4x - 3$ whose graph is shown below. Which one of the following is the slope of the tangent line at $(-2, 1)$? (1)

[Figure: A hand-drawn Cartesian coordinate grid on square ruling, about 12 columns wide and 12 rows tall. The horizontal x-axis and vertical y-axis are drawn heavier than the grid lines and cross a little left of the middle of the frame; the y-axis carries the label "y" beside its arrow head at the top and a small mark sits at the right-hand end of the x-axis. Tick dots are placed at every grid crossing along both axes but NO numeric labels are printed anywhere on the figure. The cubic curve of $f(x) = x^3 + x^2 - 4x - 3$ is drawn as one heavy continuous line: it enters at the bottom-left of the frame, rises steeply and crosses the x-axis about two units to the left of the origin, turns over at a local maximum about two units above the x-axis (roughly one and a half units left of the y-axis), falls and crosses the x-axis again just under one unit left of the origin, cuts the y-axis at about three units below the origin, bottoms out at a local minimum about five units below the x-axis (roughly one unit right of the y-axis), then rises steeply, crosses the x-axis about two units right of the origin and leaves through the top of the frame.]

- A. -4 B. $\checkmark$ 4
C. -3 D. 3

Answer: B

The derivative is $f'(x) = 3x^2 + 2x - 4$. Substituting $x = -2$, we find the slope of the tangent line to be $f'(-2) = 3(4) + 2(-2) - 4 = 12 - 8 = 4$.

40. If $p \Rightarrow \neg q$ is False, then for any proposition r which one of the following statements is True? (1)

- A. $(p \wedge q) \Rightarrow (r \wedge \neg r)$ B. $\checkmark$ $(p \wedge \neg q) \Leftrightarrow (\neg r \wedge r)$
C. $(\neg p \vee r) \Leftrightarrow (\neg q \wedge r)$ D. $(p \vee q) \Rightarrow (p \wedge r)$

Answer: B

An implication is false only when the antecedent is true and the consequent is false; thus p is True and q is True. Using these truth values, option B evaluates to $(T \text{ and } F) \Leftrightarrow F$, which simplifies to $F \Leftrightarrow F$, which is always True.

41. Which one of the following is the contra positive of the statement: "If 9 is divisible by 2, then 5 is an even number"? (1)

- A. If 5 is not an even number, then 9 is divisible by 2.
B. If 9 is not divisible by 2, then 5 is not an even number.
C. $\checkmark$ **If 5 is not an even number, then 9 is not divisible by 2.**
D. If 5 is an even number, then 9 is divisible by 2.

Answer: C

The contrapositive of the conditional statement $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$. This translates to: 'If 5 is not an even number, then 9 is not divisible by 2.'

42. Let a function f be continuous on $[a, b]$ and differentiable in (a, b) . Which one of the following is correct about f ? (1)

- A. $\checkmark$ **$f'(x) = 0$, for all $x \in (a, b)$, then f is a constant on $[a, b]$.**
B. If $f'(x) \leq 0$ for all $x \in (a, b)$, then f is increasing on (a, b) .
C. If $f(a) = f(b)$ and $f(x) > f(a)$ for all $x \in (a, b)$, then f has got a minimum value at $c \in (a, b)$ where $f'(c) = 0$.
D. If $f'(c) = 0$ and f'' is continuous at c such that $f''(c) > 0$, then f has a local maximum at c .

Answer: A

By the Mean Value Theorem, if the derivative of a function is zero everywhere on an open interval, then the function is constant on the closed interval.

43. A textile factory produces shirts and calculates the revenue r , cost c and profit p as functions of the number of shirts. The functions are as follows: $r(x)$ = the revenue from selling x shirts, $c(x)$ = the cost of producing the x shirts, $p(x) = r(x) - c(x)$ = the profit from producing and selling x shirts. If $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$, where x represents thousands of shirts produced, then which one of the following statements is correct about the loss or profit gained at the given production level? (1)

- A. ✓ **Maximum profit occurs at about $x = 3414$** B. Maximum loss occurs at about $x = 2510$
C. Maximum profit occurs at about $x = 6828$ D. Maximum loss occurs at about $x = 1586$

Answer: A

The profit function is $p(x) = -x^3 + 6x^2 - 6x$. Its derivative $p'(x) = -3x^2 - 4x + 2$ equals 0 at $x = 2 + \sqrt{2} \approx 3.414$, which represents 3414 shirts and is a local maximum.

44. If $z_1 = 5 - 3i$ and $z_2 = 3 - 4i$, then $\frac{z_1}{z_2}$ is equal to _____. (1)

- A. $\frac{3}{25} + \frac{11}{25}i$ B. ✓ $\frac{27}{25} + \frac{11}{25}i$
C. $\frac{27}{25} - \frac{11}{25}i$ D. $\frac{27}{25} + \frac{29}{25}i$

Answer: B

To divide the complex numbers, multiply the numerator and denominator by the conjugate of the denominator: $(5 - 3i)(3 + 4i) / (3^2 + 4^2) = (15 + 20i - 9i + 12) / 25 = 27/25 + (11/25)i$.

45. Which one of the following pairs contains the modulus and argument of the complex number $1 + \sqrt{3}i$ respectively? (1)

- A. 2 and $\frac{\pi}{6}$ rad B. 1 and $\frac{\pi}{3}$ rad
C. ✓ **2 and $\frac{\pi}{3}$ rad** D. 1 and $\frac{\pi}{6}$ rad

Answer: C

The modulus is $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$. Since the complex number is in the first quadrant, the argument is $\theta = \arctan(\sqrt{3}/1) = \pi/3$ rad.

46. What is the third derivative of $f(x) = \frac{1}{3}x^6 + \frac{1}{2}x^4 - 3x^3 + 2x^2 + 4x - 1$? (1)

- A. $10x^4 + 6x^2 - 18x + 4$ B. $20x^3 + 12x^2 - 18x - 4$
C. ✓ **$40x^3 + 12x - 18$** D. $40x^3 + 12x^2 - 18x + 4$

Answer: C

Differentiating $f(x)$ successively: $f'(x) = 2x^5 + 2x^3 - 9x^2 + 4x + 4$, $f''(x) = 10x^4 + 6x^2 - 18x + 4$, and the third derivative is $f'''(x) = 40x^3 + 12x - 18$.

47. Which of the following is true about the graph of the function given by $f(x) = \frac{-8}{x^2 - 4}$? (1)

- A. $(2, 0)$ is the x-intercept. B. It is symmetric about the origin.
C. Its vertical asymptote is $x = 4$. D. ✓ **Its horizontal asymptote is $y = 0$.**

Answer: D

As x approaches positive or negative infinity, the values of $f(x)$ approach 0, which means $y = 0$ is a horizontal asymptote of the function.

48. If $f(x) = \ln(2 + \sin x)$, then $f'(x)$ is equal to (1)

- A. $\frac{-\cos x}{1 + \sin x}$ B. ✓ $\frac{\cos x}{2 + \sin x}$
C. $\frac{\csc x}{1 + \sin x}$ D. $\frac{\sec x}{1 + \sin x}$

Answer: B

Using the chain rule, the derivative of $f(x) = \ln(u)$ is u'/u . For $u = 2 + \sin(x)$, we have $u' = \cos(x)$, giving $f'(x) = \cos(x) / (2 + \sin(x))$.

49. Which one of the following functions is NOT invertible? (1)

- A. ✓ $g = \{(x, y) : y = x^2 + 1\}$, where $g : \mathbb{R} \rightarrow \mathbb{R}$
 B. $f = \{(x, y) : y = 4x - 3\}$, where $f : \mathbb{R} \rightarrow \mathbb{R}$
 C. $k = \{(x, y) : y = e^{2x}\}$, where $k : \mathbb{R} \rightarrow (0, \infty)$
 D. $h = \{(x, y) : y = \log_3 2x\}$, where $h : (0, \infty) \rightarrow \mathbb{R}$

Answer: A

A function is invertible on a domain if and only if it is one-to-one. The function $g(x) = x^2 + 1$ from $\mathbb{R}$ to $\mathbb{R}$ is not one-to-one because, for example, $g(1) = g(-1) = 2$.

50. Which one of the following is a possible solution for $\frac{2x^2-5x+2}{2x^2-x} = 1 - \frac{1+x}{2x-1}$? (1)

- A. 0
 B. ✓ 2
 C. -1
 D. $\frac{1}{2}$

Answer: B

Factoring the numerator of the LHS gives $(2x - 1)(x - 2)$. For x different from $1/2$ and 0 , the equation simplifies to $(x - 2)/x = (x - 2)/(2x - 1)$. This yields $x = 2$ as a valid solution.

51. A water tank is in the form of a right circular cylinder with base radius 2m and height 3m. If water is pumped into the tank at the rate of $0.5 \text{ m}^3/\text{min}$, what is the rate at which the water level is rising? (1)

- A. $\frac{8}{\pi} \text{ m/min}$
 B. ✓ $\frac{1}{8\pi} \text{ m/min}$
 C. $\frac{4\pi}{3} \text{ m/min}$
 D. $\frac{3\pi}{8} \text{ m/min}$

Answer: B

The volume of water in the cylindrical tank is given by $V = \pi r^2 h = 4\pi h$. Differentiating both sides with respect to time t gives $\frac{dV}{dt} = 4\pi \frac{dh}{dt}$, which yields $\frac{dh}{dt} = \frac{0.5}{4\pi} = \frac{1}{8\pi} \text{ m/min}$.

52. What is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 1$ on the interval $[0, 2]$? (1)

- A. $\sqrt{\frac{2}{3}}$
 B. $\frac{2}{3}$
 C. $\frac{\sqrt{2}}{3}$
 D. ✓ $\frac{2}{3}\sqrt{3}$

Answer: D

According to the Mean Value Theorem, we need to find $c \in (0, 2)$ such that $f'(c) = \frac{f(2)-f(0)}{2-0}$. Solving $3c^2 = \frac{7-(-1)}{2} = 4$ yields $c^2 = \frac{4}{3}$, which gives $c = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$.

53. Which one of the following triplets of values of x , y and z contains all solutions for $\begin{cases} 4x - y + z = -5 \\ 2x + 2y + 3z = 10 \\ 5x - 2y + 6z = 1 \end{cases}$? (1)

- A. ✓ $x = -1, y = 3, z = 2$
 B. $x = 3, y = -1, z = 2$
 C. $x = 3, y = 2, z = -1$
 D. $x = 2, y = -1, z = 3$

Answer: A

Substituting the values of Option A ($x = -1, y = 3, z = 2$) into the system of equations, we get $4(-1) - 3 + 2 = -5$, $2(-1) + 2(3) + 3(2) = 10$, and $5(-1) - 2(3) + 6(2) = 1$, all of which are true.

54. What is the sixth term in the expanded form of $(x + 2y)^8$? (1)
- A. $1120x^4y^4$ B. ✓ $1792x^3y^5$
 C. $448x^5y^3$ D. $1024xy^7$

Answer: B

The sixth term in the expansion of $(x + 2y)^8$ is given by $\binom{8}{5}x^{8-5}(2y)^5 = 56 \cdot 32x^3y^5 = 1792x^3y^5$.

55. Among a group of 200 students who are asked whether or not they like Mathematics and/or History, it is found that 80 of them like Mathematics only, 20 like History only and 40 like both. If a student is randomly selected from the group, what is the probability that the student likes neither Mathematics nor History? (1)
- A. 0.5 B. ✓ 0.3
 C. 0.8 D. 0.7

Answer: B

The number of students who like Mathematics, History, or both is $80 + 20 + 40 = 140$. Thus, the number of students who like neither is $200 - 140 = 60$, giving a probability of $\frac{60}{200} = 0.3$.

56. Let f , g and h be differentiable functions such that $f(x) = g(x) + c$, for some $c \in \mathbb{R}$. If $f'(x) = e^{-x} - 2$ and $\frac{d}{dx}(f(x) + g(x) - h(x)) = e^{-x}$, then $h'(x) =$ _____. (1)
- A. $2e^{-x} + 2$ B. ✓ $e^{-x} - 4$
 C. $e^{-x} - 2 + g'(x)$ D. $2 - g'(x)$

Answer: B

Since $f(x) = g(x) + c$, we have $f'(x) = g'(x) = e^{-x} - 2$. Differentiating the second equation gives $f'(x) + g'(x) - h'(x) = e^{-x}$, so $2(e^{-x} - 2) - h'(x) = e^{-x}$, which simplifies to $h'(x) = e^{-x} - 4$.

57. If $f(x) = x^{3/2} + \frac{8}{x} + 7$, then $\lim_{x \rightarrow 4} \frac{f(x) - f(4)}{x - 4} =$ _____. (1)
- A. $\frac{3}{2}$ B. 1
 C. $\frac{7}{2}$ D. ✓ $\frac{5}{2}$

Answer: D

The limit represents the derivative of $f(x)$ evaluated at $x = 4$. Differentiating $f(x) = x^{3/2} + 8x^{-1} + 7$ gives $f'(x) = \frac{3}{2}x^{1/2} - 8x^{-2}$, which evaluates to $f'(4) = 3 - \frac{1}{2} = \frac{5}{2}$.

58. Let $f(x) = \begin{cases} \frac{x^3+8}{x+2}, & \text{if } x \neq -2 \\ 12, & \text{if } x = -2 \end{cases}$. What is the maximum possible interval on which f is continuous? (1)
- A. $(-2, \infty)$ B. $(-\infty, -2)$
 C. $(-\infty, -2) \cup (-2, \infty)$ D. ✓ $(-\infty, \infty)$

Answer: D

The limit of $f(x)$ as $x \rightarrow -2$ is $\lim_{x \rightarrow -2} (x^2 - 2x + 4) = 12$, which equals $f(-2)$. Therefore, f is continuous at $x = -2$ and thus continuous on the entire real line, $(-\infty, \infty)$.

59. The value of $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{(k+2)(k+3)} \right)$ is equal to _____. (1)
- A. $\frac{4}{3}$ B. ✓ $\frac{1}{3}$
 C. 0 D. $\frac{2}{3}$

Answer: B

Using partial fraction decomposition, the general term is $\frac{1}{k+2} - \frac{1}{k+3}$. This forms a telescoping series whose sum is $\lim_{n \rightarrow \infty} \left(\frac{1}{3} - \frac{1}{n+3} \right) = \frac{1}{3}$.

60. Assuming that the population of Ethiopia increases at the rate of 3% per year and the current population is 102 million, which one of the following is the best approximate of the population of Ethiopia 10 years from now? (If you need, you may take: $1.03^{10} = 1.344$, $1.03^9 = 1.305$, $1.03^5 = 1.159$) (1)

A. ✓ 137 million
B. 125 million
C. 141 million
D. 133 million

Answer: A

The future population is calculated as $P_0(1+r)^t = 102 \times (1.03)^{10} \approx 102 \times 1.344 = 137.088$ million, which is approximately 137 million.

61. A person has 2 parents, 4 grandparents, 8 great grandparents and so on. What is the total number of his ancestors during the ten generations preceding his own? (1)

A. 4092
B. 1024
C. ✓ 2046
D. 5048

Answer: C

This forms a geometric progression with $a_1 = 2$ and $r = 2$. The sum of the first 10 terms is $S_{10} = 2 \left(\frac{2^{10}-1}{2-1} \right) = 2(1023) = 2046$.

62. If the line $y = 2x + 1$ is tangent to a circle centered at $(4, 0)$, then what are the coordinates for the intersection point of the line and the circle? (1)

A. $(\frac{4}{5}, \frac{8}{5})$
B. $(\frac{3}{4}, \frac{5}{2})$
C. $(\frac{2}{5}, \frac{8}{5})$
D. ✓ $(\frac{2}{5}, \frac{9}{5})$

Answer: D

The line perpendicular to $y = 2x + 1$ through the center of the circle $(4, 0)$ is $y = -\frac{1}{2}(x - 4)$. Solving the system $y = 2x + 1$ and $y = -\frac{1}{2}x + 2$ yields the point of tangency $(2/5, 9/5)$.

63. What is the tangent of the angle formed by the lines $\ell_1 : 2x + 3y = 5$ and $\ell_2 : x - 2y = 4$? (1)

A. ✓ $-\frac{7}{4}$
B. $\frac{1}{3}$
C. $-\frac{1}{4}$
D. $\frac{2}{3}$

Answer: A

The slopes of the lines are $m_1 = -\frac{2}{3}$ and $m_2 = \frac{1}{2}$. The tangent of the angle is given by $\frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-2/3 - 1/2}{1 - 1/3} = -\frac{7}{4}$.

64. Consider the following linear programming model: objective function $Z = 3x + 5y$ subject to $3x + y \leq 6$, $x + y \geq 1$, $x + 2y \leq 6$, $x \geq 0$, $y \geq 0$. Which one of the following is the minimum value of Z ? (1)

A. ✓ 3
B. 0
C. It has no minimum value.
D. 5

Answer: A

The vertices of the feasible region are $(1, 0)$, $(0, 1)$, $(2, 0)$, $(0, 3)$, and $(1.2, 2.4)$. Evaluating the objective function $Z = 3x + 5y$ at these points yields a minimum value of 3 at $(1, 0)$.

65. A problem to be solved using linear programming method is given as follows: "A coffee packer blends coffee from Harar and from Sidama to prepare two kinds products, viz., "Super" and "Deluxe" brands. Each kilogram of Super coffee contains 0.5 kg of Harar coffee and 0.5 kg of Sidama coffee, whereas each kilogram Deluxe coffee contains 0.25 kg of Harar coffee and 0.75 kg of Sidama coffee. The packer has 120 kg of Harar coffee and 160 kg of Sidama coffee in the store. Moreover, the plan is to get 20 Birr profit on each kilogram of Super coffee and a profit of 30 Birr on each kilogram of Deluxe coffee. In order to maximize profit the packer needs to know the amount (in kg) of each brand of coffee that should be blended." If x = the amount of Super coffee in kilograms and y = the amount of Deluxe coffee in kilograms, then which one of the following standard forms of linear programming models should be used to solve the above problem? (1)

- A. Maximize $P = 20x + 30y$ subject to: $0.75x + 0.25y \leq 120, 0.5x + 0.5y \leq 160, x \geq 0, y \geq 0$.
 B. Maximize $P = 30x + 20y$ subject to: $0.5x + 0.5y \leq 120, 0.25x + 0.75y \leq 160, x \geq 0, y \geq 0$.
 C. Maximize $P = 30x + 20y$ subject to: $0.5x + 0.75y \leq 120, 0.5x + 0.25y \leq 160, x \geq 0, y \geq 0$.
 D. **Maximize $P = 20x + 30y$ subject to: $0.5x + 0.25y \leq 120, 0.5x + 0.75y \leq 160, x \geq 0, y \geq 0$.** ✓

Answer: D

The objective is to maximize profit $P = 20x + 30y$. The constraints for Harar and Sidama coffee are $0.5x + 0.25y \leq 120$ and $0.5x + 0.75y \leq 160$ respectively, along with the non-negativity constraints.

— END OF MARKING KEY —

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